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**Constrained Optimization Methods in Health Services Research – An Introduction: Report 1 of the ISPOR Optimization Methods Emerging Good Practices Task Force**

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19 **Abstract**

20 Providing health services with the greatest possible value to patients and society given the constraints  
21 imposed by patient characteristics, health care system characteristics, budgets, etc. relies heavily on the  
22 design of structures and processes. Such problems are complex and require a rigorous and systematic  
23 approach to identify the best solution. Constrained optimization is a set of methods designed to identify  
24 efficiently and systematically, the best solution (the optimal solution) to a problem characterized by a  
25 number of potential solutions in the presence of identified constraints. This report identifies: 1) key  
26 concepts and the main steps in building an optimization model; 2) the types of problems where optimal  
27 solutions can be determined in real world health applications and 3) the appropriate optimization  
28 methods for these problems. We first present a simple graphical model based upon the treatment of  
29 “regular” and “severe” patients, which maximizes the overall health benefit subject to time and budget

30 constraints. We then relate it back to how optimization is relevant in health services research for  
31 addressing present day challenges. We also explain how these mathematical optimization methods relate  
32 to simulation methods, to standard health economic analysis techniques, and to the emergent fields of  
33 analytics and machine learning.

34 **Keywords:** Decision making, care delivery, policy, modeling

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## 35 1. Introduction

36 In common vernacular, the term “optimal” is often used loosely in health care applications to refer to any  
37 demonstrated superiority among a set of alternatives in specific settings. Seldom is this term based on  
38 evidence that demonstrates such solutions are, indeed, *optimal* – in a mathematical sense. By “optimal”  
39 solution we mean the *best possible solution* for a given problem given the complexity of the system inputs,  
40 outputs/outcomes, and constraints (budget limits, staffing capacity, etc.). Failing to identify an “optimal”  
41 solution represents a missed opportunity to improve clinical outcomes for patients and economic  
42 efficiency in the delivery of care.

43  
44 Identifying optimal health system and patient care interventions is within the purview of mathematical  
45 optimization models. There is a growing recognition of the applicability of constrained optimization  
46 methods from operations research to health care problems. In a review of the literature [1], note more  
47 than 200 constrained optimization and simulation studies in health care. For example, constrained  
48 optimization methods have been applied in problems of capacity management and location selection for  
49 both healthcare services and medical supplies [2-5].

50 Constrained optimization is an interdisciplinary subject, cutting across the boundaries of mathematics,  
51 computer science, economics and engineering. Analytical foundations for the techniques to solve the  
52 constrained optimization problems involving continuous, differentiable functions and equality constraints  
53 were already laid in the 18<sup>th</sup> century [6]. However, with advances in computing technology, constrained  
54 optimization methods designed to handle a broader range of problems trace their origin to the  
55 development of the simplex algorithm--the most commonly used algorithm to solve linear constrained  
56 optimization problems--in 1947 [7-11]. Since that time, a variety of constrained optimization methods  
57 have been developed in the field of operations research and applied across a wide range of industries. This  
58 creates significant opportunities for the optimization of health care delivery systems and for providing  
59 value by transferring knowledge from fields outside the health care sector.

60 In addition to capacity management, facility location, and efficient delivery of supplies, patient scheduling,  
61 provider resource scheduling, and logistics are other substantial areas of research in the application of  
62 constrained optimization methods to healthcare [12-16]. Constrained optimization methods may also be  
63 very useful in guiding clinical decision-making in actual clinical practice where physicians and  
64 patients face constraints such as proximity to treatment centers, health insurance benefit designs,  
65 and the limited availability of health resources.

66 Constrained optimization methods can also be used by health care systems to identify the optimal  
67 allocation of resources across interventions subject to various types of constraints [17-23]. These methods  
68 have also been applied to disease diagnosis [24, 25], the development of optimal treatment algorithms  
69 [26, 27], and the optimal design of clinical trials [28]. Health technology assessment using tools from  
70 constrained optimization methods is also gaining popularity in health economics and outcomes research  
71 [29].

72 Recently, the ISPOR Emerging Good Practices Task Force on Dynamic Simulation Modeling Applications  
73 in Health Care Delivery Research published two reports in *Value in Health* [30, 31] and one in  
74 *Pharmacoeconomics* [32] on the application of dynamic simulation modeling (DSM) to evaluate problems  
75 in health care systems. While simulation can provide a mechanism to evaluate various scenarios, by  
76 design, they do not provide optimal solutions. The overall objective of the ISPOR Emerging Good  
77 Practices Task Force on Constrained Optimization Methods is to develop guidance for health services  
78 researchers, knowledge users and decision makers in the use of operations research methods to optimize  
79 healthcare delivery and value in the presence of constraints. Specifically, this task force will (1) introduce  
80 constrained optimization methods for conducting research on health care systems and individual-level  
81 outcomes (both clinical and economic); (2) describe problems for which constrained optimization

82 methods are appropriate; and (3) identify good practices for designing, populating, analyzing, testing and  
 83 reporting results from constrained optimization models.

84 The ISPOR Emerging Good Practices Task Force on Constrained Optimization Methods will produce two  
 85 reports. In this first report, we introduce readers to constrained optimization methods. We present  
 86 definitions of important concepts and terminology, and provide examples of health care decisions where  
 87 constrained optimization methods are already being applied. We also describe the relationship of  
 88 constrained optimization methods to health economic modeling and simulation methods. The second  
 89 report will present a series of case studies illustrating the application of these methods including model  
 90 building, validation, and use.

91 **2. Definition of Constrained Optimization**

92  
 93 Constrained optimization is a set of methods designed to efficiently and systematically find the best  
 94 solution to a problem characterized by a number of potential solutions in the presence of identified  
 95 constraints. It entails maximizing or minimizing an objective function that represents a quantifiable  
 96 measure of interest to the decision maker, subject to constraints that restrict the decision maker’s freedom  
 97 of action. Maximizing/minimizing the objective function is carried out by systematically selecting input  
 98 values for the decision from an allowed set and computing the objective function, in an iterative manner,  
 99 until the decision yields the best value for the objective function, a.k.a optimum. The decision that gives  
 100 the optimum is called the “optimal solution”. In some optimization problems, two or more different  
 101 decisions may yield the same optimum. Note that, *programming* and *optimization* are often used as  
 102 interchangeable terms in the literature, e.g., linear programming and linear optimization. Historically,  
 103 programming referred to the mathematical description of a plan/schedule, and optimization referred to  
 104 the process used to achieve the optimal solution described in the program.

105  
 106 The components of a constrained optimization problem are its objective function(s), its decision  
 107 variable(s) and its constraint(s). The **objective function** is a function of the decision variables that  
 108 represents the quantitative measure that the decision maker aims to minimize/maximize. **Decision**  
 109 **variables** are mathematical representation of the constituents of the system for which decisions are being  
 110 taken to improve the value of the objective function. The **constraints** are the restrictions on decision  
 111 variables, often pertaining to resources. These restrictions are defined by equalities/inequalities involving  
 112 functions of decision variables. They determine the allowable/feasible values for the decision variables. In  
 113 addition, **parameters** are constant values used in objective function and constraints, like the multipliers  
 114 for the decision variables or bounds in constraints. Each parameter represents an aspect of the decision-  
 115 making context: for example, a multiplier may refer to the cost of a treatment.

116 **3. A Simple Illustration of a Constrained Optimization Problem**

117  
 118 Imagine you are the manager of a health care center, and your aim is to benefit as many patients as  
 119 possible. Let us say, for the sake of simplicity, you have two types of patients-- regular and severe patients,  
 120 and the demand for the health service is unlimited for both of these types. Regular patients can achieve  
 121 two units of health benefits and severe ones can achieve three units. Each patient, irrespective of severity,  
 122 takes 15 minutes for consultation; only one patient can be seen at any given point in time. You have one  
 123 hour of total time at your disposal. Regular patients require \$25 of medications, and severe patients  
 124 require \$50 of medications. You have a total budget of \$150. What is the greatest health benefit this  
 125 center can achieve given these inputs and constraints?

126 At the outset, this problem seems straightforward. One might decide on four regular patients to use up all  
 127 the time that is available. This will achieve eight units of health benefit while leaving \$50 as excess budget.  
 128 An alternate approach might be to see as many severe patients as possible since treating each severe  
 129 patient generates more per capita health benefits. Three patients (totaling \$150) would generate 9 health

130 units leaving 15 minutes extra time unused. There are other combinations of regular and severe patients  
131 that would generate different levels of health benefits and use resources differently.

132 This is graphically represented in Figure 1, with regular patients on the x-axis and the severe patients on  
133 the y-axis. Line CF is the time constraint limiting total time to one hour. Line BG is the budget constraint  
134 limiting to \$150. Any point to the south-west of these constraints (lines) respectively, will ensure that time  
135 and budget do not exceed the respective limits. The combination of these together with non-negativity of  
136 the decision variables, gives the feasible region.

137 The lines AB-BD-DF-FA form the boundary of the feasibility space, shown shaded in the figure. In  
138 problems that are three or more dimensional, these lines would be hyperplanes. To obtain the optimal  
139 solution, the dashed line is established, the slope depends on the relative health units of the two decision  
140 variables (i.e., the number of regular and severe patients seen). This dashed line moves from the origin in  
141 the north-east direction as shown by the arrow. The optimal solution is two regular patients and two  
142 severe patients. This approach uses the entire one-hour time as well as the \$150 budget. Since regular and  
143 severe patients achieve two- and three-unit health benefits, respectively, we are able to achieve 10 units of  
144 health benefit and still meet the time and budget constraints.

145 No other combination of patients is capable of achieving more benefits while still meeting the time and  
146 budget constraints. Note that not all resource constraints have to be completely used to attain the optimal  
147 solution. This hypothetical example is a small-scale problem with only two decision variables; the number  
148 of regular and severe patients seen. Hence, they can be represented graphically with one variable on each  
149 axis.

150 With the difficulty in representing larger problems graphically, we turn to mathematical approaches, such  
151 as the simplex algorithm to find the solutions. The simplex algorithm is a structured approach of  
152 navigating the boundary (represented as lines in two dimensions and hyperplanes in three or more  
153 dimensions) of the feasibility space to arrive at the optimal solution. Table 1 summarizes the main  
154 components of the example and notes several other dimensions of complexity (linear vs nonlinear,  
155 deterministic vs stochastic, static vs dynamic, discrete/integer vs continuous) that can be incorporated  
156 into constrained optimization models.

157

158 **Figure 1. Graphical Representation of Solving a Simple Integer Programming Problem**

159

160 The mathematical formulation of the model is as follows:

161

162 Max  $f_R x_R + f_L x_L$  (objective function)  
 163 subject to  $c_R x_R + c_L x_L \leq B$  (budget constraint)  
 164  $t_R x_R + t_L x_L \leq T$  (time constraint)  
 165  $x_R, x_L \geq 0$  and integer (decision variables)

166

167 Where:

168  $c_R, c_L$  = cost of regular and severe patients, respectively

169  $B$  = total budget available

170  $t_R, t_L$  = time to see regular and severe patients, respectively

171  $T$  = total time available

172  $f_R, f_L$  = health benefits of regular and severe patients, respectively

173  $x_R, x_L$  = number of regular and severe patients, respectively

174

175 In the current version of the problem, the parameters are:

176  $f_R = 2$  health benefit units,  $f_L = 3$  health benefit units

177  $c_R = \$25$ ,  $c_L = \$50$ ,  $B = \$150$

178  $t_R = 0.25$  hours,  $t_L = 0.25$  hours,  $T = 1$  hour

179

180 So the model is as follows:

181

182 Max  $2x_R + 3x_L$  (objective function)  
 183 subject to  $25x_R + 50x_L \leq 150$  (budget constraint)  
 184  $0.25x_R + 0.25x_L \leq 1$  (time constraint)  
 185  $x_R, x_L \geq 0$  and integer

186

187 As described above, Figure 1 illustrates the graphical solution to this model. However, problems with  
 188 higher dimensionality must use mathematical algorithms to identify the optimal solution. The problem  
 189 described above falls into the category of *linear optimization*, because although the constraints and the  
 190 objective function are linear from an algebraic standpoint, the decision variables must be in the form of  
 191 integers. As it will be discussed further in section 5, there are other optimization modelling frameworks,  
 192 such as combinatorial, nonlinear, stochastic and dynamic optimization.

193 As the algorithms for *integer optimization* problems can take much longer to solve computationally than  
 194 those for linear optimization problems, one alternative is to set the integer optimization problem up and  
 195 solve it as a linear one. If fractional values are obtained, the nearest feasible integers can be used as the  
 196 final solution. This should be done with caution, however. First, rounding the solution to the nearest  
 197 integers can result in an infeasible solution or, and second, even if the rounded solution is feasible, it may  
 198 not be the optimal solution to the original integer optimization problem. *Nonlinear optimization* is  
 199 suitable when the constraints or the objective function are non-linear. In problems, where there is  
 200 uncertainty, such as the estimated health benefit of each patient might receive in the above example,  
 201 *stochastic optimization techniques* can be used.

202 *Dynamic optimization (known commonly as dynamic programming)* formulation might be useful when  
 203 the optimization problem is not static, that the problem context and parameters change in time and there  
 204 is an interdependency among the decisions at different time periods (for instance, when decisions made at  
 205 a given time interval, say number of patients to be seen now, affects the decisions for other time periods,

206 such as the number of patients to be seen tomorrow). Table 1 summarizes the model components in the  
 207 hypothetical problem, relates it to health services with examples and identifies the specific terminology.

208 **Table 1. Model Summary and Extensions**

209

210 **4. Problems That Can Be Tackled with Constrained Optimization Approaches**

211

212 In this section, we list several areas within health care where constrained optimization methods have been  
 213 used in health services. The selected examples do not represent a comprehensive picture of this field, but  
 214 provide the reader a sense of what is possible. In Table 2, we compare problems using the terminology of  
 215 the previous section, with respect to decision makers, decisions, objectives, and constraints.

216 **Table 2. Examples of Health Care Decisions for which Constrained Optimization is**  
 217 **Applicable**

218 **5. Steps in a Constrained Optimization Process**

219

220 An overview of the main steps involved in a constrained optimization process [33] is described here and  
 221 presented in Table 3. Some of the steps are common to other types of modeling methods. It is important to  
 222 emphasize that the process of optimization is iterative, rather than comprising a strictly sequential set of  
 223 steps.

224 **a) Problem structuring**

225

226 This involves specifying the objective, i.e. goal, and identifying the decision variables, parameters and the  
 227 constraints involved. These can be specified using words, ideally in non-technical language so that the  
 228 optimization problem is easily understood. This step needs to be performed in collaboration with all the  
 229 relevant stakeholders, including decision makers, to ensure all aspects of the optimization problem are  
 230 captured. As with any modeling technique, it is also crucial to surface key modeling assumptions and  
 231 appraise them for plausibility and materiality.

232 **b) Mathematical formulation**

233

234 After the optimization problem is specified in words, it needs to be converted into mathematical notation.  
 235 The standard mathematical notation for any optimization problem involves specifying the objective  
 236 function and constraint(s) using decision variables and parameters. This also involves specifying whether  
 237 the goal is to maximize or minimize the objective function. The standard notation for any optimization  
 238 problem, assuming the goal is to maximize the objective, is as shown below:

239 Maximize  $z=f(x_1, x_2, \dots x_n, p_1, p_2, \dots p_k)$

240 subject to

241  $c_j(x_1, x_2, \dots x_n, p_1, p_2, \dots p_k) \leq C_j$

242 for  $j=1,2,..m$

243 where,  $x_1, x_2, \dots x_n$  are the decision variables,  $f(x_1, x_2, \dots x_n)$  is the objective function; and  $c_j(x_1, x_2, \dots x_n, p_1,$   
 244  $p_2, \dots p_k) \leq C_j$  represent the constraints. Note that the constraints can include both inequality and equality  
 245 constraints and that the objective function and the constraints also include parameters  $p_1, p_2, \dots p_k$ , which  
 246 are not varied in the optimization problem. Specification of the optimization problem in this mathematical  
 247 notation allows clear identification of the type (and number) of decision variables, parameters and the  
 248 constraints. Describing the model in mathematical form will be useful to support model development.

249 **c) Model development**

250

251 The next step after mathematical formulation is model development. Model development involves solving  
 252 the mathematical problem described in the previous step, and often performed iteratively. The model  
 253 should estimate the objective function and the left hand side (LHS) values of the constraints, using the  
 254 decision variables and parameters as inputs. The complexity of the model can vary widely. Similar to  
 255 other types of modeling, the complexity of the model will depend on the outputs required, the level of  
 256 detail included in the model, whether it is linear or non-linear, stochastic or deterministic, static or  
 257 dynamic.

#### 258 **d) Perform model validation**

259  
 260 As with any modeling, it is important to ensure that the model developed represents reality with an  
 261 acceptable degree of fidelity [33]. The requirements of model validation for optimization are more  
 262 stringent than for, for example, simulation models, due to the need for the model to be valid for all  
 263 possible combinations of the decision variables. Thus, appropriate caution needs to be taken to ensure  
 264 that the model assumptions are valid and that the model produces sensible results for the different  
 265 scenarios. At the very least, the validation should involve checking of the face validity (i.e. experts evaluate  
 266 model structure, data sources, assumptions, and results), and verification or internal validity (i.e. checking  
 267 accuracy of coding).

#### 268 **e) Select optimization method**

269 This step involves choosing the appropriate optimization method, which is dependent on the type of  
 270 optimization problem that is addressed. Optimization problems can be broadly classified, depending  
 271 upon the nature of the objective functions and the constraints—for example, into linear vs non-linear,  
 272 deterministic vs stochastic, continuous vs discrete, or single vs multi-objective optimization. For instance,  
 273 if the objective function and constraints consist of linear functions only, the corresponding problem is a  
 274 linear optimization problem. Similarly, in deterministic optimization, the parameters used in the  
 275 optimization problem are fixed while in stochastic optimization, uncertainty is incorporated. Optimization  
 276 problems can be continuous (i.e. decision variables are allowed to have fractional values) or discrete (for  
 277 example a hospital ward may be either open or closed; the number of CT scanners which a hospital buys  
 278 must be a whole number).

279 Most optimization problems have a single objective function, however when optimization problems have  
 280 multiple conflicting objective functions, they are referred to as multi-objective optimization problems. The  
 281 optimization method chosen needs to be in line with the type of optimization problem under  
 282 consideration. Once the optimization problem type is clear (e.g. discrete or nonlinear), a number of texts  
 283 may be consulted for details on solution methods appropriate for that problem type [33-36].

284 Broadly speaking, optimization methods can be categorized into *exact approaches* and *heuristic*  
 285 *approaches*. Exact approaches iteratively converge to an optimal solution. Examples of these include  
 286 simplex methods for linear programming and the Newton method or interior point method for non-linear  
 287 programming [34, 37]. Heuristic approaches provide approximate solutions to optimization problems  
 288 when an exact approach is unavailable or is computationally expensive. Examples of these techniques  
 289 include relaxation approaches, evolutionary algorithms (such as genetic algorithms), simulated annealing,  
 290 swarm optimization, ant colony optimization, and tabu-search. Besides these two approaches (i.e. exact or  
 291 heuristic), other methods are also available to tackle large-scale problems as well (e.g. decomposition of  
 292 the large problems to smaller sub-problems).

293 There are software programs that help with optimization; interested readers are referred to the website of  
 294 INFORMS ([www.informs.org](http://www.informs.org)) for a list of optimization software. The users need to specify, and more  
 295 importantly understand, the parameters used as an input for these optimization algorithms (e.g., the  
 296 termination criteria such as the level of convergence required or the number of iterations).

297 **f) Perform optimization/sensitivity analysis**

298 Optimization involves systematically searching the feasible region for values of decision variables and  
 299 evaluating the objective function, consecutively, to find a combination of decision variables that achieve  
 300 the maximum or minimum value of the objective function, using specific algorithms. Once the  
 301 optimization algorithm has finished running, in some cases, the identified solution can be checked to  
 302 verify that it satisfies the “optimality conditions” (i.e. Karush-Kuhn-Tucker conditions) [38], which are the  
 303 mathematical conditions that define the optimality. Once the optimality is confirmed, the results need to  
 304 be interpreted.

305 First, the results should be checked to see if there is actually a feasible solution to the optimization  
 306 problem, i.e. whether there is a solution that satisfies all the constraints. If not, then the optimization  
 307 problem needs to be adjusted, (e.g., relaxing some constraints or adding other decision variables) in order  
 308 to broaden the feasible solution space. If a feasible optimal solution has been found, the results need to be  
 309 understood – this involves interpretation of the results to check whether the optimal solution, i.e., values  
 310 of decision variables, constraints and objective function makes sense.

311 It is also good practice to repeat the optimization with different sets of starting decision variables to  
 312 ensure the optimal solution is the global optimum rather than local optimum. Sometimes, there may be  
 313 multiple optimal solutions for the same problem (i.e. multiple combinations of decision variables that  
 314 provide the same optimal value of objective function). For multi-objective optimization problems (i.e.  
 315 problems with two or more conflicting objectives), Pareto optimal solutions are constructed from which  
 316 optimal solution can be identified based on the subjective preferences of the decision maker [39, 40].

317 It is good practice to run the optimization problem using different values of parameters, in order to verify  
 318 the robustness of the optimization results. Sensitivity analysis is an important part of building confidence  
 319 in an optimization model, addressing the structural and parametric uncertainties in the model by  
 320 analyzing how the decision variables and optimum value react to changes in the parameters in the  
 321 constraints and objective function, which ensures that the optimization model and its solution are good  
 322 representations of the problem at hand.

323 Sometimes a solution may be the mathematically optimal solution to the specified mathematical problem,  
 324 but may not be practically implementable. For example, the “optimal” set of nurse rosters may be  
 325 unacceptable to staff as it involves breaking up existing teams, deploying staff with family responsibilities  
 326 on night shifts, or reducing overtime pay to level where the employment is no longer attractive. Analysts  
 327 should resist the temptation to spring their optimal solution on unsuspecting stakeholders, expecting  
 328 grateful acceptance: rather, those affected by the model should be kept in the loop through the modeling  
 329 process. The optimal solution may come as a surprise: it is important to allow space in the modeling  
 330 process to explore fully and openly concerns about whether the “optimal” solution is indeed the one the  
 331 organization should implement.

332 **g) Report results**

333  
 334 The final optimal solution, and if applicable, the results of the sensitivity analyses should be reported. This  
 335 will include the results of the optimum ‘objective function’ achieved and the set of ‘decision variables’ at  
 336 which the optimal solution is found. Both the numerical values (i.e. the mathematical solution) and the  
 337 physical interpretation, i.e., the non-technical text describing the meaning of numerical values, should be  
 338 presented. The optimal solution identified can be contextualized in terms of how much ‘better’ it is  
 339 compared to the current state. For example, the results can be presented as improvement in benefits such  
 340 as QALYs or reduction in costs.  
 341

342 It is often necessary to report the optimization method used and the results of the ‘performance’ of the  
 343 optimization algorithm, e.g., number of iterations to the solution, computational time, convergence level,  
 344 etc. This is important as it helps users understand whether a particular algorithm can be used “online” in a  
 345 responsive fashion, or only when there is significant time available, e.g. in a planning context. Dashboards  
 346 can be useful to visualize these benefits and communicate the insights gained from the optimal solution  
 347 and sensitivity analyses.  
 348

#### 349 **h) Decision making**

350

351 The final optimal solution and its implications for policy/service reconfiguration should be presented to all  
 352 the relevant stakeholders. This typically involves a plan for amending the ‘decision variables’, (e.g., shift  
 353 patterns, screening frequency--see Table 2 for examples of decision variables--to those identified in the  
 354 optimal solution). Before an optimal solution can be implemented, it will require getting the ‘buy-in’ from  
 355 the decision makers and all the stakeholders, e.g., frontline staff such as nurses, hospital managers, etc., to  
 356 ensure that the numerical ‘optimal’ solution found can be operationalized in a ‘real’ clinical setting. It is  
 357 important to have the involvement of decision makers throughout the whole optimization process to  
 358 ensure that it does not become a purely numerical exercise, but rather something that is implemented in  
 359 real life. After the decision is made, data should still be collected to assess the efficiency and demonstrate  
 360 the benefits of the implementation of the optimal solution.  
 361

362

363 If decision makers are not directly involved in model development they may choose not to implement the  
 364 “optimal” solution as it comes from the model. This is because the model may fail to capture key aspects  
 365 of the problem (for example, the model may maximize aggregate health benefits but the decision maker  
 366 may have a specific concern for health benefits for some disadvantaged subgroup). This does not  
 367 (necessarily) mean that the optimization modeling has not been useful – enabling a decision maker to see  
 368 how much health benefit must be sacrificed to satisfy her equity objective may prove to be beneficial  
 369 towards the overall objective. After the decision is made the story does not come to an end: data should  
 370 continue to be collected to demonstrate the benefits of whatever solution is implemented, as well as  
 371 guiding future decision making.

372

373 Table 3 presents the two different stages in optimization i.e. the modeling stage and optimization stage,  
 374 highlighting that model development is necessary before optimization can be performed. The goal of  
 375 constrained optimization is to identify an optimal solution that maximizes or minimizes a particular  
 objective subject to existing constraints.

#### 376 **Table 3. Steps in an Optimization Process**

377

### 378 **6. Relationship of Constrained Optimization to Related Fields**

379

380 The use of constrained optimization can be classified into two categories. The first category is the use of  
 381 constrained optimization as a decision-making tool. The simple illustration in section 3 and all the  
 382 examples in section 4 are considered to fall under this category. The second category is the use of  
 383 constrained optimization as an auxiliary analysis tool. In this category, optimization is an embedded tool  
 384 and the results of which are often not the end results of a decision problem, but rather they are used as  
 385 inputs for other analysis/modeling methods (e.g. optimization used in the multiple criteria decision  
 386 making; in calibrating the inputs for health economic or dynamic simulation models; in machine learning  
 387 and other statistical analysis methods like solving regression models or propensity score matching).

388 As a decision-making tool, optimization is complementary to other modeling methods such as health  
 389 economic modeling, simulation modeling and descriptive, predictive (e.g. machine learning) and

390 prescriptive analytics. Most modeling methods typically only evaluate a few different scenarios and  
 391 determine a good scenario *within* the available options. In contrast, the aim of optimization methods is to  
 392 efficiently identify the *best* solution overall, given the constraints. In the absence of using optimization  
 393 methods, a brute force approach, in which all possible options are sequentially evaluated and the best  
 394 solution is identified among them, might be possible for some problems. However, for most problems, it is  
 395 too complex and too time consuming to identify and evaluate all possible options. Optimization methods  
 396 and heuristic approaches might use efficient algorithms to identify the optimal solution quickly, which  
 397 would otherwise be very difficult and time consuming.

398 Also, model development using these other methods might be necessary before optimization, especially in  
 399 situations where the objective function or constraints cannot be represented in a simple functional form.  
 400 Thus, all models currently used in health care such as health economic models, dynamic simulation  
 401 models and predictive analytics (including machine learning) can be used in conjunction with  
 402 optimization methods.

403 **a) Constrained Optimization Methods Compared with Traditional Health Economic**  
 404 **Modeling in Health Technology Assessments**

405  
 406 Constrained optimization methods differ substantially from health economic modeling methods  
 407 traditionally used in health technology assessment processes [41]. The main difference between the two  
 408 approaches is that traditional health economic modeling approaches, such as Markov models, are built to  
 409 estimate the costs and effects of different diagnostic and treatment options. If decision makers are basing  
 410 their judgements on modeling results, they may not formally consider the constraints and resource  
 411 implications in the system. Constrained optimization methods provide a structured approach to optimize  
 412 the decision problem and to present the best alternatives given an optimization criterion, such as  
 413 constrained budget or availability of resources.

414 These differences have major implications. There is an opportunity to learn from optimization methods to  
 415 improve Health Technology Assessment (HTA) processes [42-46]. Optimization is a valuable means of  
 416 capturing the dynamics and complexity of the health system to inform decision making for several  
 417 reasons. Constrained optimization methods can:

418 i. Explicitly take budget constraints into account - Informed decision making about resource  
 419 allocation requires an external estimate of the decision-maker's willingness to pay for a unit of  
 420 health outcome – the threshold. Decision making based on traditional health economic models  
 421 then relies on the principle that by repeatedly applying the threshold to individual HTA decisions,  
 422 optimization of the allocation of health resources will be achieved.

423  
 424 However, the focus of health economics (HE) is usually about relative efficiency without explicit  
 425 consideration of budget because many jurisdictions do not explicitly implement a constrained  
 426 budget nor do they employ mechanisms to evaluate retrospectively cost-effectiveness of medical  
 427 technologies currently in use.

428 ii. Address multiple resource constraints in the health system, such as resource capacity: Constrained  
 429 optimization methods also allow consideration of the effect of other constraints in the health  
 430 system, such as capacity or short-term inefficiencies. Capacity constraints are usually neglected in  
 431 health economic models. In HE models, the outcomes are central to decision makers while the  
 432 process to arrive at these outcomes is most of the time ignored.

433  
 434 For health policy makers and health care planners, such capacity considerations are critical and  
 435 cannot be neglected. Likewise, some technologies are known for short-term inefficiencies, e.g.,  
 436 large equipment such as PET-MR imaging, are usually not taken into consideration. It takes a

437 certain amount of time before a new device operates efficiently, and such short-term inefficiencies  
 438 do influence implementation [47].

439 iii. Account for system behavior and decisions over time: Traditional health economic models are  
 440 often limited to informing a decision of a single technology at a single point in time. Health  
 441 economic models with a clinical perspective, such as a whole disease model [48, 49], or a  
 442 treatment sequencing model, may allow the full clinical pathway to be framed as a constrained  
 443 optimization problem that accounts for both intended and unintended consequences of health  
 444 system interventions over time with feedback mechanisms in the system.  
 445

446 Each combination of decisions within the pathway can be a potential solution, constrained by the  
 447 feasibility of each decision, e.g., the licensed indication for various treatments within a clinical  
 448 pathway. These whole disease and treatment sequencing models can evaluate alternative guidance  
 449 configurations and report the performance in terms of an objective function (cost per QALY, net  
 450 monetary benefit) [50, 51].

451 iv. Inform decision makers about implementability of solutions that are recommended: Health  
 452 economic models are not typically constrained – it is assumed that resources are available as  
 453 required and are thus affordable, similarly the evidence used in the models come from controlled  
 454 clinical settings, which are idealized settings compared to real clinical setting. An advantage of  
 455 constrained optimization is the ability to obtain optimal solutions to decision problems and have  
 456 sensitivity analyses performed. Such analyses inform decision makers about alternate realistic  
 457 solutions that are feasible and close to the optimal solution.

458 Thus, in some sense, classic health economics models are ‘hypothetical’ to illustrate the potential value as  
 459 measured by a specific outcome with respect to cost, whereas optimization is focused on what can be  
 460 achieved in an operational context. This suggests constrained optimization methods have great value for  
 461 informing decisions about the ability to implement a clinical intervention, program, or policy as they  
 462 actually consider these constraints in the modeling approach.

463  
 464 **b) Constrained Optimization Methods Compared with Dynamic Simulation Models**

465 Dynamic simulation modeling methods (DSMs), such as system dynamics, discrete event simulation and  
 466 agent based modeling are used to design and develop mathematical representations, i.e., formal models, of  
 467 the operation of processes and systems. They are used to experiment with and test interventions and  
 468 scenarios and their consequences over time in order to advance the understanding of the system or  
 469 process, communicate findings, and inform management and policy design [30-32, 52-54]. These  
 470 methods have been broadly used in health applications [55-57].  
 471

472 Unlike constrained optimization methods, DSMs do not produce a specific solution. Rather they allow for  
 473 the evaluation of a range of possible or feasible scenarios or intervention options that may or may not  
 474 improve the system’s performance. Constrained optimization methods, in general, seek to provide the  
 475 answer to which of those options is the “best”. Hence, the types of problems and questions that can be  
 476 addressed with DSMs [30-32] are different from those that are addressed with optimization methods.  
 477 However, both types of methods can be complementary to each other in helping us to better understand  
 478 systems.

479 Traditionally, constrained optimization methods have served two distinct purposes in DSM development.  
 480 1) model calibration – fitting suitable model variables to past time series is discussed elsewhere [30-32];  
 481 2) evaluating a policy’s performance/effect relative to a criterion or set of criteria. However, the  
 482 complexity of DSMs compared to simple analytic models may render exact constrained optimization  
 483 approaches cumbersome, inappropriate and potentially infeasible due to the large search space e.g., using  
 484 methods of optimal control.

485 Due to this complexity, alternatives to exact approaches such as heuristic search strategies are available.  
 486 Historically, these types of methods have been used in system dynamics and other DSMs. Due to their  
 487 heuristic nature, there is no certainty of finding the “best” or optimal parameter set rather “good enough”  
 488 solutions. Hence, the ranges assigned need careful consideration in order to get “good” solutions, i.e.,  
 489 prior knowledge of sensible ranges both from knowledge about the system and knowledge gained from  
 490 model building.

491 Optimization is used as part of system dynamics to gain insight about policy design and strategy design,  
 492 particularly when the traditional analysis of feedback mechanisms becomes risky due to the large numbers  
 493 of loops in a model [58]. Similar procedures to evaluate policies and strategies can be can be utilized in  
 494 discrete event simulation (DES) and agent based modeling (ABM), e.g., simulated annealing algorithms  
 495 and genetic algorithms.

496 **c) Constrained Optimization Methods as Part of Analytics**

497 Constrained optimization methods fall within the area of analytics as defined by the Institute for  
 498 Operations Research and the Management Sciences (INFORMS, <https://www.informs.org/Sites/Getting-Started-With-Analytics>). Analytics can be classified into: descriptive, predictive and prescriptive analytics  
 499 (Figure 2), and discussed below. Constrained optimization methods are a special form of prescriptive  
 500 analytics.  
 501

- 502 i. Descriptive analytics concern the use of historical data to describe a phenomenon of interest—  
 503 with a particular focus on visual displays of patterns in the data. Descriptive analytics is  
 504 differentiated from descriptive analysis which uses statistical methods to test hypotheses about  
 505 relationships among variables in the data. Health services research typically uses theory and  
 506 concepts to identify hypotheses, and historical data are used to test these hypotheses using  
 507 statistical methods. Examples may include natural history of aging, disease progression,  
 508 evaluation of clinical interventions, policy interventions, and many others. Traditional health  
 509 services for the most part falls within the area of descriptive analytics.
- 510 ii. Predictive analytics and machine learning focus on forecasting the future states of disease or  
 511 states of systems. With the increased volume and dimensions of health care data, especially  
 512 medical claims and electronic medical record data, and the ability to link to other information  
 513 such as feeds from personal devices and socio demographic data, big data methods such as  
 514 machine learning are garnering increased attention [59].  
 515 Machine learning methods, such as predictive modeling and clustering, have an important  
 516 intersection with constrained optimization methods. Machine learning methods are valuable  
 517 for addressing problems involving classification, as well as data dimension reduction issues.  
 518 And maybe most importantly, optimization often needs forecasts and estimates as inputs,  
 519 which can be obtained from the results of machine learning algorithms. A discussion of  
 520 machine learning methods is beyond the scope of this paper.

521 However, the interested reader will find a detailed introduction elsewhere [60, 61]. Machine  
 522 learning has the ability to “mine” data sets and discover trends or patterns. These are often  
 523 valuable to establish thresholds or parameter values in optimization models, where it is  
 524 otherwise difficult to determine the values. Constrained optimization can also leverage the  
 525 ability of machine learning to reduce high dimensionality of data, say with thousands or  
 526 millions of variables to key variables.

- 527 iii. Prescriptive analytics uses the understanding of systems, both the historical and future based  
 528 on historical (descriptive) and predictive analytics respectively to determine future course of  
 529 action/decisions. Traditional (without optimization) clinical trials and interventions fall under  
 530 the category of prescriptive analytics (“Change what will happen” in figure). Constrained  
 531 optimization is a specialized form of prescriptive analytics, since it helps with determining the  
 532 *optimal* decision or course of action in the presence of constraints  
 533 (<https://www.informs.org/Sites/Getting-Started-With-Analytics/Analytics-Success-Stories>).  
 534

**535 Figure 2. Descriptive, Predictive, and Prescriptive Analytics.**

536

**537 7. Summary and Conclusions**

538

539 This is the first report of the ISPOR Constrained Optimization Methods Emerging Good Practices Task  
540 Force. It introduces readers to the application of constrained optimization methods to health care systems  
541 and patient outcomes research problems. Such methods provide a means of identifying the best policy  
542 choice or clinical intervention given a specific goal and given a specified set of constraints. Constrained  
543 optimization methods are already widely used in health care in areas such as choosing the optimal location  
544 for new facilities, making the most efficient use of operating room capacity, etc.

545 However, they have been less widely used for decision making about clinical interventions for patients.  
546 Constrained optimization methods are highly complementary to traditional health economic modeling  
547 methods and dynamic simulation modeling—providing a systematic and efficient method for selecting the  
548 best policy or clinical alternative in the face of large numbers of decision variables, constraints, and  
549 potential solutions. As health care data continues to rapidly evolve in terms of volume, velocity, and  
550 complexity, we expect that machine learning techniques will also be increasingly used for the development  
551 of models that can subsequently be optimized.

552 In this report, we introduce readers to the vocabulary of constrained optimization models and outline a  
553 broad set of models available to analysts for a range of health care problems. We illustrate the  
554 formulation of a linear program to maximize the health benefit generated in treating a mix of “regular”  
555 and “severe” patients subject to time and budget constraints and solve the problem graphically. Although  
556 simple, this example illustrates many of the key features of constrained optimization problems that would  
557 commonly be encountered in health care.

558 In the second task force report, we describe several case studies that illustrate the formulation, estimation,  
559 evaluation, and use of constrained optimization models. The purpose is to illustrate actual applications of  
560 constrained optimization problems in health care that are more complex than the simple example  
561 described in the current paper and make recommendations on emerging good practices for the use of  
562 optimization methods in health care research.

563

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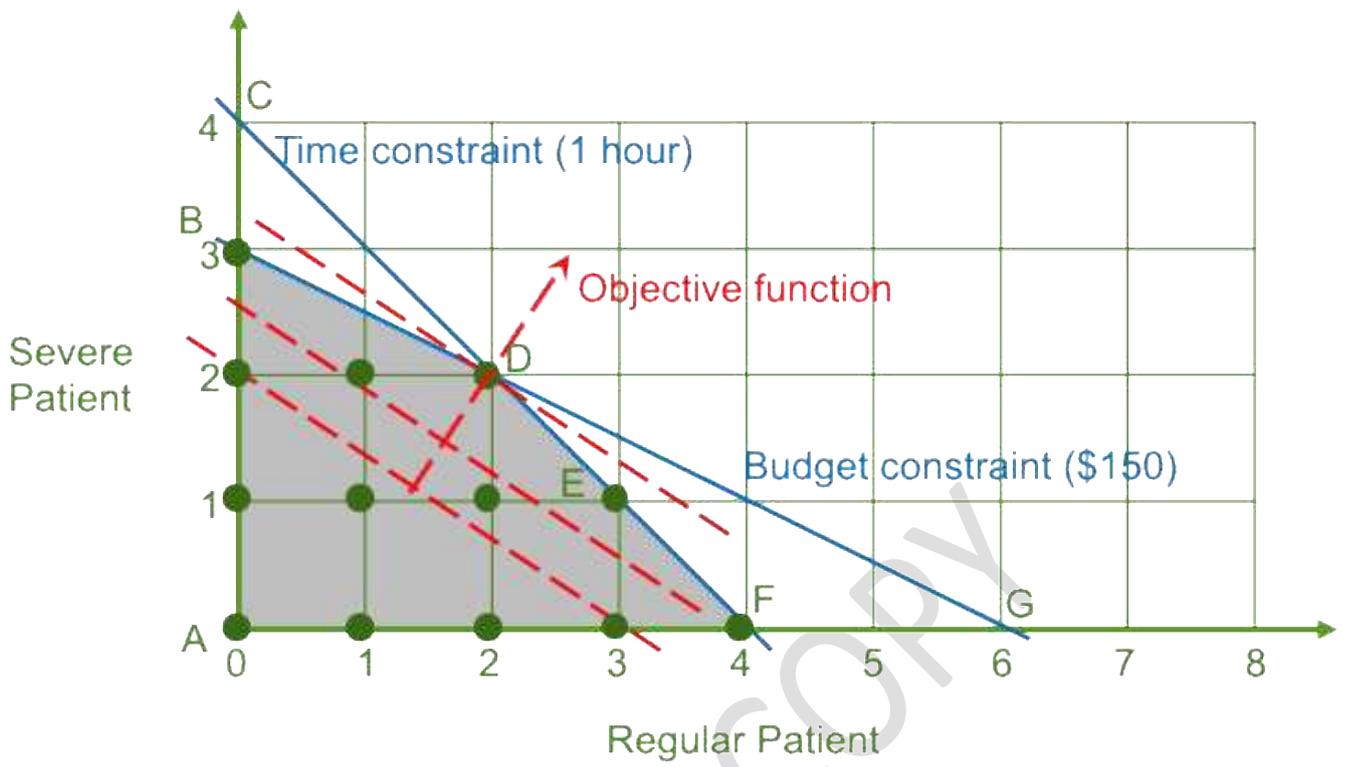
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685 **Figure 1. Graphical Representation of Solving a Simple Integer Programming Problem**

686

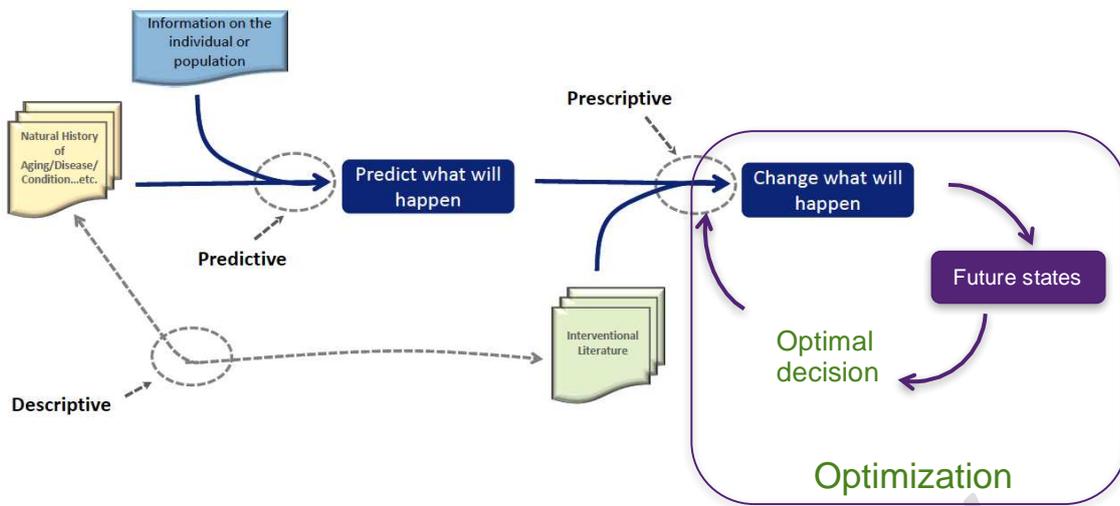


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690 **Figure 2. Descriptive, Predictive, and Prescriptive Analytics.**



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Wilson, ISPOR 2014

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**Table 1. Model Summary and Extensions**

606

	Hypothetical problem	Real-life Health Services	Terminology
<b>Aim</b>	Maximize health/health care benefits	Maximize health/health care benefits	Objective function
<b>Options available</b>	Regular or severe patients	Service lines, case mix, service mix, etc.	Decision variables
<b>Constraints</b>	Total cost $\leq$ \$15 Total time $\leq$ 1 hour	Budget constraint Time constraint Resource constraint (e.g. staff, beds, etc.)	Constraints
<b>Evidence base</b>	Cost of each patient, health benefits of each patient and the time taken for consultation	Costs, health benefits, and other relevant data associated with each intervention to be selected	Model (to determine the objective function and constraints)
<b>Complexity</b>	<p><i>Static</i> The problem does not have a time component; decision made in one time period does not affect decisions made in another</p> <p><i>Deterministic</i> All the information is assumed to be certain (e.g. Cost of each patients, health benefits of each patient and the time taken for consultation)</p> <p><i>Linear</i> (i.e. each additional patient costs the same and achieves same health benefits)</p> <p><i>Integer/discrete</i> The decision variables (number of patients) can only take discrete and integer values</p>	<p><i>Dynamic</i> The optimization problem and parameters may change in different time points, and the decision made at any point in time can affect decisions at later time points (e.g. there can be a capacity constraint defined on 2 months, whereas the planning cycle is 1 month)</p> <p><i>Stochastic</i> Know that the information is uncertain (i.e. uncertainty in the costs and benefits of the interventions)</p> <p><i>Non-linear</i> (objective function or constraints may have a non-linear relationship with the model parameters, e.g. total costs and QALYs typically have a non-linear relationship with the model parameters)</p> <p><i>Continuous</i> The decision variables can take fractional values (e.g. number of hours)</p>	Optimization method

697 **Table 2. Examples of Health Care Decisions for which Constrained Optimization is**  
 698 **Applicable**

<b>Type of health care problem</b>	<b>Typical decision makers</b>	<b>Typical decisions</b>	<b>Typical objectives</b>	<b>Typical constraints</b>
Resource allocation within and across disease programs	Health authorities, insurance funds	List of interventions to be funded	Maximize population health	Overall health budget, other legal constraints for equity
Resource allocation for infectious disease management	Public health agencies, health protection agencies	Optimal vaccination coverage level	Minimize disease outbreaks and total costs	Availability of medicines, disease dynamics of the epidemic
Allocation of donated organs	Organ banks, transplant service centers	Matching of organs and recipients	Maximize matching of organ donors with potential recipients	Every organ can be received by at most one person
Radiation treatment planning	Radiation therapy providers	Positioning and intensity of radiation beams	Minimizing the radiation on healthy anatomy	Tumor coverage and restriction on total average dosage
Disease management models	Leads for a given disease management plan	Best interventions to be funded, best timing for the initiation of a medication, best screening policies	Identify the best plan using a whole disease model, maximizing QALYs	Budget for a given disease or capacity constraints for healthcare providers
Workforce planning/ Staffing / Shift template optimization	Hospital managers, all medical departments (e.g., ED, nursing)	Number of staff at different hours of the day, shift times	Increase efficiency and maximize utilization of healthcare staff	Availability of staff, human factors, state laws (e.g., nurse-to-patient ratios), budget
Inpatient scheduling	Operation room/ ICU planners	Detailed schedules	Minimize waiting time	Availability of beds, staff
Outpatient scheduling	Clinical department managers	Detailed schedules	Minimize over- and under-utilization of health care staff	Availability of appointment slots
Hospital facility location	Strategic health planners	Set of physical sites for hospitals	Ensure equitable access to hospitals	Maximum acceptable travel time to reach a hospital

699

700

701 **Table 3. Steps in an Optimization Process**

<b>Stage</b>	<b>Step</b>	<b>Description</b>
<b>Modeling</b>	Problem structuring	Specify the objective and constraints, identify decision variables and parameters, and list and appraise model assumptions
	Mathematical formulation	Present the objective function and constraints in mathematical notation using decision variables and parameters
	Model development	Develop the model; representing the objective function and constraints in mathematical notation using decision variables and parameters
	Model validation	Ensure the model is appropriate for evaluating all possible scenarios (i.e. different combinations of decision variables and parameters)
<b>Optimization</b>	Select optimization method	Choose an appropriate optimization method and algorithm based on the characteristics of the problem
	Perform optimization/sensitivity analysis	Use the optimization algorithm to search for the optimal solution and examine performance of optimal solution for reasonable values of parameters
	Report results	Report the results of optimal solution and sensitivity analyses
	Decision making	Interpret the optimal solution and use it for decision making

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