The Company You Keep: Qualitative Uncertainty in Providing a Club Good

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Abstract

Clubs are typically experience goods. Potential members cannot ascertain precisely beforehand their quality (dependent endogenously on the club's facility investment and number of users, itself dependent on its prices). Members with unsatisfactory initial experiences discontinue visits. We show that a monopoly profit maximiser never offers a free trial period for such goods. For quality functions homogeneous of degree of at least minus one, a welfare maximiser, motivated by distributional concerns to mitigate disappointed consumers' losses, always does. We demonstrate the robustness of this finding by showing that: (i) without qualitative uncertainty (thus, no disappointed customers), neither welfarist nor monopolist offers free trials; (ii) If the planner pursues an objective mixing welfare maximisation with profit maximisation, the likelihood of free trials increases with the weight put on welfare maximisation. Regarding club quality and usage, the monopolist provides a socially excessive level of quality to repeat buyers when the quality function is homogeneous of degree zero. With non-homogeneous quality functions, the monopolist permits too little club usage; quality may or may not be socially excessive.

JEL Classification: D4, D6, D8, H, L12

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1 Introduction

Club goods - shared congestible goods (e.g., transport, health, education and leisure facilities, telephone systems) - are important and pervasive. Their literature is extensive[1], but little theoretical research on their industrial organisation exists. Thus, theory does not yet give much guidance on such classic policy-relevant issues as whether imperfect competition leads to excessive investment in club quality, or excessive club prices relative to the welfare optimum. This paper studies such important issues by comparing the provision and pricing rules of a monopolist and a welfare maximiser. We use a two-period

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[1] Cornes and Sandler (1996) and Sandler and Tschirhart (1997) survey key issues.

model to show that a monopolist is likely to over-provide quality and allow too little club use relative to a welfare optimum.

We emphasize particularly the qualitative uncertainty consumers face as club goods are basically experience goods. *Ex-ante*, a potential club user is uncertain how agreeable club membership will be. So, a parent evaluating a private school could have objective information on staff-pupil ratios, its exam league table position and number of sport teams it fields, yet not know if her child will thrive in its particular disciplinary ethos. Again, in a private leisure club, the water in the swimming pool might be too tepid or too enervating, the food more than she can stomach, or other club users just not her type. Such customers must try the club good before really knowing what they buy and if they wish to continue buying (stay members). The existing club literature largely ignores this important aspect of qualitative uncertainty to deal with other important issues, such as multi-jurisdictional many-club competition in large economies, congestion externalities or tiered pricing (see, e.g., Wooders 1978, 1999; Cornes and Sandler 1996; Scotchmer 1985; Glazer et al. 1997).

A few club papers study qualitative uncertainty, but from the standpoint that potential members know their tastes for the club good and not the quality they will experience as they cannot predict the club use of others (Sandler, Sterbenz and Tschirhart 1985; Sterbenz and Sandler 1992) or if they will be allowed to join (Hillman and Swan 1979, 1983). In these papers, club membership and consumption are exogenous. Our model has similarities with these analyses as our consumers' *ex-ante* uncertainty about their tastes makes aggregate club demand and quality random *ex-ante*. But there are major differences. First, in our model, club membership and usage are determined *endogenously* via consumer self-selection, given the provider's pricing and facility investment strategy. Second, we explore both market and welfare-optimal club good provision. Third, our analysis is dynamic: consumers who consume the club good in period 1 can stop doing so in period 2 if they find it not to their taste. Lastly, consumers are not rationed explicitly if club demand is high. Rather, via the standard club mechanism, implicit rationing is then via queuing (e.g., less teacher time per pupil or longer journey times), which reduces the club good's quality.

As our club good is an experience good that generates the frequency of future use by potential members, this paper relates to the literature on experience goods and repeat buying. Crémer (1984), Liebeskind and Rumelt (1989), Hoerger (1993), Krähmer (2003), Villas-Boas (2006) and Bergemann and Välimäki (2006), among others, analyse how qualitative uncertainty associated with experience goods affects buyers' learning and inter-temporal pricing by an imperfectly competitive firm. However, none compares the behaviour of monopoly and welfare-maximising suppliers of the experience good, as we do.[2] This comparison lets us address classic concerns like: (i) Will the monopolist overcharge for a 'congestion-prone' good (e.g., cf. Mills 1981); (ii) Will the monopolist

[2] No-one analyses explicitly club goods with specific features like those noted above (although Crémer (1984) mentions a club as an example of an experience good). Barbieri and Malueg (2014) provide a recent example and survey of a related literature on how uncertainty impacts the private provision of a pure public good.

under or over-invest in quality (e.g., cf. Spence 1975)?

The most relevant papers for us, of those noted, are Crémer (1984) and Bergemann and Välimäki (2006),[3] which study monopoly specifically. Crémer shows that a monopolist will not offer first-time buyers of an experience good an introductory price but charges repeat buyers a lower price . Bergemann and Välimäki show that a monopolist supplying an experience good faces two types of markets: a mass market (where buyers are willing to buy at the full information monopoly price) and a niche one (with uninformed buyers who are not) where pricing strategies differ. In the mass market, prices decline over time; in the niche one, higher prices follow lower ones.

Inevitably, as argued in section 2.5.2 below, a distribution-sensitive welfarist treats repeat customers relatively worse compared to first-time customers than does a monopolist. This is because repeat customers have relatively favourable period-1 experiences and are *ex-post* relatively better off compared to period-1 consumers as a whole (some of whose unsatisfactory period-1 experiences dissuade them from consuming the club good in period 2). In our model, by assumption, any facility investment is made in period 1 and persists over both periods. So, a welfarist's main instrument for redistribution between consumers who try the club good and are disappointed and those who are not is the price it charges in period 1 relative to that in period 2. In the limit, by charging a zero period-1 price, it can shift the entire burden of paying for the club facility onto only those with satisfactory period-1 experience with the club. Conversely, for any given facility investment, the monopolist just wants to maximise revenues. By charging all those who try the good initially, it can reduce period-2 price, perhaps inducing some of those with bad trials that it would otherwise lose, to stay. Alternatively, at any given period-2 price, it can increase investment to the same end. Our following results should be seen in this light.

We show (*Proposition* 1) that a monopolist providing an experience club good will not make a *free* "introductory offer" letting consumers "try for free before buying" (although it could offer a *low* introductory price), but a welfare maximiser might. Specifically, we study the class of club quality functions homogeneous in the club facility investment and usage. We show (*Proposition* 3, *Corollary* 1) that a welfarist *always* offers a *free* trial period for all degrees of homogeneity leading to feasible outcomes (necessitating homogeneity of at least minus unity). This strong result highlights the welfarist's redistributive motive noted above.

Regarding the club's provision of quality, we show the following: (i) under plausible assumptions, if the degree of homogeneity exceeds minus unity, a monopolist always invests in a greater level of facility provision per period-2 use of the club than does a welfarist (*Corollary* 2); (ii) in the much-discussed case of a quality function homogeneous of degree zero, this translates to a monopolist: (a) supplying a higher quality to repeat

[3] Liebeskind and Rumelt (1989), Hoerger (1993), Krähmer (2003) and Villas-Boas (2005) have different angles to our paper. Liebeskind and Rumelt and Hoerger study quality uncertainty given adverse selection on the producers' side. Krähmer and Villas-Boas analyze how consumers' learning with quality uncertainty impacts on oligopolists' pricing strategies. Others, e.g. Courty (2003) and Liu and Schiraldi (2014), study monopoly intertemporal provision of a private experience good (a non-durable and durable good, respectively) but do not let consumers repeat buy.

buyers than does a welfare maximiser (*Proposition* 4); (b) supplying the level of quality that a break-even welfarist would with fully-informed consumers.

Predictions about each supplier's behaviour reflect a joint hypothesis about the supplier's objective and its assumptions about the environment. A change in the assumed environment usually alters the behaviour of a given supplier with an unchanged objective. Likewise, suppliers with different objectives will usually behave differently in the same assumed environment. Thus, inevitably, predicted differences between the welfarist's and monopolist's behaviour will be due both to them having different objectives and to them confronting qualitative uncertainty. So, to try to disentangle the effect of the difference in the club supplier's objectives from that of qualitative uncertainty, we also study a benchmark model with demand uncertainty but no qualitative uncertainty. We expect a profit-maximising monopolist to do whatever is necessary and feasible in the pursuit of profit in all environments. However, this pursuit will manifest itself differently in different environments. Similarly, we expect a utilitarian welfare maximiser's behaviour to be influenced by distributional concerns in all circumstances, again demonstrating this in different ways in different environments. We show that both the monopolist and, under reasonable assumptions, the welfarist will post positive prices in both periods of this benchmark model. Nevertheless, although their behaviour is superficially similar, the welfarist is still influenced by distributional concerns while a monopolist is not at all. This leads to greater predictability in the monopolist's behaviour.

We further test the robustness of our conclusion that it is redistributive concerns that lead to a welfarist pricing at zero in the introductory period when there is qualitative uncertainty by considering when the club good supplier is neither pure welfare maximiser nor pure profit maximiser. Suppose it seeks to maximise an arbitrarily weighted sum of aggregate consumer welfare and producer surplus, subject to a no-loss constraint. We show (*Proposition* 6): (i) if there is positive producer surplus at the optimum, if consumer income is sufficiently low in the trial period for the club good (in a sense made precise below), then the optimal club price is zero in the trial period; (ii) if consumer income is sufficiently high in the trial period, then the optimal club price is positive in that period. Moreover, the greater the weight a planner puts on consumer welfare relative to producer surplus, the more likely is the club price to be zero in the trial period.

Our theoretical results on quality are consistent with the rather sparse empirical evidence, mainly for education and healthcare (e.g., Marlow 2000; Gaynor 2006). This evidence suggests that, in a market for a shared good with endogenous price and quality, quality can increase with the degree of monopoly (although this does not in itself say that the monopoly level is excessive). But, for education at least, these empirical models are often predicated not on profit maximisation but on bureaucratic budget maximisation. So, the greater expenditure on providing the shared good found empirically in non-competitive conditions need not be on quality-enhancing activities.

Section 2 presents our two-period model of qualitative uncertainty and analyzes first-time visitors' period-2 club membership decisions. We determine membership endogenously, depending on the provider's price-quality strategy, and do comparative statics on its sensitivity to price and quality. In section 2.2, we study the monopolist's price and investment choices and, in 2.3, a social welfare maximiser's. We compare their equilibrium pricing and investment in 2.4-2.5. In 2.6, we consider demand uncertainty but not qualitative uncertainty (although much of the formal details are put in an Online Appendix). In 2.7, we analyze some consequences of a planner seeking to maximise a weighted average of consumer welfare and producer surplus. Section 3 presents our conclusions. An Appendix contains all proofs and derives a key equation that drive some of our main findings.

2 The Model

We consider a two-period model of club membership in an economy with one private good and one club good ("a club" for short) with a sole supplier.[4] Only the private good is essential. There are n *ex-ante* identical consumers, n being very large. The assumption of n being large is made to avoid having to analyze consumers internalizing their own individual club usage decisions on aggregate usage in a Cournotian way. Consumers are initially uncertain about the club's quality. They must join and *experience* the club to learn their evaluation of its quality, which then becomes their private information. To find his evaluation, a consumer must visit the club a fixed number of times (normalised at unity) in period 1, irrespective of the supplier. We assume all consumers learn their own evaluation of the good *perfectly* and *instantly* after their period-1 visits, implying that *ex-ante* homogenous consumers become heterogeneous in their valuations *ex-post* once they join. Given his experience, a consumer then decides to remain a period-2 member or to quit, and how many visits to make if he stays. Note, the issue of consumers first joining the club in period 2 does not arise. Given the period-1 price, as they are identical *ex-ante,* either all join the club in period 1 to learn their evaluation or nobody joins. Likewise, as consumers learn their evaluations of the club quality perfectly after the first visit, if they remain members this is because they prefer to do so given their valuations.[5]

Assume that a typical consumer has a strictly concave time-separable utility function with per period utility $U(x_i, v_i, c(\varepsilon, y, V_i))$, where: x_i is his period i's private good consumption, $i = 1, 2$; v_i is his total club visits in period i; y is the quantity of the club good (equivalently, its facility size) that, once provided, does not depreciate; V_i is the total visits by all members in period i; ε is a random-valued parameter capturing the 'qualitative uncertainty' and $c(\varepsilon, y, V)$ is the *quality* or *congestion* function, with $c(\varepsilon, y, V_i)$ increasing in ε. For simplicity, we take the function $U(.)$ to be quasi-linear:

$$U(x_i, v_i, c(\varepsilon, y, V_i)) \equiv u(x_i) + \varepsilon v_i C(y, V_i)$$

[4]Fraser (1996) has a similar set-up on excludable public goods. Lind (2016) considers the issue of the optimal and equilibrium number of organizations to supply a differentiated public good to a heterogeneous population. However, he does not consider any aspect of uncertainty.

[5]Similarly, there is no basis for the supplier to randomly ration customers in period 1 and then allow their entry in period 2. Note that our assumption of perfect learning after the intitial visit is quite usual in the literature on experience goods (see e.g. Jing (2011)). Incorporating *gradual* learning in our model will be interesting and is part of our future research plans (see our concluding comments).

with $u(x_i)$ being strictly concave.

An example clarifies the three influences on c. In a health club with a pool, everyone prefers a 50m pool to a 25m one, though it costs more (a larger y). This is like vertical differentiation. But, depending on their realised ε, some swimmers might find a given pool temperature too high, some too low and others just right. This is like horizontal differentiation. Lastly, all agree that, from their viewpoint, fewer swimmers (a smaller V) are better than more - again like vertical differentiation.

Assumption A1 specifies the distribution of ε (A3, introduced in section 2.1, gives more details of its support); A2 says that club quality increases in the facility size but decreases in crowding:

A1 The parameter ε is distributed over the interval $[\underline{\varepsilon}\ ,\ \overline{\varepsilon}]$ with density function $f(\varepsilon)$ and CDF $F(\varepsilon)$. The supplier knows f and F, but not any individual's realisation of ε.

A2 $C_y(y, V) > 0; C_V(y, V) < 0; C(0, V) > 0$.

The club good is supplied by a profit-maximizing monopolist acting as a Stackelberg leader to choose provision level y and prices p_i for periods $i = 1, 2$ at the start of period 1. We assume linear pricing[6] and a constant unit cost of unity to provide the club good.

Let $\overline{V}$ denote the aggregate number of visits made in period 1: $\bar{V} = n = V_1$. Letting M_i, $i = 1, 2$, be the period i income of consumers, the budget constraints of a member in periods 1 and 2 are then, respectively:

$$M_1 - p_1 = x_1; \quad M_2 - p_2 v_2 = x_2$$

For most of our analysis, it is unnecessary to specify the relationship between M_1 and M_2. However, we take the first period as the trial period in which consumers can instantly learn their valuations, ε, after a single usage of the club good. So, it might be plausible to take period 2 to be of longer duration with $M_2 > M_1$ and to let consumers' period-2 purchases of the club good not only reflect any higher income that they might have during that period but also to vary according to the favourableness of their evaluation, ε. As shown in section 2.6, it turns out that allowing consumers to optimise over their period-2 purchases of the club good, rather than restricting them to consume only either one or zero units of it in period 2, has important implications for distribution and, hence, for a welfare-maximiser's pricing of the club good.

Unless we state otherwise, the sequence of events is:

- *Period* 1. The leader sets y, p_1 and p_2. People then decide to join (or not) the club and make a visit. After experiencing it, they become heterogeneously (and

[6] As period-1 visits are fixed, a consumer effectively has to pay a lump sum (a membership fee) to join the club and try the club good. So, pricing has the flavour of intertemporal two-part pricing.

privately) informed about its quality, based on which they decide whether to stay in the club or to exit.[7]

- *Period* 2. If a customer remains with the club, he decides how many visits to make in period 2, given his private valuation of it.

As each consumer's realisation of ε is private information, a provider has no more information on period 2's demand at the start of period 2 than at the start of period 1. So, it cannot gain from setting p_2 at the start of period 2 rather than period 1.

2.1 The members' problem in period 2

2.1.1 The exit decision and club membership

For convenience, denote v_2 by v and V_2 by V from now on until we consider a benchmark case without qualitative uncertainty. Suppose each member treats V (determined endogenously later) as parametric and chooses v to maximise period 2 utility subject to the budget constraint.[8] For a given p_2 and y, we assume that both a supplier and consumers can infer the V that will occur in an equilibrium. Additionally, given the large number of consumers, V is taken to equal its expected value (or decision makers take it as so when making their decisions), *i.e., there is no aggregate uncertainty.*

A typical member then solves the following period 2 problem:

$$\max_{v} \quad u(M_2 - p_2 v) + \varepsilon v C(y, V)$$

Letting u' be the first derivative of utility function u, the first-order condition (FOC) yields:

$$-p_2 u'(M_2 - p_2 v) + \varepsilon C(y, V) \leqq 0 \quad \text{for } v \geqq 0 \tag{1}$$

with $-p_2 u'(M_2) + \varepsilon C(y, V) \leqq 0$ if $v = 0$. Now, with quality taken as parametric, $-p_2 u'(M_2) + \varepsilon C(y, V)$ is increasing in ε. The following assumption ensures that $f(\varepsilon)$ has sufficiently wide support.

A3 $\underline{\varepsilon} < 0$, so $\underline{\varepsilon} < p_2 u'(M_2)/C(y, V)$; $\overline{\varepsilon}$ is sufficiently large so that $\overline{\varepsilon} > p_2 u'(M_2)/C(y, V)$ holds for all possible values of p_2 and C.

The significance of $\underline{\varepsilon} < 0$ is that, given A2, there are realisations of ε that leave consumers sufficiently disappointed to be unwilling to consume the club good at *any*

[7] We do not (explicitly) treat exit or congestion fees, for, as we see later, neither the monopolist nor the welfarist provider wants to discourage consumers from trying out the good at the start of the game. However, the model can support the interpretation of there being an implicit exit fee.

[8] Formally, this requires n to represent a continuum. But, as Acemoglou and Jensen (2010, note 5) observe, even if the game is not large with infinitessimal players, we can still look for an equilibrium in which each player takes the aggregate, V, as given. None of our ensuing results change if we treat the continuum case explicitly, but the mathematical notation is much complicated.

non-negative price. Then, given assumptions A2 and A3, for a given M_2, y and V, there exists an $\varepsilon^* \in [\underline{\varepsilon}, \overline{\varepsilon})$ such that

$$-p_2 u'(M_2) + \varepsilon C(y, V) \gtreqless 0 \text{ according as } \varepsilon \gtreqless \varepsilon^*.$$

Call $\varepsilon^* \in [\underline{\varepsilon}, \overline{\varepsilon})$ the *marginal quality valuation* - i.e., ε^* solves

$$-p_2 u'(M_2) + \varepsilon^* C(y, V) = 0. \tag{2}$$

So, ε^* just leaves the consumer indifferent between choosing some club consumption and not. Clearly, ε^* is a function of p_2 and y (as well as other parameter values, such as M_2). Note that the number of visits at the marginal quality valuation is zero: $v(\varepsilon^*) = 0$.

Lemma 1 shows how period-2 club membership is determined depending upon the realization of ε.

LEMMA 1 (*Single Crossing*) *Members with $\varepsilon \geqq \varepsilon^*$ stay in the club, those with $\varepsilon < \varepsilon^*$ exit.*

A member who stays in the club has visits $v = v(\varepsilon, p_2, y, V)$ solving

$$-p_2 u'(M_2 - p_2 v(\varepsilon, p_2, y, V)) + \varepsilon C(y, V) = 0 \tag{3}$$

Thus, *ex-ante* (when seen from period 1), for a given p_2 and y, the expected number of visits by a member is given by

$$\int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon, p_2, y, V) dF(\varepsilon). \tag{4}$$

Denoting $v(\varepsilon, p_2, y, V)$ by only $v(\varepsilon)$ from now on, unless otherwise necessary, the expected number of visits in *aggregate* is therefore given by

$$V = n \int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon) dF(\varepsilon) \tag{5}$$

Lemma 2 shows the existence and uniqueness of an equilibrium in expected visits for a given p_2 and y.

LEMMA 2 *For a given y and p_2, a unique equilibrium in expected period 2 visits exists.*

2.1.2 Some comparative statics

The following comparative statics for consumers' responses to magnitudes that they take as parametric are used to solve the leader's problem:

LEMMA 3 (i) $\partial V/\partial y > 0$; (ii) $\partial V/\partial p_2 < 0$; (iii) $\partial \varepsilon^*/\partial y < 0$; (iv) $\partial v(\varepsilon)/\partial \varepsilon > 0$; (v) defining the visit elasticity of quality by $\eta_v \equiv \frac{V}{C}\frac{\partial C}{\partial V}$ (< 0, since $C_V < 0$), $\partial \varepsilon^*/\partial p_2 \left\{\gtreqless\right\} 0$ as $1 \left\{\gtreqless\right\} |\eta_v|$.

Thus: (i) aggregate (and individual) visits increase with the level of facility provision; (ii) an increase in p_2 reduces aggregate (and individual) visits; (iii) more people stay with the club if the level of provision increases; (iv) period 2 demand for the club good increases with the 'favourableness' of period 1's experience; (v) the effect of p_2 on club membership depends on the responsiveness of individuals to quality. This last result is of independent interest. We show below that p_2 can be set at a level where a further increase would produce either a rise or no change in club membership, depending on the club provider's objective.

2.2 The Monopolist's problem

The monopolist acts as a Stackelberg leader, choosing (y, p_1, p_2) to maximise its profit knowing that members behave as described above, where $p_i \in [0, \infty), i = 1, 2$ and $\infty >> y \geqq 0$.[9] So we assume, like several other papers on dynamic monopoly pricing with demand uncertainty, that it can commit deterministically to its pricing and quality strategies in advance.[10] Price commitment is a commonly used strategy (e.g., 'limited-time' offers) and is often implied by the repeated nature of transactions where the seller/provider is trying to build up a reputation for quality and prices. We confine attention to a pure strategy equilibrium.[11]

The monopolist maximises subject to the constraint that agents join the club in the first period. One may argue that a monopolist might wish to deny club access in period 1 to some of the *ex-ante* identical consumers so as to raise the club's period-1 quality for any given level of y, and thereby earn a higher p_1 and/or p_2. However, it cannot do so in our setting. This is because of the standard assumption in entrepreneurial club models that the supplier posts prices and allows all consumers who wish to purchase the club good at those prices to do so. Then, with *ex-ante* identical consumers, if one wishes to join the club in period 1, all will wish to do so. Without this assumption, we would have to consider an explicit rationing mechanism for the monopolist. The possibility of being rationed then would have to be incorporated in consumers' maximisation and participation constraints also.[12]

[9] One can think that there is a choke-off price $\overline{p}$ such that demand is zero above that price. Likewise, if investment y is set arbitrarily large, the provider needs to set price arbitraily large as well - which would then make its payoff go to zero.

[10] See, e.g., Courty (2003), Lewis and Sappington (1994), Van Cayseele (1991), Jing (2011) and Liu and Schiraldi (2014).

[11] Full commitment then implies we can rule out "ratcheting effects" à la Laffont-Tirole - which normally produce mixed strategy equilibria in non-commitment games.

[12] However, this requires introducing capacity limits along with *ex-ante* buyer heterogeneity - neither of which is a feature of our model. For this purpose, one can refer to the literature on advance purchase discounts with demand uncertainty where rationing is important (e.g. Dana (1998), Möller and Watanabe (2010), Nocke, Peitz, and Rosar (2010). This literature analyses how such discounts price discriminate between buyers of different valuations where the price discriminationg strategies depend upon the rationing rules.

Given all the above, the monopolist's maximisation problem is:

$$\max_{p_1,p_2,y} n\left\{p_1+\delta\int_{\varepsilon^*}^{\overline{\varepsilon}} p_2 v(\varepsilon)dF(\varepsilon)\right\}-y$$

subject to the participation constraint (PC):

$$u(M_1-p_1)+C(y,\overline{V})E(\varepsilon)+\delta\int_{\varepsilon^*}^{\overline{\varepsilon}}\left[u(M_2-p_2v(\varepsilon))+\varepsilon v(\varepsilon)C(y,V)\right]dF(\varepsilon)\geqq$$
$$u(M_1)+\delta u(M_2)(1-F(\varepsilon^*)) \tag{6}$$

where $E(\varepsilon)=\int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\varepsilon f(\varepsilon)d\varepsilon$ and $\delta, 1\geq\delta>0$ is the discount factor. We assume that the monopolist can earn strictly positive profits and satisfy PC. Using subscript 'i,' $i=1,2$, to denote the corresponding period and superscript 'm' to denote magnitudes for the monopolist, let $\mathcal{L}^m$ be the Lagrangian, λ^m the multiplier, $u_1^{m\prime}=u'(M_1-p_1)$ and $u_2^{m\prime}=u'(M_2-p_2v(\varepsilon))$ in the monopoly case,

$$\begin{aligned}\mathcal{L}^m &= n\{p_1+\delta\int_{\varepsilon^*}^{\overline{\varepsilon}}p_2v(\varepsilon)dF(\varepsilon)\}-y+\lambda^m[u(M_1-p_1)+C(y,\overline{V})E(\varepsilon)\\ &\quad +\delta\int_{\varepsilon^*}^{\overline{\varepsilon}}\left[u(M_2-p_2v(\varepsilon))+\varepsilon v(\varepsilon)C(y,V)\right]dF(\varepsilon)-u(M_1)-\delta\int_{\varepsilon^*}^{\overline{\varepsilon}}u(M_2)dF(\varepsilon^*)]\end{aligned}$$

After simplification, the FOC's for this maximisation problem are:[13]

$$\frac{\partial\mathcal{L}^m}{\partial p_1^m} = n-\lambda^m u_1^{m\prime}\leqq 0 \quad \text{for} \quad p_1^m\geqq 0 \tag{7}$$

$$\begin{aligned}\frac{\partial\mathcal{L}^m}{\partial p_2^m} &= n\delta[\int_{\varepsilon^*}^{\overline{\varepsilon}}\left\{v(\varepsilon)+p_2^m\frac{\partial v(\varepsilon)}{\partial p_2}\right\}dF(\varepsilon)]+\\ &\quad \lambda^m\delta\int_{\varepsilon^*}^{\overline{\varepsilon}}\{\varepsilon v(\varepsilon)C_{2V}\frac{\partial V}{\partial p_2}-v(\varepsilon)u_2^{m\prime}\}dF(\varepsilon)\leqq 0 \text{ for } p_2^m\geqq 0\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial\mathcal{L}^m}{\partial y^m} &= n[C_{1y}(y,\overline{V})E(\varepsilon)+\delta\int_{\varepsilon^*}^{\overline{\varepsilon}}\varepsilon v(\varepsilon)\{C_{2y}+C_{2V}\frac{\partial V}{\partial y}\}dF(\varepsilon)]+\\ &\quad n\delta u_1^{m\prime}\int_{\varepsilon^*}^{\overline{\varepsilon}}p_2\frac{\partial v(\varepsilon)}{\partial y}dF(\varepsilon)\leqq u_1^{m\prime} \quad \text{for} \quad y^m\geqq 0.\end{aligned} \tag{9}$$

We conclude the following from these FOCs. First, from equation (7), $\lambda^m\geqq n/u_1^{m\prime}>0$. Hence, the participation constraint binds. So, as expected, consumers get no rent in

[13] The monopolist's strategy space is closed and bounded and its objective and constraint functions are continuous, so equilibria exist characterised by these FOCs. It is also easily seen that these FOCs identify an equilibrium in pure strategies. Consumers cannot deviate and improve welfare by not joining the club: their utility outside the club is their reservation expected utility from membership. Given consumers do not deviate, the monopolist maximises profit while satisfying these FOCs.

equilibrium, whether $p_1^m > 0$ or not. Second, $p_2^m > 0$ holds; otherwise, the demand for period-2 club visits would be infinite and this maximisation would have no solution. But can p_1^m be zero? I.e., could the monopolist make a "free introductory offer" (interpreted as a "free trial period") on the club good? Proposition 1 shows the answer is no. This is because the monopolist wants to extract surplus from period-1 users to make repeat buying more attractive in period 2.

PROPOSITION 1 *The monopolist does not make a free introductory offer on the club good - i.e.,* $p_1^m > 0$.

Note, although the monopolist does not make a *free* introductory offer, it can still make an introductory offer - i.e., have $p_1^m \leq p_2^m$: a result that differs from Crémer (1984), where the monopolist *never* made an introductory offer to first-time buyers. If it charges $p_1^m \geq p_2^m$ then it does so perhaps to induce some of those customers with relatively bad trial experience not to exit, or (as if) to reward the consumers who stay with the club, or both.[14] If, on the other hand, it charges $p_1^m < p_2^m$, it does so to extract more surplus from those (*ex-post*) high-valuation buyers.

Remark 1 (Remarks on $p_1^m > 0$) [15]*Note that* $p_1^m > 0$ *also holds when individuals are not treated as infinitessimally small, but as discrete. Consider the analogous model with discrete individuals and taste parameters drawn from a discrete distribution, say* $\phi(\varepsilon)$. *As the Appendix shows, all that we use to prove Proposition 1 is that the participation constraint holds with equality, so there is full rent extraction, at the monopoly optimum. This will be true if* v, p_1 *and* p_2 *are continuous variables. Given these, it is easy to formulate the participation constraint with discrete individuals and to show that Proposition 1 holds in this environment. This participation constraint is*

$$u(M_1 - p_1) + C(y, \overline{V})E(\varepsilon) + \delta \sum_{\varepsilon \geqq \varepsilon^*} \{u(M_2 - p_2 v(\varepsilon)) + \varepsilon v(\varepsilon) C(y, V)\} \phi(\varepsilon) \geqq u(M_1) + \delta u(M_2) \sum_{\varepsilon \geqq \varepsilon^*} \phi(\varepsilon)$$

where now $V = \sum_{\varepsilon \geqq \varepsilon^*} v(\varepsilon)\phi(\varepsilon)$. *Retracing the steps of the proof of Proposition 1 in the Appendix when this binds shows that* $p_1^m > 0$ *holds with discrete individuals too. Additionally, it is worth stressing that* $p_1^m > 0$ *prevails even without any consideration of capacity constraints and/or ex-ante demand heterogeneity. For example, with a binding capacity constraint and (say) the efficient rationing rule, if ex-ante identical consumers are to be rationed (at random), the monopolist may have an incentive to raise* p_1^m *and adjust* p_2^m *accordingly in order to satisfy the intertemporal participation constraint that makes* $p_1^m > 0$. *On the other hand, if consumers are heterogeneous ex-ante, then with a binding capacity constraint and the efficient rationing rule, consumers with the highest willingness to purchase will be served, leaving the less 'eager' consumers being rationed - which again implies* $p_1^m > 0$. *This then gives rise to intra- as well as inter-temporal price*

[14] It is as if the leavers then pay an (implicit) 'exit' fee as they can no longer benefit from the lowered period-2 price.

[15] We thank an anonymous referee for suggesting inclusion of this discussion.

discrimination. What is interesting about our result is that, in contrast to the literature on monopoly pricing with demand uncertainty (see footnote 12), $p_1^m > 0$ prevails even without any capacity limit and/or any ex-ante demand heterogeneity.

Now, $p_2^m > 0$ implies (8) holds as an equality. Substituting $\lambda^m = n/u_1^{m\prime}$, using (25) in the Appendix for $\partial V/\partial p_2$ and simplifying, we get (see the Appendix)

$$\left[u_1^{m\prime}\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{u_2^{m\prime}}{p_2 u_2^{m\prime\prime}}dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}}u_2^{m\prime}v(\varepsilon)dF(\varepsilon)\right][\eta_v + 1] = 0 \qquad (10)$$

As $[u_1^{m\prime}\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{u_2^{m\prime}}{p_2 u_2^{m\prime\prime}}dF(\varepsilon) - \int_{\varepsilon^*}^{\overline{\varepsilon}}u_2^{m\prime}v(\varepsilon)dF(\varepsilon)] < 0$, the monopolist's period-2 pricing rule implies:

OBSERVATION 1 *The monopolist sets $p_2^m > 0$ such that $|\eta_v| = 1$.*

$|\eta_v| = 1$ is a marginal revenue = 0 condition analogous to ones found elsewhere - e.g., in the efficiency wage hypothesis. Having chosen y and p_1, the monopolist picks a p_2 that maximises VC, the quality-adjusted aggregate expected visits. This maximises consumers' willingness to pay for the club good in period 2. So, when the monopolist chooses the p_2^m that leads consumers to maximise their willingness to pay for the club good at a given y and p_1^m, it also maximises its expected revenues whilst extracting all of the expected surplus from consumers. Lemma 3(v) shows that $\partial\varepsilon^*/\partial p_2 = 0$ if $|\eta_v| = 1$. Thus, the p_2 the monopolist chooses also has the feature that a marginal variation in that price would have no impact on the number of people buying the club good. However, it would alter the amounts bought by infra-marginal members.

Lastly, we cannot rule out at this stage the possibility that y^m can be zero.

2.3 A benchmark: social welfare maximisation (under an identical informational constraint)

As a benchmark, consider provision of the club good by a benevolent social welfare maximiser. Like the monopolist, she knows members' behaviour, as described in sections 2.1.1 and 2.1.2, but cannot observe agents' *ex-post* valuation of the good. (So, she cannot engage in discriminatory pricing *ex-post*.) She uses this information while solving the following social welfare maximisation problem:

$$\max_{p_1,p_2,y} \quad n[u(M_1 - p_1) + C(y,\overline{V})E(\varepsilon) + \delta\int_{\varepsilon^*}^{\overline{\varepsilon}}\{u(M_2 - p_2 v(\varepsilon)) + \varepsilon v(\varepsilon)C(y,V)\}dF(\varepsilon) + \delta\int_{\underline{\varepsilon}}^{\varepsilon^*}u(M_2)dF(\varepsilon)]$$

subject to

$$n[p_1 + \delta\int_{\varepsilon^*}^{\overline{\varepsilon}}p_2 v(\varepsilon)dF(\varepsilon)] \geqslant y \qquad (11)$$

where (11) requires that the expected revenue from the club good must cover its provision cost.[16] We assume that the social welfare can be strictly positive while satisfying the above expected revenue constraint. The optimal values of the choice variables ε^*, p_2, etc., here will thus generally differ from the corresponding values for the monopolist.

Let subscript 'i,' $i = 1, 2$, denote the time-period (as in the monopoly case) and the superscript 's' denote magnitudes in the welfarist's regime (*except with C where, to save clutter, we omit the superscript s*). With $\mathcal{L}^s$ the Lagrangian for the welfarist's optimization and the same conventions for utility derivatives as with monopoly, the FOCs are:[17]

$$\frac{\partial \mathcal{L}^s}{\partial p_1^s} = -u_1^{s\prime} + \lambda^s \leqq 0 \text{ for } p_1^s \geqq 0 \tag{12}$$

$$\begin{aligned}\frac{\partial \mathcal{L}^s}{\partial p_2^s} = n\delta\Big[\int_{\varepsilon^{*s}}^{\overline{\varepsilon}} \{-v(\varepsilon)u_2^{s\prime} + \varepsilon v(\varepsilon) C_{2V}\frac{\partial V}{\partial p_2^s}\} dF(\varepsilon) + \\ \lambda^s \int_{\varepsilon^{*s}}^{\overline{\varepsilon}} \{v(\varepsilon) + p_2^s \frac{\partial v(\varepsilon)}{\partial p_2^s}\} dF(\varepsilon)\Big] \leqq 0 \text{ for } p_2^s \geqq 0\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial \mathcal{L}^s}{\partial y^s} = n[C_{1y}E(\varepsilon) + \delta \int_{\varepsilon^{*s}}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\{C_{2y} + C_{2V}\frac{\partial V}{\partial y^s}\} dF(\varepsilon)] + \\ \lambda^s n\delta \int_{\varepsilon^{*s}}^{\overline{\varepsilon}} p_2^s \frac{\partial v(\varepsilon)}{\partial y^s} dF(\varepsilon) - \lambda^s \leqq 0 \text{ for } y^s \geqq 0\end{aligned} \tag{14}$$

By the same argument as with monopoly, $p_2^s > 0$ in equilibrium. So, (13) holds with equality. Second (after substituting for $\partial V/\partial y$), as

$$\{C_{2y} + C_{2V}\frac{\partial V}{\partial y}\} = \frac{C_{2y}}{1 + nC_{2V}\int_{\varepsilon^{*s}}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_2^{s\prime\prime}} dF(\varepsilon)} > 0 \tag{15}$$

(14) indicates $\lambda^s > 0$ - i.e., the revenue constraint binds.[18] This, with $p_2^s > 0$, implies

[16] We can reasonably ignore the participation constraint (6) for, if it binds with a profit- maximizer making positive profits, it will certainly be slack with a welfarist that just breaks even and leaves some surplus with consumers.

[17] As with monopoly (footnote 13), the welfarist's strategy space is compact and its objective and constraint functions are continuous. So, equilibria exist satisfying these FOCs. Consumers get positive expected surplus in the welfarist equilibrium. They cannot deviate to improve welfare by not joining the club, which would yield their reservation expected utility. As consumers do not deviate, the welfarist cannot do better than satisfy these FOCs.

[18] To show that (14) indicates $\lambda^s > 0$, suppose not, thus $\lambda^s = 0$ by Kuhn-Tucker theorem. Then (14) collapses to $n[\overline{v}C_{1y}E(\varepsilon) + \delta \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\{C_{2y} + C_{2V}\frac{\partial V}{\partial y^s}\} dF(\varepsilon)] < 0$. This cannot be, given $C_{2y} + C_{2V}\frac{\partial V}{\partial y^s} > 0$ and $C_y > 0$. So, $\lambda^s > 0$ must hold.

$y^s > 0$ at the welfarist's optimum. Hence, (14) holds with strict equality. Thus, members receive some rents at the welfarist's optimum (as opposed to none under monopoly). However, even with $\lambda^s > 0$, (12) can have a corner solution, which we show formally in Proposition 3, Corollary 1, with homogeneous $C(.)$. Hence, we have

OBSERVATION 2 *The welfarist could make a free introductory offer on the club good (i.e., have $p_1^s = 0$).*

Observation 2 helps us to prove the following in the Appendix:

PROPOSITION 2 *If the welfarist makes a free introductory offer, then she also sets $p_2^s > 0$ so that $\mid \eta_v \mid > 1$ holds. Also, there is then 'overprovision' of the good in the Samuelson rule sense that willingness to pay for the marginal investment in the club facility is less than its cost.*

Recall that the Samuelson rule is a condition for the optimal provision of a shared good (see e.g. Cornes and Sandler 1996). It states that optimal provision occurs where the total willingness to pay for an incremental unit of the good (its "marginal benefit") just equals the marginal cost of getting that unit. Conventionally, it is said that there is "overprovision" of the good if its supply is taken to a point where its marginal benefit is less than its marginal cost. There is "underprovision" if the converse holds. However (as seen below), a case indentified as "overprovision" by this rule, with marginal benefit less than marginal cost, can also be one of zero provision.

Further, (13) holding with equality can be re-arranged to yield

$$n\delta[-\{\eta_v + 1\}\int_{\varepsilon^{*s}}^{\overline{\varepsilon}} \varepsilon v(\varepsilon) dF(\varepsilon) + \lambda^s \int_{\varepsilon^{*s}}^{\overline{\varepsilon}} \{v(\varepsilon) + p_2^s \frac{\partial v(\varepsilon)}{\partial p_2^s}\} dF(\varepsilon)] = 0 \tag{16}$$

As $\eta_v + 1 < 0$ at the welfarist's optimum if it makes a free introductory offer, it follows that $\int_{\varepsilon^{*s}}^{\overline{\varepsilon}} \{v(\varepsilon) + p_2^s \frac{\partial v(\varepsilon)}{\partial p_2^s}\} dF(\varepsilon) < 0$. I.e., other things equal, the welfarist could increase its expected revenue by lowering p_2. The rationale is simple: as $|\eta_v| > 1$, quality is very sensitive to visits at the optimum and it will wish to discourage visits, other things equal. It can do so by raising p_2^s to above the profit-maximizing level, given its choice of y^s and p_1^s. The fact that a welfarist chooses to operate where consumers' expected willingness to pay for the good is not maximised, given y, reflects the second-best nature of its problem. As club use is fixed in period 1, the only role of p_1^s is to finance facility investment, y. If p_1^s is set to zero, this financing of y will come exclusively from p_2^s. If so, then it is as if the welfarist is redistributing from those with high-realised valuation, who consume the club good in period 2, to those who do not (i.e., those with low-realised valuation).

2.4 Monopoly versus welfarist equilibrium

At the monopoly equilibrium $p_1^m > 0$ and $p_2^m > 0$, although $y^m = 0$ is possible; at the social optimum $p_2^s > 0$ and $y^s > 0$, while $p_1^s = 0$ is possible. Note that all the choice variables cannot *simultaneously* be strictly positive for both the monopolist and the welfarist.[19] So, there are only three possible ways in which the monopolist's equilibrium can differ from the social optimum:

1. *Regime* (a). $p_1^m > 0$, $p_2^m > 0, y^m > 0$; $p_1^s = 0$, $p_2^s > 0, y^s > 0$;
2. *Regime* (b). $p_1^m > 0$, $p_2^m > 0, y^m = 0$; $p_1^s = 0, p_2^s > 0, y^s > 0$; and
3. *Regime* (c). $p_1^m > 0$, $p_2^m > 0, y^m = 0$; $p_1^s > 0$, $p_2^s > 0, y^s > 0$.

2.5 The characterisation of different regimes

We now explore which one(s) of the above regimes is (are) likely to occur and their implications. Given ambiguities of the existing literature on monopoly provision of quality of a *private* good with both price and quality endogenous (e.g. Spence 1975; Sheshinsky 1976), we especially wish to identify when we can rank the monopoly provision of quality for the club good relative to that in the welfare optimum.

2.5.1 A homogeneous quality function

We first study cases when the quality function, $C(y, V)$, is homogeneous. Formally, a function $C(y, V)$ is homogeneous of degree k (abbreviated "h.o.d.k." here) if it satisfies the following equation:

$$C(ty, tV) = t^k C(y, V) \text{ for all } t > 0 \tag{17}$$

Although $C(y, V)$ might not be homogeneous, homogeneity is a convenient simplification for visualising the consequences of different extents of *qualitative* returns to scale. In the club context, an h.o.d.0 quality function is often discussed (as are homogeneous technologies more generally in the theory of production).[20] When $C(.)$ is h.o.d.0, changing y and V in the same proportion keeps both facility provision per use of the club and the club's perceived quality constant. If $k = -1$, doubling y and V keeps facility provision per use of the club constant but halves the quality as perceived by customers. Our general result is the following:

PROPOSITION 3 *Suppose that the quality function $C(y, V)$ is homogeneous of degree k. Then: (i) regime (c) cannot occur for any k; (ii) regime (b) occurs if and only*

[19] If all variables were positive simultaneously for both the monopolist and welfarist, we can show that their FOCs become exactly identical. In this model, that will only be possible when both profits and welfare are zero at the optimum. (This follows for, in this model, the monopolist's and the welfarist's problems can be considered as *dual* to one another.) But we have already ruled out this uninteresting case by assuming that the monopolist and welfarist make strictly positive payoffs.

[20] See Barro and Romer (1987), Fraser (2000; 2012) and Kolm (1974), among others, on implications of homogeneous club quality or congestion functions.

if $k=-1$; (iii) only regime (a) occurs for all k satisfying $k+1>0$.[21]

Proposition 3 shows that: (i) the only degrees of homogeneity of $C(.)$ that lead to a feasible solution satisfy $k+1\geq 0$; (ii) with sufficiently large qualitative scale diseconomies, the monopolist finds it suboptimal to invest in the club facility. For example, if $k=-1$, as crowding per se causes customers such detriment, it would then find it more profitable to not spend on the facility and keep visits low if it wishes to uphold quality. But, an even more striking and important implication of Proposition 3 is the following Corollary:

COROLLARY 1 *A welfarist always offers a free trial period (i.e., sets $p_1^s=0$) for all degrees of homogeneity of $C(.)$ that lead to a feasible solution.*

Corollary 1 shows that, with a homogeneous $C(.)$, a welfarist's behaviour contrasts starkly with a monopolist's, which *never* offers a free trial (*Proposition* 1).

If consumers were risk-neutral and bought at most one unit of a zero-production-cost private experience good each period, a monopolist would extract all of consumer surplus by charging in period 1 only. Conversely, a welfarist would price at zero each period. But, if the experience good is congestible and period-2 congestion is reducible by using p_2 to choke off some demand, other things unchanged, both a monopolist and a welfarist have an incentive to set $p_2>0$. If, additionally, the supplier can increase the good's quality by investing in the facility, then the welfarist might have an incentive to have $p_1>0$ to raise funds for that purpose.

In the much-discussed case with $C(.)$ h.o.d.0, as noted above, $C(y,V)=c(y/V)$ and $c'(y/V)>0$, for some function $c(.)$. Quality then just depends on the facility investment per use of the club. For example, patients at a health clinic might find the quality of care depends on the average time doctors spend with each patient and average drug and equipment spending per treatment, or a parent might think that the quality of school lessons is determined by just the staff-pupil ratio. The club good is then like a purely private good producible under constant returns. Consumers do not care whether it is produced in one or any number of facilities if the same ratio of facility investment to usage is maintained in each. Unsurprisingly, then, despite varying period-2 use of the club by members, the welfarist still chooses $p_1=0$ and finances the private-like club good of unvarying quality via $p_2>0$. What is remarkable, however, is that this behaviour by the welfarist extends to other degrees of homogeneity, when the club good is not purely private-like.

In the context of Proposition 2, Proposition 3 also means that a welfarist over-supplies the club facility in the Samuelson rule sense for all feasible degrees of homogeneity k (i.e., homogeneity of degree greater than or equal to minus 1). Conversely, in regime (a), as $y^m>0$, (2.9) means that the monopolist's provision satisfies Samuelson's rule. If $k=-1$, regime (b) holds. Then, (2.9) indicates that, generically, the monopoly overprovides

[21] It can be checked that, for $k<-1$, there is no monopoly equilibrium, so a comparison between the monopoly equilibrium and welfarist's optimum is irrelevant.

under this rule, although $y^m = 0$. This seems paradoxical. But, it just implies that *any* provision by the monopolist would be socially excessive, given the configuration of its other choice variables, because the marginal cost of provision exceeds the marginal benefit, even at a zero level of facility provision. This comparison is also significant for exemplifying the well-known fact that Samuelson's rule need not have a straightforward implication in terms of levels of provision of a shared good.[22]

When $C(.)$ is h.o.d.0, the fundamental elasticity condition $|\eta_v| = 1 \iff V^m C_V^m + C^m = 0$ at the monopoly equilibrium has a simple intuitive interpretation. Defining the facility investment per period-2 club visit by $z \equiv y/V$, the condition becomes $c(z^m) - z^m c'(z^m) = 0$: that for maximizing $c(z)/z$ with respect to z. The maximizing z^m is unique if $c'' < 0$. Granted this, the monopolist makes the investment in the quality of a visit that maximises the return per unit investment in quality. Fraser (2000, 2005) shows that this unique z solving $c(z) - zc'(z) = 0$ is what a welfarist, constrained to break even supplying fully-informed consumers in a single-period setting, would choose with the same quality function. So, in the h.o.d.0 case with qualitative uncertainty, a monopolist supplies the full information welfare-maximising club quality to period-2 consumers but the welfare maximiser does not. The monopolist's behaviour mimics that in Bergemann and Välimäki's mass market, but in a club context.

In the arbitrary k-degree homogeneous case, the quality function satisfies $C(y,V) = V^k c(y/V)$ for some function $c(.)$. We can show that the monopolist then always wishes to offer a higher level of facility provision per visit than does the welfarist if the facility provision elasticity of quality, $zc'(z)/c(z)$, $z \equiv y/V$, is monotonic in the facility provision per visit, z. First, we show in the next Lemma that $zc'(z)/c(z)$ is decreasing in z at both the monopolist's and welfarist's equilibrium. Hence, if it is monotonic, it must be decreasing everywhere.

LEMMA 4 *If there are diminishing returns to an investment in the facility provision (i.e., $c'' < 0$) and the facility provision elasticity of quality, $zc'(z)/c(z)$, is monotonic in z, then it is decreasing everywhere.*

The following corollary, showing that the monopolist will always invest in a greater level of facility per period-2 visit than the welfarist, follows from Lemma 4 and the fact that their equilibria satisfy $z^s c'(z^s)/c(z^s) > k + 1 = z^m c'(z^m)/c(z^m)$.

COROLLARY 2 *If the conditions of Lemma 4 are satisfied, $z^m > z^s$.*

In the h.o.d.0. case, the following proposition follows immediately from Lemma 4 and Corollary 2.

PROPOSITION 4 *If $C(.)$ is h.o.d.0. and the elasticity of quality w.r.t. facility provision is monotonic, then $C(y^m, V^m) = c(y^m/V^m) > c(y^s/V^s) = C(y^s, V^s)$: the monopolist invests in socially excessive quality provision for period 2.*

[22] In a different setting (specifically, with no qualitative uncertainty and with technological external scale economies or diseconomies in providing club goods), Kennedy (1990) shows that a club facility is under-provided in a competitive market. Conversely, a monopolist's provision is efficient as it internalises the cost externality.

This result occurs because the monopolist both wishes to extract rent from those who try in period 1 but not buy in period 2 (hence it sets $p_1^m > 0$) and to provide an incentive for those period-1 tryers to remain period-2 buyers. It can do this by ensuring a high period-2 quality, which relaxes the participation constraint. The welfarist, conversely, is concerned about equity as well as efficiency. It is interested in equalizing the actual utility of stayers and leavers as nearly as possible. So, it prefers to not charge in period 1, though this means relatively less funds are available for facility provision to enhance period-2 quality.

By pricing in this way, the welfarist operates a limited system of random redistributive taxation: only consumers with good enough period-1 experiences are 'taxed' to pay for the club good. Their "tax" increases with the favourableness of their experience as their club use increases in ε. Indeed, were transfers possible, the welfarist might wish to make *ex-post* equalising transfers to those who choose to not use the club in period 2 due to bad period-1 experiences. It is limited for, by assumption, transfers are impossible. So, setting $p_1^s = 0$ is the best it can do.

Surprisingly, this scenario is reminiscent of the literature on monopoly pricing with asymmetric information where high prices signal high product quality (e.g. Milgrom and Roberts 1986; Bagwell and Riordan 1991; Judd and Riordan 1994)). In such models, to signal quality, a monopolist may charge a price well above the full-information profit-maximizing one. Our no-signalling model can have an observationally equivalent implication: when the club good's quality is yet unknown, a monopolist credibly provides a higher quality club good than a welfarist would by charging a higher period-1 price than the latter (i.e., $p_1^m > p_1^s = 0$). Simultaneously, this contrasts sharply with Lewis and Sappington (1994). In the latter, to signal quality, the monopolist can costlessly and strategically provide information about the product via, e.g., free samples or car test drives that are like our welfarist's 'free' introductory offer. This monopolist's incentive in Lewis and Sappington is due to her wanting to maximise profits through price discrimination and so improved private information by buyers allows her to segment the market according to buyers' valuation. Such a tendency to extract rents from buyers through explicit price discrimination does not arise in our model as all consumers are homogenous *ex-ante*. Nevertheless, our model can have the flavour of intertemporal price discrimination if the monopolist charges a higher second-period than first-period price - i.e., if $p_2^m > p_1^m (> 0)$.

An Example Suppose $C(y,V) = [(y/V)+\gamma]^{\vartheta}$, for some scalars $\gamma < 0$ and $\vartheta \in (0,1)$. Then, it is easy to show that $y^m/V^m = \frac{\gamma}{\vartheta-1} > y^s/V^s$.[23] So $C(y^m,V^m) = [(y^m/V^m)+\gamma]^{\vartheta} > C(y^s,V^s) = [(y^s/V^s)+\gamma]^{\vartheta}$.

[23] From (2.10), $-(y^m/V^m)[\gamma+(y^m/V^m)]^{\vartheta-1}\vartheta + [\gamma+(y^m/V^m)]^{\vartheta} = 0 \iff -(y^m/V^m)\vartheta + [\gamma+(y^m/V^m)] = 0 \iff (y^m/V^m)(1-\vartheta)+\gamma = 0 \iff y^m/V^m = \frac{\gamma}{\vartheta-1}$. Likewise, from Proposition 2, $-(y^s/V^s)[\gamma+(y^s/V^s)]^{\vartheta-1}\vartheta + [\gamma+(y^s/V^s)]^{\vartheta} < 0$, which simplifies to $y^s/V^s < \frac{\gamma}{\vartheta-1}$.

2.5.2 A nonhomogeneous quality function

From Proposition 3, we know that homogeneity of $C(.)$ severely restricts the possibility of regimes (b) and (c) occurring. So, we will now suppose that $C(y,V)$ is not homogeneous and that these regimes are possible. What might be the characteristics of these regimes? We will make the following reasonable assumption:

A4 $C_{VV} \leqq 0$ (increasing marginal disutility of congestion); $C_{Vy} \geqq 0$ (increased facility provision ameliorates the negative impact of increased club usage).[24]

In comparing monopoly and welfarist regimes now, the visit elasticity of quality plays the same pivotal role as in the homogeneous case (cf. the proofs of Propositions 3-4 and Lemma 4). In regime (b), the inequality $|\eta_v^s| > |\eta_v^m| = 1$ holds as $|\eta_v^m| = 1$ by Observation 1 and $|\eta_v^s| > 1$ by Proposition 2. Conversely, under regime (c), $|\eta_v^s| = 1$ (combining equations (2.12) and (2.13) when $p_1^s > 0$) - i.e., under regime (c), $|\eta_v^s| = |\eta_v^m| = 1$ holds. We can use this, together with the properties of η_v when C is non-homogeneous, to show that monopoly will plausibly result in *less than socially optimal period-2 use* of the club in regimes (b) and (c).

To see how the elasticity $\eta_v \equiv VC_V(y,V)/C(y,V)$ behaves in response to changes in y and V, we can totally differentiate and rearrange to obtain

$$d\eta_v = C^{-2}\left[\left(CC_V + CVC_{VV} - V(C_V)^2\right)dV + \left(CVC_{Vy} - VC_VC_y\right)dy\right] \qquad (18)$$

By our assumptions, $\left(CC_V + CVC_{VV} - V(C_V)^2\right) < 0$ and $\left(CVC_{Vy} - VC_VC_y\right) > 0$. Thus, other things equal, an increase in V will decrease η_v (make it more negative), while increasing y will increase it. In both these regimes $y^m = 0$. So, to compare the monopolist and the welfarist's behavior in them, we let $dy = y^s > 0 = y^m$. Then, to satisfy $V^mC_V(0,V^m)/C(0,V^m) = -1 \geq V^sC_V(y^s,V^s)/C(y^s,V^s)$ and (2.18), we must have $V^s > V^m$. This establishes the following:

PROPOSITION 5 *Under regimes (b) and (c) and A4, the aggregate second period visits to the club under monopoly are less than the socially optimal level:* $V^m < V^s$.

$C_{Vy} \geqq 0$ in A.4 can be violated and yet we get $V^s > V^m$ in regimes (b) and (c). E.g., if $C(y,V) = h(y)/g(V)$ for some positive increasing functions h and g, then $C_{Vy} = -h'(y)g'(V)/g(V)^2 < 0$. Yet, direct calculation shows that $CVC_{Vy} - VC_VC_y = 0$ in this case and, so, we must have $V^s > V^m$ as before.

Under Proposition 5's conditions, as period-1 aggregate club visits under both monopoly and the social optimum are equal, aggregate expected visits over the two periods are greater at the welfare optimum than under monopoly. But we cannot compare period-2 quality levels in regimes (b) and (c) for, although $y^s > 0 = y^m$, $V^s > V^m$ might still mean $C(0,V^m) > C(y^s,V^s)$ holds. Still, as total period-1 visits ($\overline{V}$) are the same under

[24] Note, if $C(0,V) > 0$ did not hold, regimes (b)-(c) could not occur: the monopolist would not get any period-2 customers and the participation constraint could not be met if $y^m = 0$.

monopoly and welfarism, $C\left(y^s, \overline{V}\right) > C\left(0, \overline{V}\right)$ holds - implying that the welfarist offers a higher period-1 quality than the monopolist in these regimes. This is consistent with the suggestion that, compared with the welfarist, the monopolist is more focused on treating retained customers well, even if at the expense of disappointed period-1 customers. Such arguments imply that the monopolist could offer a higher quality to repeat customers, yet a lower quality to first-time and once-only customers, than does the welfarist. So, unlike in a single-period model, we cannot say unambiguously that the monopolist will over- or under-supply quality.

2.6 Behaviour with demand uncertainty, but not qualitative uncertainty.[25]

To assess how sensitive our results are to the experience nature of the club good, we now examine a model with a similar set-up to that studied so far, except that there is no qualitative uncertainty. Instead, we assume that there is demand uncertainty: each consumer now knows their ε from the start of period 1 whilst the supplier only knows the distribution of ε, again denoted $F(\varepsilon)$.

2.6.1 Binary decisions in both periods

First, we study when consumers can consume only 1 or 0 unit of the club good in each period.

For any given y and p_1 chosen by the supplier and aggregate use of the club good in period 1, now denoted V_1, consumers, who now know their ε, will buy the club good if and only if

$$u(M_1 - p_1) + \varepsilon C(y, V_1) \gtreqqless u(M_1) \Longleftrightarrow \varepsilon \gtreqqless [u(M_1) - u(M_1 - p_1)] / C(y, V_1)$$

A marginal consumer valuation for this period-1 problem, now denoted ε_1^*, is defined by

$$\varepsilon_1^* = [u(M_1) - u(M_1 - p_1)] / C(y, V_1) \equiv \Delta u_1 / C(y, V_1)$$

So, $\varepsilon_1^* = 0$ if $p_1 = 0$. Also, V_1 must satisfy $V_1 = n \int_{\varepsilon_1^*}^{\overline{\varepsilon}} dF(\varepsilon)$. Thus, substitution defines ε_1^* implicitly by

$$\varepsilon_1^* \equiv \Delta u_1 / C\left(y, n \int_{\varepsilon_1^*}^{\overline{\varepsilon}} dF(\varepsilon)\right) \tag{19}$$

We now use this to obtain the comparative statics of ε_1^* with respect to p_1 and y. Again letting the period 1 congestion function $C(y, V_1) \equiv C_1$, let C_{1V} and C_{1y} denote

[25] We are indebted to the anonymous referee whose comments motivated us to include subsections 2.6 and 2.7 in this paper.

respectively the partial derivatives with respect to V_1 and y. Then we can show that

$$\begin{aligned}
\partial\varepsilon_1^*/\partial p_1 &= \left[1 - \Delta u_1 \left(C_1\right)^{-2} C_{1V} f\left(\varepsilon_1^*\right)\right]^{-1} u^{/}\left(M_1 - p_1\right)/C_1 > 0 \\
\partial\varepsilon_1^*/\partial y &= -\left[1 - \Delta u_1 \left(C_1\right)^{-2} C_{1V} f\left(\varepsilon_1^*\right)\right]^{-1} \Delta u_1 \left(C_1\right)^{-2} C_{1y} < 0
\end{aligned} \tag{2.20}$$

Consumers learn nothing in going from period 1 to period 2. If they are restricted to purchasing either 1 or 0 units of the club good in period 2, they have the same problem in period 2 as in period 1. So, we can define period-2 magnitudes p_2, Δu_2, ε_2^*, C_2 and V_2 analogously to those for period 1 to get

$$\begin{aligned}
\partial\varepsilon_2^*/\partial p_2 &= \left[1 - \Delta u_2 \left(C_2\right)^{-2} C_{2V} f\left(\varepsilon_2^*\right)\right]^{-1} u^{/}\left(M_2 - p_2\right)/C_2 > 0 \\
\partial\varepsilon_2^*/\partial y &= -\left[1 - \Delta u_2 \left(C_2\right)^{-2} C_{2V} f\left(\varepsilon_2^*\right)\right]^{-1} \Delta u_2 \left(C_2\right)^{-2} C_{2y} < 0
\end{aligned} \tag{2.21}$$

When the club supplier is a monopolist, it maximises discounted net revenues of

$$\begin{aligned}
& p_1 n \int_{\varepsilon_1^*}^{\overline{\varepsilon}} dF\left(\varepsilon\right) + \delta p_2 n \int_{\varepsilon_2^*}^{\overline{\varepsilon}} dF\left(\varepsilon\right) - y = \\
& p_1 n \left[F\left(\overline{\varepsilon}\right) - F\left(\varepsilon_1^*\right)\right] + \delta p_2 n \left[F\left(\overline{\varepsilon}\right) - F\left(\varepsilon_2^*\right)\right] - y
\end{aligned} \tag{2.22}$$

where ε_1^* and ε_2^* are, respectively, functions of p_1 and y, and p_2 and y. To avoid too many notational changes and clutter, let superscript "m" now also denote the optimal values (just as in the qualitative uncertainty monopoly case).The FOCs for this maximisation at the optimal values $(\varepsilon_1^{*m}, \varepsilon_2^{*m}, p_1^m, p_2^m)$ are then

$$\left[F\left(\overline{\varepsilon}\right) - F\left(\varepsilon_1^{*m}\right)\right] - p_1^m f\left(\varepsilon_1^{*m}\right) \partial\varepsilon_1^*/\partial p_1 \leqq 0 \qquad (2.23\ (i))$$

$$\delta\left[F\left(\overline{\varepsilon}\right) - F\left(\varepsilon_2^{*m}\right)\right] - p_2^m \delta f\left(\varepsilon_2^{*m}\right) \partial\varepsilon_2^*/\partial p_2 \leqq 0 \qquad (2.23\ (ii))$$

$$-p_1^m n f\left(\varepsilon_1^{*m}\right) \partial\varepsilon_1^*/\partial y - p_2^m n \delta f\left(\varepsilon_2^{*m}\right) \partial\varepsilon_2^*/\partial y - 1 \leqq 0 \qquad (2.23\ (iii))$$

It is clear immediately that p_1^m and p_2^m must be positive, or else we have contradictions of (2.23)(i)-(ii). It is also clear why the monopolist will not price at zero in either period: it has nothing to gain from doing so and would simply forego revenues. Also, we see that these two FOCs are actually identical. This means that, if $M_1 = M_2$, then $p_1^m = p_2^m > 0$ will solve (2.23)(i)-(ii) holding with equality. The only thing that links the two periods for the monopolist is the non-depreciating club facility investment, y^m, for which it equates the discounted marginal revenues to marginal cost if $y^m > 0$ ((2.23)(iii)).

If the club good supplier is instead a welfarist seeking to maximise the total utility of those who buy and do not buy the club good over the two periods, subject to breaking even, then its formal optimisation problem is

$$\begin{aligned}
\underset{p_1,p_2,y}{Max} \qquad & nu\left(M_1-p_1\right)\left[F\left(\overline{\varepsilon}\right)-F\left(\varepsilon_1^{*s}\right)\right]+nC_1^s\int_{\varepsilon_1^{*s}}^{\overline{\varepsilon}}\varepsilon dF\left(\varepsilon\right)+\\
& nu\left(M_1\right)\left[F\left(\varepsilon_1^{*s}\right)-F\left(\underline{\varepsilon}\right)\right] \;\; +nu\left(M_2-p_2\right)\left[F\left(\overline{\varepsilon}\right)-F\left(\varepsilon_2^{*s}\right)\right]+\\
& nC_2^s\int_{\varepsilon_2^{*s}}^{\overline{\varepsilon}}\varepsilon dF\left(\varepsilon\right)+nu\left(M_2\right)\left[F\left(\varepsilon_2^{*s}\right)-F\left(\underline{\varepsilon}\right)\right]\\
\text{s.t.} \qquad & p_1n\left[F\left(\overline{\varepsilon}\right)-F\left(\varepsilon_1^{*s}\right)\right]+p_2n\left[F\left(\overline{\varepsilon}\right)-F\left(\varepsilon_2^{*s}\right)\right]\geqq y
\end{aligned} \tag{2.24}$$

We can proceed as before to formulate the corresponding Lagrangian and derive the FOCs (see the Online Appendix). These optimising conditions show that, again, if $M_1 = M_2$, then $p_1^s = p_2^s$ will hold. Hence, if $y^s > 0$, the welfarist will levy the same positive price in the two periods to finance it. Of course, as the monopolist's and welfarist's FOCs differ, so will the prices p_i^m and p_i^s, $i = 1, 2$, that solve them.

However, the most important observation is that, with the supplier facing demand uncertainty in the absence of qualitative uncertainty, there is no difference between the pricing behaviour of the two types of suppliers in this special case. Although they will generally charge diffferent prices from each other, neither discriminates between the consumers in the two periods. This is a striking difference from the behaviour we highlight with qualitative uncertainty, when consumers differ between the two periods as they go from not knowing their ε to being fully informed about it.

2.6.2 Consumers can optimise over second period visits

For direct comparability with our earlier model of qualitative uncertainty, we next suppose consumers can purchase 1 unit of the club good in period 1, but can optimise over their period-2 club visits constrained only by their budget constraints. The structure of demand in period 2 is the same for a supplier as in the model with qualitative uncertainty: consumers also know their period-2 tastes in that model. When the supplier is a monopolist, it now maximises discounted expected profits over two periods given by

$$p_1n\left[F\left(\overline{\varepsilon}\right)-F\left(\varepsilon_1^{*}\right)\right]+\delta p_2n\int_{\varepsilon_2^{*}}^{\overline{\varepsilon}}v\left(\varepsilon\right)dF\left(\varepsilon\right)-y \tag{2.25}$$

Let $\widehat{}$ on variables denote optima with variable period-2 visits. It is again easy to show (see the Online Appendix) that $\widehat{p}_1^m > 0$ and $\widehat{p}_2^m > 0$ hold. Likewise, it is again evident that only the non-depreciating $\widehat{y}^m$ links the two periods.

If the sole club supplier is a welfarist under the same assumptions, then it prices to maximise two-period expected welfare subject to breaking even. I.e., it maximises

$$nu\left(M_1-p_1\right)\left[F\left(\overline{\varepsilon}\right)-F\left(\varepsilon_1^{*s}\right)\right]+nC_1^s\int_{\varepsilon_1^{*s}}^{\overline{\varepsilon}}v\left(\varepsilon\right)dF\left(\varepsilon\right)+nu\left(M_1\right)\left[F\left(\varepsilon_1^{*s}\right)-F\left(\underline{\varepsilon}\right)\right]+$$

$$nu\left(M_2-p_2\right)\left[F\left(\overline{\varepsilon}\right)-F\left(\varepsilon_2^{*s}\right)\right]+nC_2^s\int_{\varepsilon_2^{*s}}^{\overline{\varepsilon}}v\left(\varepsilon\right)dF\left(\varepsilon\right)+nu\left(M_2\right)\left[F\left(\varepsilon_2^{*s}\right)-F\left(\underline{\varepsilon}\right)\right]$$

subject to

$$p_1 n \left[F(\overline{\varepsilon}) - F(\varepsilon_1^{*s})\right] + p_2 n \int_{\varepsilon_2^{*s}}^{\overline{\varepsilon}} v(\varepsilon)\, dF(\varepsilon) \geqq y \tag{2.26}$$

Although we cannot rule out $\widehat{p}_1^s = 0$ in all circumstances at the optimal choices, we can find sufficient conditions to do so. The Online Appendix shows that a sufficiently large responsiveness of the club's quality to its usage - i.e., an elasticity $\left(\widehat{\varepsilon}_1^{*s}/\widehat{C}_1^s\right)\left(\partial\widehat{C}_1^s/\partial\widehat{\varepsilon}_1^{*s}\right) \geqq 1$ - suffices to ensure $\widehat{p}_1^s > 0$ at the optimum.

The contrast between the unambigous implications for monopoly pricing, irrespective of whether or not consumers are restricted to a binary choice in period 2, and the ambiguity in the welfarist's pricing when consumers can optimise over second period club usage highlights two things. The first is that differences between the two suppliers' behaviour is because of the redistributive concerns of the welfarist. The second is that these concerns can be obscured in a model that restricts consumers to purchase only either one or zero units of the club good in each period. The following argument explains why this is so.

Definitionally, a utilitarian welfarist is concerned about distribution in all circumstances. This is as true when there is no qualitative uncertainty as when there is. In our model without qualitative uncertainty, if we restrict consumers' choice of club visits to a binary choice, we then have the lowest possible level of inequality if transfers are not allowed. There is still some inequality, for those who have a low known ε are worse off than those who have a high ε and choose to buy the club good at positive prices. However, at this minimum attainable level of inequality, if it is optimal for the welfarist to invest in the club facility, then it would choose a positive price in both periods (and certainly if $M_1 = M_2$, when it chooses $p_1 = p_2$) as there is no distributional basis for differentiating between consumers as a whole in the two periods.

If we have no qualitative uncertainty but allow consumers to optimise over visits, this introduces more inequality: those with a larger ε are better off for that reason, but also because of the ability to increase their purchases of the desirable club good. This causes the welfarist to engage in more redistribution as compared with the situation when consumers only have a binary choice over the purchase of the club good. It is not able to redistribute to those with low ε by reducing p_2 – this would just encourage those with high ε to buy even more of the desirable club good in period 2 and thereby increase inequality. So, the only way for the welfarist to engage in more redistribution/favour those with low ε relative to those with high ε is for it to reduce p_1 (and/or increase p_2). In principle, it might wish to reduce p_1 all the way to zero even with no qualitative uncertainty if we allow consumption of the club good to differ between consumers in a non-binary way.

2.7 Behaviour when the welfarist has a different objective

So far, we have adopted the standard approach to welfare maximisation in a productive economy exposited in texts such as Cornes and Sandler (1996) - namely, maximise the social welfare function subject to an economy-wide resource or transformation function.

Here, with productive activities involving simply the transformation of fixed endowments into the private consumption good or the club facility one-for-one, the transformation function can be taken as the break-even constraint. However, it is possible that a planner would put some weight on producer as well as consumer interests. This might be so, for example, if consumers and the producer are distinct and consumers do not own the producer. To allow for this possibility, we revert to a model with qualitative uncertainty to consider the outcome if the planner seeks to maximise a weighted sum of consumer welfare and producer surplus, subject to a no-loss constraint in production. We show that distributional concerns again dictate the planner's behaviour.

For arbitrary $\alpha \; \epsilon \; (0,1]$, let the weighted sum of consumer welfare and producer surplus that the planner maximises (with endogenous marginal valuation ε_2^*) be

$$\alpha\{n\left[u\left(M_1-p_1\right)\right]+C(y,n)E\left(\varepsilon\right)+\ \delta\int_{\varepsilon_2^*}^{\overline{\varepsilon}}\left[u\left(M_2-p_2v\left(\varepsilon\right)\right)+\varepsilon v\left(\varepsilon\right)C(y,V)\right]dF\left(\varepsilon\right)$$
$$+\delta\int_{\underline{\varepsilon}}^{\varepsilon_2^*}u\left(M_2\right)dF\left(\varepsilon\right)\}+(1-\alpha)\left\{n\left[p_1+\delta\int_{\varepsilon_2^*}^{\overline{\varepsilon}}p_2v\left(\varepsilon\right)dF\left(\varepsilon\right)-y\right]\right\}$$

subject to

$$n\left[p_1+\delta\int_{\varepsilon_2^*}^{\overline{\varepsilon}}p_2v\left(\varepsilon\right)dF\left(\varepsilon\right)-y\right]\geqq 0 \tag{2.27}$$

Let superscript 'sw' now show optimal values. From the relevant Lagrangian, optimal values satisfy the no-loss constraint and the following FOCs:

$$-\alpha u'\left(M_1-p_1^{sw}\right)+(1-\alpha)+\lambda^{sw}\leqq 0 \tag{2.28(i)}$$

$$\alpha n\delta\left\{\int_{\varepsilon_2^{*sw}}^{\overline{\varepsilon}}\left[-v\left(\varepsilon\right)u'\left(M_2-p_2^{sw}v\left(\varepsilon\right)\right)+\varepsilon v\left(\varepsilon\right)C_{2V}^{sw}\frac{\partial V_2^{sw}}{\partial p_2}\right]dF\left(\varepsilon\right)\right\} \tag{2.28(ii)}$$
$$+n\delta\left(1-\alpha+\lambda^{sw}\right)\int_{\varepsilon_2^{*sw}}^{\overline{\varepsilon}}\left[p_2^{sw}\frac{\partial v\left(\varepsilon\right)}{\partial p_2}+v\left(\varepsilon\right)\right]dF\left(\varepsilon\right)\leq 0$$

$$n\alpha\left\{C_{1y}^{sw}E\left(\varepsilon\right)+\delta\int_{\varepsilon_2^{*sw}}^{\overline{\varepsilon}}\left[\varepsilon v\left(\varepsilon\right)\left(C_{2y}^{sw}+C_{2V}^{sw}\frac{\partial V_2^{sw}}{\partial y}\right)\right]\right\} \tag{2.28(iii)}$$
$$+\left(1-\alpha+\lambda^{sw}\right)\left[n\delta\int_{\varepsilon_2^{*sw}}^{\overline{\varepsilon}}p_2^{sw}\frac{\partial v\left(\varepsilon\right)}{\partial y}-1\right]\leqq 0$$

Evidently, these FOCs are identical to those for the original welfarist problem, except the Lagrange multiplier in the latter, λ^s, is replaced by $\left(1-\alpha+\lambda^{sw}\right)/\alpha$.

Our principal result in this scenario, stated as Proposition 6 below, shows, essentially, that $p_1^{sw}=0$ is more likely the larger the weight the planner puts on consumer welfare relative to producer surplus. This reiterates our observation that distributional considerations determine the welfarist's pricing behaviour.

PROPOSITION 6 *(i) Let $M_\alpha \equiv (u')^{-1}[(1-\alpha)/\alpha]$, where $(u')^{-1}$is the inverse function for u'. Then $p_1^{sw} > 0$ if $M_1 > M_\alpha$.(ii) If the producer gets positive surplus, then $p_1^{sw} = 0$ if $M_1 \leqq M_\alpha$.*

Propositon 6 all makes sense in terms of redistribution. If the first-period income, M_1, is sufficiently large, the planner is not impelled to charge $p_1^{sw} = 0$ to partially compensate those who have such unfavourable experiences in period 1 that they do not wish to purchase the club good in period 2. Rather, it will either take advantage of the returns to scale in providing a shared good and charge even those customers in order to increase y, or/and it will reduce p_2^{sw}. Further, as M_α is increasing in α, the greater the weight put on consumer welfare relative to producer surplus, the less likely is $M_1 > M_\alpha$ and $p_1^{sw} > 0$ for sure. It follows that we are more likely to have a corner solution $p_1^{sw} = 0$ the greater is the weight put on consumer welfare relative to that put on producer surplus. This reinforces our previous conclusion that we get $p_1^s = 0$ for distributional reasons - in particular, because the welfarist seeks to compensate as best it can those with an adverse initial experience with the club good.

For a given α and $(1-\alpha)$, the cut-off income level M_α is fixed. So, other things equal, with this weighted maximand, if there is positive producer surplus, $p_1^{sw} = 0$ will hold if first-period income is sufficiently low.

3 CONCLUSIONS AND EXTENSIONS

The literature has not studied the experience good aspect of clubs. We show that it can lead to very contrasting investment and pricing strategies for a monopoly profit maximiser and a welfare maximiser.

In our model, potential members visit the club a fixed number of times in period 1 to learn perfectly their evaluation of its quality. Based on this, they decide if to continue as members and their number of visits, or to exit for good. So, pricing strategies announced in period 1 and the investment in the shared facilities are crucial in determining the club's ultimate membership and social welfare.

In this scenario, whether a club provider offers an introductory discount to consumers with no prior knowledge of its quality depends on its type. A welfare maximiser might give consumers an "introductory free trial period". It does so definitely if the quality function is homogeneous. This reduces the welfare disparity between those who try the product, find it unsatisfactory and exit the club, and those who like it and stay. Conversely, a monopolist focused on extracting as much rent as possible from consumers *never* makes a *free* introductory offer. So, all its customers, stayers and leavers, contribute to any cost of facility provision and to profits.

Our results are consistent with Spence (1975), Sheshinsky (1976) and others that show a monopoly might over-provide quality. They are also consistent with results in models of monopoly pricing with experience goods and repeat purchases where the monopolist does not offer introductory prices to first time buyers (e.g., Crémer (1984) and Bergemann and Välimäki (2006)). But such papers do not study explicitly, as we do, the implications of peculiar features of clubs - the congestion externality and endogenous determination of quality arising both from members' utilization choice and the provider's pricing and facility provision strategy.

We model the club good as an experience good subject to negative network externalities. A large literature after Katz and Shapiro (1985) studies positive and negative network externalities. We know of no papers analyzing their qualitative uncertainty/experience aspects. Maybe Lambertini and Orsini (2001) is closest to our focus. They do not study the experience aspects of network goods but do show that a monopolist is likely to over-supply quality with a positive externality network good.

Archetypal clubs like leisure facilities are not the only ones with experience characteristics. For example, a school's ethos and teachers' dedication affect its quality. Also, different parents might take contrasting stances on the balance between concentration on the "3 Rs" and, say, pastoral care. In the same vein, many welfare states try to ensure equal opportunity to *ex-ante* identically-treated individuals by providing a fixed amount of primary and secondary education free at the point of delivery. Only those with a preference or aptitude for education have to pay for extra in the tertiary system. The predictions of our welfarist analysis with *ex-ante* identical consumers mimic this scenario. This rationalises observing partial tax financing of such goods and partial financing by user charges, a rationalisation starkly different from that based on *ex-ante* differences between consumers.

In our two-period model, *ex-ante* uncertain consumers learn their valuations of the club good perfectly after the first purchase. Our model can be extended into a multi-period framework where consumers may need to experience the good a few times to learn their true valuations, the timing of learning differing due to their heterogeneity. The provider will then face a mixture of consumers, some maybe knowing their own types whilst others do not. It then may want not to commit to any pricing strategies *ex-ante*. Analysing the potential dynamics between price commitment, level of facility provision and behaviour-based pricing strategies is then a meaningful direction for future research.

A further natural question to ask is whether our results extend to other environments, such as the multi-jurisdictional competition one of the Tiebout hypothesis.[26] We know of no theoretical analysis of Tiebout under uncertainty, much less one with qualitative uncertainty when a club good is the local service subject to jurisdictional competition. We believe that our results are likely to be robust to such a setting.

A crucial component of Tiebout's hypothesis is consumers' free mobility between different jurisdictions. Given this mobility, if there is a wide range of jurisdictions available, individuals reveal their true preferences for the 'public good' by choosing to reside in the one where provision most closely matches their preferences. Then, all those living in the same jurisdiction essentially have the same preferences. However, this outcome is

[26]We are indebted to the Associate Editor who suggested that we discuss implications of the Tiebout hypothesis for our analysis.

based on the Tiebout idea that consumers exactly know their valuation for the public good (or the public good that they will be consuming).

In our model, the club good is an experience good. This violates the Tiebout assumption of full information. If any club good provided by any competing monopoly is an experience good, consumers cannot necessarily pick the 'right' club for them *ex-ante*, even with perfect residential mobility. This is because consumers will have to 'try before they know what they buy' for any club. (See below for a description of equilibrium.) So, as long as there are no other costs involved - e.g., transportation costs - we conjecture that knowing this, local monopolists will still choose the same pricing strategy as we have analysed in this paper.

To illustrate this, suppose that a very large population of *ex-ante* homogenous consumers inhabit a two-period world, as in this paper. Suppose, moreover, there are a number of possible jurisdictions and consumers are initially assigned to jurisdictions randomly – i.e., each consumer is a random drawing from the same taste distribution and each has an equal chance of being assigned to any jurisdiction. Then, by the multivariate law of large numbers, the initial populations and taste compositions of the jurisdictions will be equal almost surely. Suppose there is one club supplier in each jurisdiction and all the local monopolists operate clubs with exactly the same quality functions and commit to exactly the same prices (p_1 and p_2) that satisfy the consumers' participation constraint in order to maximise their expected profits. All consumers will be prepared to join the club in their jurisdiction (as there is no incentive to move elsewhere on grounds of price or/and quality). Moreover, depending on their actual realisation of ε, each consumer will remain with the club or quit in period 2 in exactly the same way that we have analysed so far. If it quits a given club, it will have no incentive to join another jurisdiction's, given that will have the same characteristics as the one it has left. This is therefore an equilibrium. An immediately striking feature of such an equilibrium is that, unlike with Tiebout, it will be characterised by taste heterogeneity within jurisdictions that are just copies of each other.

Now suppose, instead, consumers are still *ex-ante* homogenous but some jurisdictions have unique resources that impact on consumers' perception of the quality of the club good they supply. Then, even if putative members are allocated to jurisdictions randomly as just described, the *ex-post* situation will be one of taste heterogeneity allied to 'product differentiation' amongst monopolists. Given that each of the goods provided by the local monopolies are still experience goods, we expect that the monopolists will adopt similar pricing rules to those studied in this paper. However, the actual monopoly club prices themselves will differ from one jurisdiction to another. Moreover, because of the differences in prices and qualities, the analysis of equilibrium will necessarily be more involved than that in this paper.

In sum, we believe that the experience good aspect of the club good in our model makes our results robust to Tiebout-type generalisations, provided different jurisdictions do not have unique characteristics that impact upon consumers' perceptions of quality. Conversely, if we did not have the experience good aspect and the qualities were completely known at the start of the game, then that would give rise to Bertrand-type competition amongst local monopolies with zero mobility costs. This would be expected to bring price(s) down to zero (or the actual cost of provision).

4 APPENDIX

Proof of Lemma 1. The marginal quality valuation, ε^*, satisfies $-p_2u'(M_2)+\varepsilon^*C(y,V)=0$ ((2.2) in the text). Given p_2 and C, $-p_2u'(M_2)+\varepsilon C(y,V)$ is increasing in ε and equals zero at $\varepsilon=\varepsilon^*$. Hence, for $\varepsilon>\varepsilon^*$, $-p_2u'(M_2-p_2v)+\varepsilon C(y,V)=0$ can be satisfied for some $v>0$. But this implies that members having $\varepsilon \geqq \varepsilon^*$ remain in the club and make positive visits (the marginal member "remains" in the club but makes zero visit). Obviously, for $\varepsilon<\varepsilon^*$, members make zero visits and exit the club as $-p_2u'(M_2)+\varepsilon C(y,V)<0$. ■

Proof of Lemma 2. For a given p_2, y and V, the club usage choice of someone with experience ε is a continuous and differentiable mapping $v(\varepsilon,p_2,y,V):[0,M_2/p_2]\rightarrow[0,M_2/p_2]$ satisfying (3).The *ex-ante* expected visits for this consumer satisfy (4) and those for all consumers must satisfy $V=n\int_{\varepsilon^*}^{\overline{\varepsilon}}v(\varepsilon,p_2,y,V)dF(\varepsilon)$ uniquely if a unique equilibrium exists. Define the aggregate expected visit mapping $V(p_2,y,V)$ by $V(p_2,y,V)=n\int_{\varepsilon^*}^{\overline{\varepsilon}}v(\varepsilon,p_2,y,V)dF(\varepsilon):[0,nM_2/p_2]\rightarrow[0,nM_2/p_2]$. This mapping is also continuous and differentiable. By differentiating,
$n\partial\left[\int_{\varepsilon^*}^{\overline{\varepsilon}}v(\varepsilon,p_2,y,V)dF(\varepsilon)\right]/\partial V=-n\left(C_V\left(y,V\right)/C\left(y,V\right)\right)\int_{\varepsilon^*}^{\overline{\varepsilon}}p_2\frac{u_2'}{u_2''}dF\left(\varepsilon\right)<0$, using (3) and Leibnitz's rule. So, $V(p_2,y,V)$ is monotonically decreasing in V and takes its maximum value at $V(p_2,y,0)$, where $nM_2/p_2>V(p_2,y,0)>0$, with the first inequality following from the fact that the private good is essential. As $nM_2/p_2>V(p_2,y,0)>V(p_2,y,nM_2/p_2)$, the graph of $V(p_2,y,V)$ against V must cross the 45^0 line uniquely from above at a point where $V(p_2,y,V)=V$. Thus, a unique equilibrium in expected visits exists for a given p_2 and y. ■

Proof of Lemma 3. (i) Differentiation of (5) with respect to y, (using Leibnitz's rule) yields

$$\frac{\partial V}{\partial y}=n[\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{\partial v(\varepsilon)}{\partial y}dF(\varepsilon)-v(\varepsilon^*)\frac{\partial\varepsilon^*}{\partial y}]=n\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{\partial v(\varepsilon)}{\partial y}dF(\varepsilon) \tag{20}$$

since $v(\varepsilon^*)=0$. Differentiating (3) with respect to v and y and integrating over ε, we obtain

$$\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{\partial v(\varepsilon)}{\partial y}dF(\varepsilon)=-[C_{2y}+C_{2V}\frac{\partial V}{\partial y}]\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{\varepsilon}{p_2^2u_2''}dF(\varepsilon) \tag{21}$$

Hence

$$\frac{\partial V}{\partial y}=-n\left\{C_{2y}\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{\varepsilon}{p_2^2u_2''}dF(\varepsilon)\right\}/\left\{1+nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{\varepsilon}{p_2^2u_2''}dF(\varepsilon)\right\}>0. \tag{22}$$

So, as the level of provision increases, both individual and aggregate visits increase.

(ii) Differentiating (5) with respect to p_2, we find

$$\frac{\partial V}{\partial p_2} = n[\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial p_2} dF(\varepsilon) - v(\varepsilon^*)\frac{\partial \varepsilon^*}{\partial p_2}] = n\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial p_2} dF(\varepsilon) \tag{23}$$

Differentiation of (3) with respect to v and p_2 yields

$$\frac{\partial v(\varepsilon)}{\partial p_2} = \frac{\left\{-\varepsilon C_{2V}\frac{\partial V}{\partial p_2} - u_2'' p_2 v(\varepsilon) + u_x\right\}}{p_2^2 u_2''} \tag{24}$$

Thus, integrating and rearranging to isolate $\partial V/\partial p_2$,

$$\frac{\partial V}{\partial p_2} = -n\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2} - \frac{u_x}{p_2^2 u_2''}\right] dF(\varepsilon)/\left[1 + nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_2''} dF(\varepsilon)\right] < 0 \tag{25}$$

(iii) Differentiating (2) w.r.t. y, using (22) and rearranging yields

$$\frac{\partial \varepsilon^*}{\partial y} = -C_{2y}\varepsilon^*/C_2\left[1 + nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_2''} dF(\varepsilon)\right] < 0. \tag{26}$$

(iv) By (2.3) in the text, $p_2^2 u_2'' \partial v(\varepsilon)/\partial\varepsilon + C_2 = 0$, so $\partial v(\varepsilon)/\partial\varepsilon = -C_2/p_2^2 u_2'' > 0$.
(v) From the condition defining the marginal quality valuation, using (25),

$$\begin{aligned}
\frac{\partial \varepsilon^*}{\partial p_2} &= \left\{u_2'(M_2) - \varepsilon^* C_{2V}\frac{\partial V}{\partial p_2}\right\}\frac{1}{C_2} \\
&= \left\{C_2 + nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}} v(\varepsilon)dF(\varepsilon)\right\}\left[1 + nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_2''} dF(\varepsilon)\right]^{-1}\frac{\varepsilon^*}{p_2} \\
&= \{1+\eta_v\}\,C_2\left[1 + nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_2''} dF(\varepsilon)\right]^{-1}\frac{\varepsilon^*}{p_2}
\end{aligned} \tag{27}$$

$\Longleftrightarrow \partial\varepsilon^*/\partial p_2 \left\{\gtrless\right\} 0$ as $1 \left\{\gtrless\right\} |\eta_v|$.

Proof of Proposition 1. As the participation constraint binds in equilibrium (whether or not $p_1^m > 0$), rewrite (6) as

$$\begin{aligned}
&\delta\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[u(M_2 - p_2 v(\varepsilon)) + \varepsilon v(\varepsilon)C(y,V) - u(M_2)\right] dF(\varepsilon) \\
=\;& u(M_1) - u(M_1 - p_1) - C(y,\overline{V})E(\varepsilon)
\end{aligned}$$

If $p_1^m = 0$, then the RHS will be strictly negative while the LHS will be strictly positive, given that for $\varepsilon > \varepsilon^*$ the person get more utility in the club than out. This would violate the participation constraint. Hence $p_1^m > 0$. ■

Derivation of equation (10). From (2.8) in the text, substituting $\lambda = n/u_1^{m\prime}$,

$$n\delta u_1^{m\prime}\left[\int_{\varepsilon^*}^{\overline{\varepsilon}} \{v(\varepsilon) + p_2^m\frac{\partial v(\varepsilon)}{\partial p_2^m}\}dF(\varepsilon)\right] + n\delta\left[\int_{\varepsilon^*}^{\overline{\varepsilon}}\{-v(\varepsilon)u_2^{m\prime} + \varepsilon v(\varepsilon)C_{2V}\frac{\partial V}{\partial p_2^m}\}dF(\varepsilon)\right] = 0$$

Rearranging and cancelling $n\delta > 0$, this becomes

$$\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(u_1^{m\prime} - u_2^{m\prime}\right) v(\varepsilon)\, dF(\varepsilon) + u_1^{m\prime} p_2^m \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\partial v(\varepsilon)}{\partial p_2^m} dF(\varepsilon) + C_{2V} \frac{\partial V}{\partial p_2^m} \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\, dF(\varepsilon) = 0$$

As $\frac{\partial V}{\partial p_2} = \frac{-n\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2} - \frac{u_2^{\prime}}{p_2^2 u_2^{\prime\prime}}\right] dF(\varepsilon)}{\left[1+nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_2^{\prime\prime}} dF(\varepsilon)\right]} \equiv -n\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2} - \frac{u_2^{\prime}}{p_2^2 u_2^{\prime\prime}}\right] dF(\varepsilon)/D$ from (25), (where $D = \left[1 + nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^2 u_2^{\prime\prime}} dF(\varepsilon)\right]$), the last equation can be written as

$$\begin{aligned}&\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(u_1^{m\prime} - u_2^{m\prime}\right) v(\varepsilon)\, dF(\varepsilon) - \\ &\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2^m} - \frac{u_2^{m\prime}}{p_2^{m2} u_2^{m\prime\prime}}\right] dF(\varepsilon) \left\{u_1^{m\prime} p_2^m + nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\, dF(\varepsilon)\right\}/D = 0\end{aligned}$$

Or

$$\begin{aligned}&\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(u_1^{m\prime} - u_2^{m\prime}\right) v(\varepsilon)\, dF(\varepsilon) \left[1 + nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon}{p_2^{m2} u_2^{m\prime\prime}} dF(\varepsilon)\right] - \\ &\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2^m} - \frac{u_2^{m\prime}}{p_2^{m2} u_2^{m\prime\prime}}\right] dF(\varepsilon) \left\{u_1^{m\prime} p_2^m + nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\, dF(\varepsilon)\right\} = 0\end{aligned}$$

Now, from the first and second terms,

$$\begin{aligned}&\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(u_1^{m\prime} - u_2^{m\prime}\right) v(\varepsilon)\, dF(\varepsilon) - u_1^{m\prime} p_2^m \int_{\varepsilon^*}^{\overline{\varepsilon}} \left[\frac{v(\varepsilon)}{p_2^m} - \frac{u_2^{m\prime}}{p_2^{m2} u_2^{m\prime\prime}}\right] dF(\varepsilon) \\ = \;&\int_{\varepsilon^*}^{\overline{\varepsilon}} \left[u_1^{m\prime} \frac{u_2^{m\prime}}{p_2^m u_2^{\prime\prime}} - u_2^{m\prime} v(\varepsilon)\right] dF(\varepsilon) \equiv (A).\end{aligned}$$

From the terms in C_{2V}, we have

$$\begin{aligned}&-\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2} u_2^{m\prime\prime}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_2^{m\prime} v(\varepsilon)\, dF(\varepsilon) + \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_2^{m\prime}}{p_2^{m2} u_2^{m\prime\prime}} dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\, dF(\varepsilon) \\ = \;&-\int_{\varepsilon^*}^{\overline{\varepsilon}} \left(\frac{\varepsilon}{p_2^{m2} u_2^{m\prime\prime}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_2^{m\prime} v(\varepsilon)\, dF(\varepsilon) + \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_2^{\prime}}{p_2^{m2} u_2^{m\prime\prime}} dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_2^{m\prime} v(\varepsilon) \frac{p_2^m}{C_2} dF(\varepsilon)\end{aligned}$$

= (using $\varepsilon = p_2^m u_2^{m\prime}/C_2$ from the FOC for an agent with period 1 experience ε)

$$\begin{aligned}&-\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{p_2^m}{C_2} \left(\frac{u_2^{m\prime}}{p_2^{m2} u_2^{m\prime\prime}}\right) dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_2^{m\prime} v(\varepsilon)\, dF(\varepsilon) \\ &+\int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{u_2^{m\prime}}{p_2^{m2} u_2^{m\prime\prime}} dF(\varepsilon) \int_{\varepsilon^*}^{\overline{\varepsilon}} u_2^{m\prime} v(\varepsilon) \frac{p_2^m}{C_2} dF(\varepsilon) = 0\end{aligned}$$

The residual terms in C_{2V} equal

$$nC_{2V}\left[\int_{\varepsilon^*}^{\overline{\varepsilon}}\left(\frac{\varepsilon}{p_2^{m^2}u_2^{m''}}\right)dF(\varepsilon)\int_{\varepsilon^*}^{\overline{\varepsilon}}u_1^{m'}v\left(\varepsilon\right)dF(\varepsilon)-\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{v(\varepsilon)}{p_2^m}dF\left(\varepsilon\right)\int_{\varepsilon^*}^{\overline{\varepsilon}}\varepsilon v\left(\varepsilon\right)dF(\varepsilon)\right]\equiv(B).$$

Amalgamating (A) and (B),

$$\begin{aligned}
& u_1^{m'}\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{u_2^{m'}}{p_2^m u_2^{m''}}dF\left(\varepsilon\right)+u_1^{m'}nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}}\left(\frac{\varepsilon}{p_2^{m2}u_2^{''}}\right)dF(\varepsilon)\int_{\varepsilon^*}^{\overline{\varepsilon}}v\left(\varepsilon\right)dF(\varepsilon)\\
= & \; u_1^{m'}\left[\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{u_2^{'}}{p_2^m u_2^{''}}dF\left(\varepsilon\right)+C_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}}\left(\frac{\varepsilon}{p_2^{m^2}u_2^{m''}}\right)dF(\varepsilon)V\right]\\
= & \left[u_1^{m'}\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{u_2^{m'}}{p_2^m u_2^{m''}}dF\left(\varepsilon\right)\right]\left[1+\eta_v\right]\;\;\text{(using } \varepsilon=p_2^m u_2^{m'}/C_2 \text{ again)}
\end{aligned}$$

Also,

$$\begin{aligned}
& -\int_{\varepsilon^*}^{\overline{\varepsilon}}u_2^{m'}v\left(\varepsilon\right)dF\left(\varepsilon\right)-nC_{2V}\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{v(\varepsilon)}{p_2^m}dF\left(\varepsilon\right)\int_{\varepsilon^*}^{\overline{\varepsilon}}\varepsilon v\left(\varepsilon\right)dF(\varepsilon)\\
= & -\int_{\varepsilon^*}^{\overline{\varepsilon}}u_2^{m'}v\left(\varepsilon\right)dF\left(\varepsilon\right)\left[1+\eta_v\right]\;\;\text{(again using } \varepsilon=p_2^m u_2^{m'}/C_2\text{)}
\end{aligned}$$

So, resubstituting, the FOC (2.8) becomes

$$\left[u_1^{m'}\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{u_2^{'}}{p_2^m u_2^{''}}dF\left(\varepsilon\right)-\int_{\varepsilon^*}^{\overline{\varepsilon}}u_2^{m'}v\left(\varepsilon\right)dF\left(\varepsilon\right)\right]\left[1+\eta_v\right]=0$$

which is (2.10) in the text.

Proof of Proposition 2. (i) First, we show that if $p_1^s=0$ then $u_1^{s'}>\lambda^s$. Suppose not, so $p_1^s=0$ yet $u_1^{s'}=\lambda^s$. Suppose the welfarist were then to increase p_1^s to $p_1^s=\varepsilon>0$, for some very small ε. To first-order, the loss of welfare in first period utility is exactly counter-balanced by the value of extra funds, λ^s. Thus the welfarist could equally well set $p_1^s=\varepsilon>0$, contradicting the unique optimality of $p_1^s=0$. Next, note from (13), since $u_1^{s'}>\lambda^s>0$,

$$n\delta[\int_{\varepsilon^*}^{\overline{\varepsilon}}\{-v(\varepsilon)u_2^{s'}+\varepsilon v(\varepsilon)C_{2V}\frac{\partial V}{\partial p_2^s}\}dF(\varepsilon)+u_1^{s'}\int_{\varepsilon^*}^{\overline{\varepsilon}}\{v(\varepsilon)+p_2^s\frac{\partial v(\varepsilon)}{\partial p_2^s}\}dF(\varepsilon)]>0$$

which, after simplification (similar to the derivation of (2.10) above), yields

$$\left[u_1^{s'}\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{u_2^{s'}}{p_2^2 u_2^{s''}}dF(\varepsilon)-\int_{\varepsilon^*}^{\overline{\varepsilon}}u_2^{s'}v(\varepsilon)dF(\varepsilon)\right]\left[\eta_v+1\right]>0$$

Since $\left[u_1^{s'}\int_{\varepsilon^*}^{\overline{\varepsilon}}\frac{u_2^{s'}}{p_2^s u_2^{s''}}dF(\varepsilon)-\int_{\varepsilon^*}^{\overline{\varepsilon}}u_2^{s'}v(\varepsilon)dF(\varepsilon)\right]<0$, hence $[\eta_v+1]<0\iff\mid\eta_v\mid>1$ as $C_{2V}<0$.

(ii) Using the fact that $u_1^{s'} > \lambda^s$ for $p_1^s = 0$, equation (14) - which holds with equality as $y^s > 0$ - can be rewritten as

$$n\left[C_{1y}E(\varepsilon) + \delta\int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon v(\varepsilon)\{C_{2y} + C_{2V}\frac{\partial V}{\partial y^s}\}dF(\varepsilon)\right] + u_1^{s'}n\delta\int_{\varepsilon^*}^{\overline{\varepsilon}} p_2^s\frac{\partial v(\varepsilon)}{\partial y^s}dF(\varepsilon) < u_1^{s'}$$

The left hand side is the (expected) marginal 'valuation' of increased facility provision. The first $n\,[.]$ term is the expected benefit from increased facility size (taking into account any direct and indirect impact on quality, the latter from any induced change in congestion), at unchanged total usage of the club. The second term, $u_1^{s'}n\delta\int_{\varepsilon^*}^{\overline{\varepsilon}} p_2^s\frac{\partial v(\varepsilon)}{\partial y^s}dF(\varepsilon)$, is the valuation of the expenditure on extra visits induced by the increased facility provision. The right hand side is the utility value of the cost incurred to increase the facility size. There is overprovision of the club good in the Samuelson rule sense since the valuation of the good induced by an increase in facility size falls short of the cost of providing that increase in the facility. ■

Proof of Proposition 3. Our strategy for this proof is to show, first, that if $C\,(y, V)$ is homogeneous, then the monopolist's behaviour under regimes (b)-(c) (i.e., $y^m = 0$) occurs *iff* $C\,(y, V)$ is homogeneous of degree -1 (abbreviated "h.o.d.-1"). We then show that $C\,(y, V)$ being h.o.d.-1 is inconsistent with the welfarist's behaviour under regime (c). So, if $C\,(y, V)$ is h.o.d.-1, then only regime (b) holds. For all other k, only regime (a) is possible. But, the monopolist's behaviour under regime (a) is only consistent with $C\,(y, V)$ being h.o.d.k , where $k + 1 > 0$.

Suppose that $C(y, V)$ is h.o.d.k., i.e.

$$C(ty, tV) = t^k C(y, V) \text{ for all } t > 0 \tag{28}$$

Then, by Euler's theorem,

$$yC_y + VC_v = kC(y, V) \tag{29}$$

At the monopoly equilibrium (omitting time subscript "2" on magnitudes): $|\eta_v^m| = 1 \Rightarrow V^m C_v^m = -C(y^m, V^m)$. Substituting in (29) then yields:

$$y^m C_y^m = (k+1)C(y^m, V^m) \tag{30}$$

Proof of part (i): *regime (c) cannot occur for any k.*

In regime (c), $p_1^m > 0$, $p_2^m > 0$, $y^m = 0$; $p_1^s > 0$, $p_2^s > 0, y^s > 0$.With $y^m = 0$ for the monopolist and $C(0, V) > 0$ (see (A2)), (30) then implies the only possible value of k, for this regime to occur is $k = -1$. However, as $p_1^s > 0$,we must have $[\frac{V^s}{C^s}C_v^s + 1] = 0 \Rightarrow V^s C_v^s = -C^s$ for the welfarist, which then yields, similar to the monopoly case, the following form of (29): $y^s C_y^s = (k+1)C(y^s, V^s) \Rightarrow y^s = 0$ if $k = -1$ thereby contradicting the fact that $y^s > 0$ in this regime.

Proof of part (ii): *regime (b) occurs if and only if $k = -1$.*

In regime (b), $p_1^m > 0$, $p_2^m > 0$, $y^m = 0$; $p_1^s = 0$, $p_2^s > 0$, $y^s > 0$.The '$\Rightarrow$'part: If $k = -1$, then (30) implies $y^m C_y^m = 0$. As $C_y(0, V) > 0$ by (A2), we must have $y^m = 0$.This means, from the monopolist's point of view, both regimes (b) and (c) are possible. However, as just shown above, with $k = -1$, for the welfarist, regime (c) is not possible. Therefore, the only candidate for a plausible regime, when $k = -1$ is regime (b). We need to verify that $y^s > 0$ is consistent with regime (b). We do that as follows. By part (i) of the proof of proposition 2, at the welfarist equilibrium in regime (b) we have

$$[\frac{V^s}{C^s}C_v^s + 1] < 0 \Rightarrow V^s C_v^s + C^s < 0 \tag{31}$$

Now, from (29), $y^s C_y^s + V^s C_v^s = kC(y^s, V^s)$. Rewrite this by adding $C(.)$ on both sides,

$$\begin{aligned} y^s C_y^s + V^s C_v^s + C(y^s, V^s) &= (k+1)C(y^s, V^s) & (32) \\ i.e., V^s C_v^s + C(y^s, V^s) &= (k+1)C(y^s, V^s) - y^s C_y^s & (33) \end{aligned}$$

Then, using (4.12),

$$\begin{aligned} (k+1)C(y^s, V^s) - y^s C_y^s &< 0 & (34) \\ i.e., (k+1)C(y^s, V^s) &< y^s C_y^s & (35) \end{aligned}$$

When $k = -1$, (4.16) $\Rightarrow$

$$y^s C_y^s > 0 \Rightarrow y^s > 0 \text{ as } C_y^s > 0 \tag{36}$$

Thus, if C is h.o.d.-1, then only regime (b) holds.

Proof of part (iii): *Only regime (a) can occur for all k satisfying $k + 1 > 0$.*

We know from parts (i)-(ii) that we can rule out regimes (b) and (c) *iff* $k + 1 \neq 0$. So, if $k + 1 \neq 0$, only regime (a) can occur and we must have $p_1^s = 0$, $y^m > 0$, and $y^s > 0$. Now, for the monopolist, $y^m C_y^m = (k+1)C^m$ (equation (30)) implies $y^m > 0 \Leftrightarrow k + 1 > 0$, by (A2). ■

Proof of Lemma 4 By definition, if $C(.)$ is homogeneous of arbitrary degree k, then $C(y, V) = V^k c(y/V)$ for some function $c(.)$. As $V^m C_V^m + C^m = 0$ at the monopoly equilibrium and $C_V = kV^{k-1}c(y/V) - yV^{k-2}c'(y/V)$ then, using $z^m \equiv y^m/V^m$, $z^m c'(z^m)/c(z^m) = k + 1$. Likewise, as $V^s C_V^s + C^s < 0$ at the welfarist equilibrium, we can show $z^s c'(z^s)/c(z^s) > k + 1$. Now, by differentiation,

$d[zC'(z)/C(z)]/dz = [C'(z)]^{-2}[zC(z)C''(z) + C'(z)\{C(z) - zC'(z)\}]$.

As $C(z^m) - z^m C'(z^m) = 0$, then $d[z^m C'(z^m)/C(z^m)]/dz < 0$ must hold. Likewise, $C(z^s) - zC'(z^s) < 0$, so $d[z^s C'(z^s)/C(z^s)]/dz < 0$ also. Therefore, if $zC'(z)/C(z)$ is monotonic, it must be decreasing everywhere. ■

Proof of Proposition 6. (i) The FOC for p_1^{sw} is $-u'(M_1 - p_1^{sw}) + \frac{(1-\alpha+\lambda^{sw})}{\alpha} \leqq 0$. Thus, $p_1^{sw} > 0$ must hold if $-u'(M_1) + \frac{(1-\alpha)}{\alpha} > 0$. This is because $\lambda^{sw} > 0$ cannot reverse the last inequality, as is required by Kuhn-Tucker. It can only be reversed by

$p_1^{sw} > 0$ ensuring $-u'(M_1 - p_1^{sw}) + \frac{(1-\alpha)}{\alpha} = 0$. Now, $-u'(M_1) + \frac{(1-\alpha)}{\alpha} > 0 \iff \frac{(1-\alpha)}{\alpha} > u'(M_1) \iff M_1 > (u')^{-1}[(1-\alpha)/\alpha] \equiv M_\alpha$, by concavity, where $(u')^{-1}$ is the inverse function of u'. So, $M_1 > M_\alpha$ guarantees $p_1^{sw} > 0$, irrespective of whether or not there is positive producer surplus at the optimum. (ii) If the producer earns positive surplus, then the no-loss constraint does not bind and the multiplier on the no-loss constraint is $\lambda^{sw} = 0$. From (2.28)(i), the FOC for p_1^{sw} becomes

$$-u'(M_1 - p_1^{sw}) + \frac{(1-\alpha)}{\alpha} \leqq 0 \iff u'(M_1 - p_1^{sw}) \geqq \frac{(1-\alpha)}{\alpha} \tag{37}$$

Now, a corner solution p_1^{sw} for implies $u'(M_1) \geqq \frac{(1-\alpha)}{\alpha}$. I.e., by concavity, a corner solution for p_1^{sw} implies $M_1 \leqq M_\alpha \equiv (u')^{-1}[(1-\alpha)/\alpha]$. ■

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