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Evaluating currency crises: 
A multivariate Markov regime switching approach

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Abstract:

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JEL: C32, F31

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Evaluating Currency Crises: A Multivariate Markov Regime Switching Approach

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August 18, 2010

Abstract

This paper provides an empirical framework to analyse the nature of currency crises by extending earlier work of Jeanne and Masson (2000) who suggest that a currency crisis model with multiple equilibria can be estimated using Markov regime switching (MRS) models. However, Jeanne and Masson (2000) assume that the transition probabilities across equilibria are constant and independent of fundamentals. Thus, currency crisis is driven by a sunspot unrelated to fundamentals. This paper further contributes to the literature by suggesting a multivariate MRS model to analyse the nature of currency crises. In the new set up, one can test for the impact of the unobserved dynamics of fundamentals on the probability of devaluation. Empirical evidence shows that expectations about fundamentals, which are reflected by their unobserved state variables, not only affect the probability of devaluation but also can be used to forecast a currency crisis one period ahead.

1 Introduction

The European Monetary System crises in 1992-93, the Mexican 'peso' crisis in 1994, the Asian 'flu' crisis in 1997, the Russian crisis in 1998, and the Brazilian crisis in 1999 raise concerns about the nature of currency crises. There are two main models for currency crises. The first generation model, introduced by Krugman (1979), is based on the idea that currency crisis is driven by bad fundamentals such as expansionary monetary and fiscal policy, which lead to

\textsuperscript{*}We would like to thank Alessandro Flamini and Andy Dickerson, Corresponding author: Kostas Mouratidis, Department of Economics; Email: k.Mouratidis@sheffield.ac.uk
a persistent loss of foreign exchange reserves that ultimately force monetary authorities to abandon the fixed exchange rate regime.

In the second generation crises model, the government is not a simple mechanism but, rather, an agent that minimises a loss function. There are two new elements in the second generation crises model. Firstly, it considers a wider range of fundamentals than the first generation crises model. Secondly, it endogenises monetary policy by introducing market expectations. The second generation crises model emphasises that the contingent nature of economic policies may produce multiple equilibria and generate self-fulfilling crises. Thus, the economy can be in equilibrium which is consistent with fixed exchange rate regime and sudden change of expectations may force policy makers to abandon the peg, thereby validating agent’s expectations. An important implication of the second generation crises model is that predicting currency crises becomes a more difficult task. This is so because currency crises may occur without any change of fundamentals. The key element that differentiates the two models of currency crises is the disconnection of fundamentals from market expectations.

Jeanne (1997, 2000), in his so-called escape-clause model, provides a theoretical reconciliation between the first and second generation crises models. Although the first generation model argues that a currency crisis is due to expansionary monetary and fiscal policies, followed by policy makers, it has not addressed the question about the introduction of these policies in the first place. Jeanne (2000) shows that, in the first generation model of currency crises, there is always a level of interest rate that monetary authorities can adopt to defend the peg. However, high interest rate will have negative macroeconomic consequences. This implies that by endogenising monetary and fiscal policies in the first generation crises model, the logic of the two models is the same. Jeanne (2000) also argues that the two models of currency crises are not mutually exclusive. For a currency to be subject to self-fulfilling speculative attack, fundamentals must firstly put the currency into the crisis zone. The precise time and occurrence of an attack is difficult to be determined only on the basis of fundamentals.

Jeanne and Masson (2000) show that strategic complementarity between market expectations, about the intended policy rule and the policy actually adopted, produces multiple equilibria. Jeanne (1997) and Jeanne and Masson (2000) argue that what drives market expectations and the economy, from one equilibrium to another, is an external event unrelated to fundamentals. This is a sunspot which reflects waves of pessimism and optimism. Jeanne and Masson (2000) show that devaluation expectations are the sum of the probabilities of devaluation in the next period, weighted by the transition probabilities that the state of economy switches from the current to future states, that is

---


2. The logic of self-fulfilling crises is based on the idea that devaluation expectation increases the cost of retaining a peg and, therefore, the desire of the policy-maker to devalue.
\[
q_t = \sum_{s=1}^{n} p_{ij} F(Z_t, Z_{t+1}^s)
\]

where

\[
F(Z_t, Z_t^s) = \Pr[Z_{t+1} < Z_t^s | Z_t = Z]
\]

\[
F_z(Z_t, Z_t^s) \leq 0
\]

\(p_{ij} = P(S_{t+1} = j | S_t = i)\) denotes the transition probability of the unobserved state variable \(S_t\). The state variable \(\{S_t\}\) is assumed to form a Markov chain on \(\Omega = \{1, \ldots, n\}\). \(F(Z_t, Z_t^s)\) captures the observed dynamics of fundamentals dynamics. Alternatively, \(S_t\) indicates the unobserved state of the probability of devaluation.

Jeanne and Masson (2000) suggest a MRS model to apply models of currency crises with multiple equilibria to data. However, the empirical work of Jeanne and Mason (2000) has two shortcomings. Firstly, it does not allow the unobserved state of probability of devaluation to be a function of fundamentals. Secondly, it evaluates currency crises using in-sample fit comparison of the MRS model and a linear model. In terms of the first shortcoming, Filardo (1994), Diebold et. al. (1994), and Filardo and Gordon (1998) show that the unobserved state variable of a Markov process may be time varying, related to a group of fundamental variables. The external uncertainty that drives the devaluation probability across different equilibria is not independent of fundamentals. For the second shortcoming, an in-sample forecast comparison of linear and non-linear models is not fair, since the latter are over-parameterised. Empirical studies such as Clements and Smith (1999) and Diebold and Nason (1990) also show that linear models often outperform non-linear models in out-of-sample forecast comparison.

Mouratidis (2008) takes on board both these shortcomings by comparing the forecast performance of a Markov switching model with time-varying transition probabilities (MRS-TVP) with both a constant transition probabilities (MRS) model and a linear benchmark. However, the MRS-TVP model used by Mouratidis (2008) only partially takes into account the impact of fundamentals on the unobserved state of probability of devaluation. This is so because the MRS-TVP model only considers the impact of the observed dynamics of fundamentals on the unobserved state of probability of devaluation. In terms of (1), the MRS-TVP model estimates the impact of \(Z_t\) on \(S_t\). However, recent economic literature shows that fundamentals may also follow a Markov process. This implies that \(S_t\) is a vector and not a scalar. Assuming that \(Z_t\) includes a single fundamental variable (say \(X_t\)), then \(S_t = [S_t^f S_t^e]\) where \(S_t^f\) is the the unobserved state of probability of devaluation and \(S_t^e\) is the unobserved state of fundamental variable \(X_t\). The later unobserved state reflects expectations about the fundamental variable \(X_t\). Phillips (1991) shows that \(S_t^f\) and \(S_t^e\) can be either independent or perfectly correlated or being in a different phase. The implication of the framework suggested by Phillips (1991) was that both the ob-
served and unobserved dynamics of fundamentals might affect the unobserved state of probability of devaluation.

The aim of this paper is to provide an empirical framework to analyse the nature of currency crises. We do so by employing the multivariate MRS model suggested by Phillips (1991). We allow both the observed dynamics and the unobserved state of fundamentals to affect the probability of devaluation. If the unobserved state of fundamentals has a significant impact on the unobserved state of probability of devaluation, then market expectation about the probability of devaluation is not unrelated to fundamentals. This is consistent with the escape-clause model suggested by Jeanne (1997, 2000).

We analyse the nature of currency crises by implementing an out-of-sample forecast comparison. We compare MRS models with independent unobserved states and MRS models with either perfectly correlated states or unobserved states that lead each other. This comparison will show whether currency crisis is driven by a sunspot or by a combination of market expectation and fundamentals. We also compare MRS models to linear models. If MRS models outperform linear models, then currency crisis is in line with the second generation model of currency crises.

Empirical results of this paper are based on the Italian currency crisis in 1992 and the speculative attack on the French franc in July 1993. There is a widespread agreement that the former currency crisis was driven by bad fundamentals while the latter was driven by a sunspot unrelated to fundamentals. De Grauwe (1997) reports that the Italian lira was overvalued by between 25 and 30% in 1992, and the choice for Italy was either to deflate its economy or to use a large realignment once the large capital inflows, that had financed the current account deficit until then, came to an end. Deterioration of Italian fundamentals before the speculative attack in 1992 raises the question whether the Italian currency crisis was in line with the first generation of currency crises model rather than with currency crises driven by sunspots. In 1993, currency markets questioned the exchange rates within the core of the European Monetary System (EMS), where there was no sign of systematic over-valuation of the Belgian or French francs and the Danish krone. Jeanne and Masson (2000) show that the speculative attack on the French franc in 1993 was driven by an external uncertainty unrelated to fundamentals.

The paper proceeds by introducing the econometric methodology adopted to analyse the Italian and French currency crises. Section 3 explains data and empirical results from application to the Italian and French currency crises in 1992 and 1993, respectively. The final section summarises and concludes.

2 Econometric Methodology

This section describes the methodology used by Jeanne and Masson (2000) to estimate currency crises models with multiple equilibria. They use a uni-variate

\footnote{Sola et al. (2007) use a general bivariate MRS model, allowing for interaction of both unobserved states and transition probabilities to be time-varying.}
MRS model to give empirical support to the view that a currency crisis is driven by market expectations unrelated to fundamentals. We also present and explain the implication of the model suggested by Phillips (1991).

2.1 Proxy of probability of devaluation

In this paper, in line with Gomez-Puig and Montalvo (1997), we use the interest rate differential as a proxy for devaluation expectations, and consequently as an indicator of credibility of a target zone exchange rate regime. This can be justified by the simple Uncovered Interest Rate Parity (UIP) model:

$$r_t^D - r_t^G = E_t \Delta s_{t+1}$$

(2)

where $r_t^D$ and $r_t^G$ are, respectively, the domestic (Italian or French) and the German interest rates. $E_t \Delta s_{t+1}$ denotes the expected rate of depreciation at time $t + 1$ given the information at time $t$. (2) is based on the assumption that foreign exchange market risk premium is small. Svensson (1992) argues that for a reasonable level of risk aversion a zero risk premium is a reasonable assumption for target zone models. Ayuso and Restoy (1992) obtain small estimates for the risk premium component of the EMS countries. This is so because the risk premium could be diversified in the EMS.

Eichengreen et al. (1996), Mouratidis (2008), and Fratzcher (2003) employ the actual exchange market pressure (EMP) as an indicator of devaluation probability. This is the weighted average of the change of exchange rate, the change of interest rate differential, and the change of foreign exchange reserves:

$$EMP_t = w_1 \Delta s_t + w_2 \Delta (r_t^D - r_t^G) - w_3 \Delta R_t$$

(3)

where the weights, $w$‘s, are calculated as the inverse of the series variance. EMP indicates that if the central bank faces a currency pressure can either devalue, or increase interest rates, and/or reduce foreign exchange reserves ($R$).

The interest rate differential is the product of the probability of devaluation and its size:

$$r_t^D - r_t^G = q_t \Delta s_{t+1}.$$ 

If $\Delta s_{t+1}$ is chosen to be a reasonable value, say a conventional 10 or 20 % (or the average of historical devaluations), and with interest rate differential much lower, it is very unlikely that the estimate of the probability, that is $q_t = \frac{r_t^D - r_t^G}{\Delta s_{t+1}}$, would even approach unity, much less exceed it. As for the exchange market

---

4Jeanne and Masson (2000) also use the interest rate differential as a proxy of devaluation probability, investigating the nature of the French currency crises in 1993.

5Giavazzi and Pagano (1988) argue that "tying their hands" the authorities of a high inflation country lower the output cost of disinflation. In view of this, the EMS was used as a mechanism to transfer credibility from Germany to other EMS countries. Therefore, we use Germany as the reference country to evaluate monetary policy of the rest EMS countries.

6The model itself, which relies on the cumulative distribution, will produce a probability bounded by 0 and 1. Jeanne (1997) shows that

$$q_t = \mu F(\gamma q_t - Z_t),$$

(4)
pressure, we define it as \( q_t = (EMP_t - w_1 \Delta s_t + w_2 \Delta R_t) / w_3 \Delta s_{t+1} \). Thus, one has a more difficult task in mapping it into a probability measure, since it also includes a variable, reserves, that has no particular relation to a devaluation probability.

### 2.2 MRS Model

Jeanne and Masson (2000) employ the following MRS model to explain the probability of devaluation in the speculative attack of the French franc in 1993:

\[
q_t = \alpha_{s_t} + \beta' Z_t + \epsilon_t
\]  

(5)

where \( Z_t \) is a vector of indicators and \( s_t \) is the unobserved state of probability of devaluation. They assume that the transition probabilities of \( s_t \) is time invariant. With 2 regimes, the transition between regimes is characterised by a \((2 \times 2)\) transition probability matrix \( p = [p_{ij}] \), with \( i, j = 1, 2 \). Each \( p_{ij} \) gives the transition probability that regime \( i \) will be followed by regime \( j \) (every column of \( p \) sums to unity). However, transition across states may be driven by an external uncertainty unrelated to fundamentals. Thus, the assumption of constant transition probabilities may be restrictive. Cipollini et al. (2008) and Mouratidis (2008) extend the work of Jeanne and Masson (2000) by allowing the transition probabilities of \( \{s_t \} \) to be a function of fundamentals. If the vector of economic fundamentals that determine the transition probabilities at time \( t \) is \( Z_t \), the time-varying transition probabilities have the following form:

\[
p_{ij,t} = \exp \left( \alpha_{s_{ij,t}} + Z'_{t-1} \beta_{ij,t} \right) / \left[ 1 + \exp \left( \alpha_{s_{ij,t}} + Z'_{t-1} \beta_{ij,t} \right) \right], \quad i, j = 1, 2.
\]  

(6)

The implication of this specification is that fundamentals can help to predict future behaviour of the unobserved state variable of probability of devaluation. Now, we further extend this approach using a bivariate MRS model to estimate the impact the unobserved state of fundamentals has on the unobserved state of probability of devaluation. The aim of this exercise is to find whether there is interaction between market expectations, concerning the probability of devaluation, and market expectations about the unobserved state of economic fundamentals.

In terms of (5) we assume that not only \( q_t \), but also \( Z_t \), follow Markov process. To make our argument clear, we further assume that \( Z_t \) includes a single variable \( (X_k) \). Phillips (1991) shows that the unobserved states \( s^0_t \) and \( s^1_t \) could be either independent or perfectly correlated. Phillips (1991) also shows that unobserved states may be in a different phase such as \( s^0_t \) leads \( s^1_t \) and vice versa.\(^8\) The interaction of unobserved states has implications on both where \( \mu \) is the probability that the policy maker is in a "soft" mood and \( \gamma \) indicates the impact of market expectation, for devaluation, on the policy maker’s benefit function.

\(^7\)For more details on this model, see Filardo (1994), Diebold et. al. (1994), and Filardo and Gordon (1998).

\(^8\)Bengonechea, Camacho and Quiros (2006) and Camacho and Quiros (2006) used a model where the unobserved states are partially correlated.
the nature of currency crises and the appropriate framework needed to model market expectation.

To illustrate the implications that the interaction of unobserved states has for the nature of currency crises consider a $2 \times 1$ vector $z_t = [q_t, X_t]'$ such that

$$z_t = \mu_t + \sum_{i=1}^{p} \Phi_i v_{t-i}$$

(7)

where $q_t = r_t^D - r_t^F$, $X_t$ a column of $Z_t$ and $v_t = [u_t^q, u_t^x]'$ is a Gaussian process with mean zero and positive-definite variance covariance matrix $\Sigma$; $\{s_t\}$ is modelled as a linear homogenous four-state Markov process with

$$s_t = \begin{cases} 1 & \text{if } s_t^x = 1 \text{ and } s_t^f = 1 \\ 2 & \text{if } s_t^x = 2 \text{ and } s_t^f = 1 \\ 3 & \text{if } s_t^x = 1 \text{ and } s_t^f = 2 \\ 4 & \text{if } s_t^x = 2 \text{ and } s_t^f = 2 \end{cases}$$

(8)

where $s_t^x$ and $s_t^f$ represent the unobserved states of $X_t$ and $q_t$, respectively. In the first model, where $s_t^x$ and $s_t^f$ are independent, the transition probability matrix is given by

$$P^A_{qsf} = P^q \otimes P^x = \begin{bmatrix} p_{11}^q p_{11}^x & p_{11}^q p_{12}^x & p_{11}^q p_{21}^x & p_{11}^q p_{22}^x \\ p_{12}^q p_{11}^x & p_{12}^q p_{12}^x & p_{12}^q p_{21}^x & p_{12}^q p_{22}^x \\ p_{21}^q p_{11}^x & p_{21}^q p_{12}^x & p_{21}^q p_{21}^x & p_{21}^q p_{22}^x \\ p_{22}^q p_{11}^x & p_{22}^q p_{12}^x & p_{22}^q p_{21}^x & p_{22}^q p_{22}^x \end{bmatrix}.$$ 

(9)

We call this model A. It is worth noting that we impose the restriction that transition probabilities across regimes are constant (i.e. $\beta_{ij,t} = 0$). This restriction does not affect our argument about the importance of fundamentals; it provides further support. This is so because if $\beta_{ij,t} = 0$, then

$$p_{ij}^q = \exp(\alpha_{q,ij,t} + Z_{t-i-1}^r \beta_{q,ij,t}) / [1 + \exp(\alpha_{q,ij,t} + Z_{t-i-1}^r \beta_{q,ij,t})] \quad (10)$$

$$p_{ij}^x = \exp(\alpha_{x,ij,t} + Z_{t-i-1}^r \beta_{x,ij,t}) / [1 + \exp(\alpha_{x,ij,t} + Z_{t-i-1}^r \beta_{x,ij,t})] \quad (11)$$

This is a general framework where fundamentals affect the probability of devaluation through two channels. The first channel concerns the impact of the observed dynamics of fundamentals given by (10). The second channel, which is the object of this paper, focuses on the impact that the unobserved state of fundamentals ($s_t^x$) has on the unobserved state of probability of devaluation ($s_t^f$). Thus, imposing the restriction $\beta_{ij,t} = 0$, we constrain the channels through which fundamentals can affect the probability of devaluation. The implication of model A is that expectations about the future state of fundamentals do not have any impact on the expectation of probability of devaluation. In this set-up currency crisis is driven by a sunspot.

One hypothesis suggested by Schwert (1989 a, b) and Campell et al. (1993) is perfect synchronisation between $s_t^f$ and $s_t^x$ ($s_t^f = s_t^x$). The unobserved state
variable $s_t$ follows a two-state Markov process with transition probability matrix\(^9\):

$$P^B_{qz} = \begin{bmatrix} p^q_{11} & p^q_{12} & 0 & 0 & p^q_{21} & p^q_{22} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ p^q_{12} & p^q_{12} & 0 & 0 & p^q_{21} & p^q_{22} \\ \end{bmatrix}$$

(12)

We call this model $B$. This model implies that the unobserved state of fundamentals provides information for the unobserved state of probability of devaluation, but this information concerns the nature, rather than, the predictability of a currency crisis. That is, although fundamentals matter for the genesis of crises, they cannot be used to predict it. The interesting element of this case is that the escape-clause model of Jeanne (2000) will be observationally equivalent to the self-fulfilling model of Obstfeld (1996).\(^10\)

Expectations about the probability of devaluation might affect the current state of fundamentals which could be modelled by imposing $s^q_t = s^q_{t-1}$. This reduces the full transition probability matrix to

$$P^C_{qz} = \begin{bmatrix} p^q_{11} & p^q_{12} & 0 & 0 & 0 & 0 \\ p^q_{11} & p^q_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix} .$$

(13)

The implication of this implicit causality is that a currency crisis is driven by a sunspot unrelated to fundamentals. We denote this new model as $C$. Although models $A$ and $C$ have led to the same view about the nature of currency crises, they differ in terms of their forecasting ability. In Model $C$ the relationship between $s^q_t$ and $s^q_t$ has two offsetting effects. Although, changes of $s^q_t$ have positive learning effects about the current state of $s^q_t$, they also increase the variability of $s^q_t$. In this set up, it will become very difficult to forecast currency crisis on the basis of fundamentals. In practice, we expect model $C$ to have better in sample-fit than model $A$, but worse out of sample forecast performance.

Finally, expected deterioration of fundamentals might affect the current state of probability of devaluation ($s^q_t = s^q_{t-1}$). The implication of this model is that expectations about the future state of fundamentals may affect the current state of probability of devaluation. If this is the case, the dynamics of the unobserved state of fundamentals will not only affect the probability of devaluation but also they will be able to predict currency crises one period ahead. Hence, sunspots are related to fundamentals and the nature of a currency crisis will be in line with the escape-clause model of Jeanne (1997, 2000). We denote this model as

\(^9\)For more details see Hamilton and Lin (1996) and Sola et al. (2002 and 2007). A similar framework is used by Bengochea, Camacho and Quiros (2006).

\(^10\)Here the term observationally equivalent means that, on the basis of the observed dynamics of fundamentals, both models (escape-clause and self-fulfilling) have the same predictive power.
D. We can model this hypothesis by reducing the full $P_{qz}$ matrix to

$$P_{qz}^D = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 
\end{bmatrix}. \tag{14}$$

The implication of (13) and (14) is that there is interaction between market expectations about the probability of devaluation and the future state of economic fundamentals. Modelling this interaction may enable us to forecast currency crises one period ahead.

3 Forecast Evaluation

This section describes the properties of optimal forecasts under the assumption that forecasters have a quadratic loss function (QLF). We also discuss forecast evaluation under various loss functions.

3.1 Properties of Optimal Forecasts

If a forecaster has a QLF and the data generation process (DGP) of predicted variable $y_t$ is linear, the optimal forecast is the conditional mean $(E_{t}y_{t+h}, h > 0)$.\footnote{11} Diebold and Lopez (1996) and Patton and Timmermann (2007) discuss the properties of rational forecasts. Provided that both the target and predicted variables are jointly stationary, the rational forecast satisfies the following properties:

- Forecast is unbiased. This implies that on average forecast errors are equal to zero $E(y_{t+h} - y_{t+h|t}) = 0$.
- Forecast errors are orthogonal to the information set, available to the forecaster when the forecast is made.
- The variance of the forecast error is a non-decreasing function of forecast horizon.
- $h$-step-ahead forecast errors are serially uncorrelated beyond an order of $h - 1$.
- The one-step-ahead forecast error is serially uncorrelated.

A test for rationality is usually based on the regression

$$y_{t+h} = \alpha + \beta y_{t+h|t} + \epsilon_{t+1} \tag{15}$$

\footnote{11} $E_{t}()$ denotes conditional expectation based on information available at time t, $h$ is the forecast horizon.
where the null hypothesis is $H_0 : \alpha = 0$ and $\beta = 1$. However, Holden and Peel (1993) show that a more satisfactory test for unbiasedness is to test for $c = 0$ in the regression

$$e_{t+h|t} = c + e_{t+1}$$ (16)

where $e_{t+h|t}$ is the forecast error at forecast horizon $h$.\textsuperscript{12} Although a test for $c = 0$ is described as a test for unbiasedness, it can be also be viewed as a test for efficiency, in the sense that forecast errors are uncorrelated with forecasts, that is

$$E_t(e_{t+h|t}, y_{t+h|t}) = 0.$$ (17)

If forecasts are unbiased, the nature of a currency crisis depends on the interaction of the unobserved states of each variable included in the multivariate MRS model. This is so because, unbiased forecasts imply that forecasters have used all the observable information at the time of forecast. Thus, any impact of fundamentals on the probability of devaluation will be through the unobserved states of fundamentals. Alternatively, if forecasts are biased and unobserved states are uncorrelated, the currency crisis is driven by a sunspot. Biased forecast implies that forecasters are irrational in the sense that they disregard information from fundamentals. Irrational forecast, in conjunction with uncorrelated unobserved state, indicates that fundamentals do not influence the probability of devaluation. Under such circumstances, currency crises are driven by market expectation unrelated to fundamentals. Finally, if forecasts are biased and the unobserved states are correlated, in line with model $D$ (i.e. $s_t^r = s_{t-1}^r$), it is important to check whether forecasts are biased over the whole out-of-sample period, or at certain points before the currency crisis. Evidence that forecasts become irrational before a currency crisis may be considered as early warning signal that a currency crisis may be imminent.\textsuperscript{13}

If forecasters have a QLF then it is straightforward to show that the RMSFE can be used as a criterion for forecast evaluation. However, Granger (1969, 1999), Christoffersen and Diebold (1997) and Patton and Timmermann (2007) show that if forecasters have an asymmetric loss function, then standard properties of optimal forecasts do not hold. Elliott et al. (2005, 2007) proposed a rationality test under asymmetric loss function in a GMM framework. Alternatively, a density forecast criterion can be used to select the best forecast among different forecasting models. Diebold et al. (1998) show how the density forecast criterion is optimal regardless of the loss function of decision maker.

Assessing whether a density forecast is correct with respect to observed outcomes is associated with a goodness-of-fit test. The two classical non-parametric approaches to test goodness-of-fit are the likelihood ratio and Pearson’s chi-

\textsuperscript{12}Under the null $H_0 : \alpha = 0$ and $\beta = 1$, if we subtract $y_{t+h|t}$ from both sides of (15) we obtain (16).

\textsuperscript{13}Irrationality before currency crisis implies that it becomes increasingly difficult to guess the probability of devaluation, based only on the observed dynamics of fundamentals. Thus, speculative attack will be very sensitive to any news about fundamentals. Under such an uncertain environment, it is a matter of time for currency crisis to occur.
squared test. An alternative group of goodness-of-fit tests is based on the probability integral transform. If \( F() \) is an estimated density forecast then \( z = F(y) \) where \( y \) is the observed outcome, has a \( U[0, 1] \) distribution. Deviation of \( z \) from \( U[0, 1] \) indicates that the estimated density forecast \( F() \) is not correct. Diebold et al. (1998) show that if a sequence of density forecasts is correctly conditionally calibrated, then sequence is iid \( U[0, 1] \). Berkowitz (2001) suggests an alternative goodness-of-fit test where instead of testing for uniformity of \( z \) it may be more fruitful to test for normality of the inverse cumulative distribution function (CDF) transformation of \( z \). Under the null, the sequence \( \{z\} \) is i.i.d. \( N(0, 1) \). The argument of Berkowitz (2001) for normality test was that more powerful tools can be employed to test for normality than uniformity. For one-step ahead forecasts, Berkowitz (2001) proposes a likelihood ratio test for the joint null hypothesis that standardised forecast errors \( z^*_t \)’s have zero mean, unit variance and are independent, allowing them to potentially follow an AR(1) process. The Berkowitz test is computed as

\[
\begin{align*}
\quad
z^*_t & = c + \rho z^*_t-1 + \varepsilon_t \\
LR_B & = -2[L(0, 1, 0) - L(\hat{\varepsilon}, \hat{\sigma}^2, \rho)]
\end{align*}
\]

where \( L(\hat{\varepsilon}, \hat{\sigma}^2, \hat{\rho}) \) is the value of the maximised log-likelihood for the AR(1) model and \( L(0, 1, 0) \) is the constrained log-likelihood. Under the null \( LR_B \sim \chi^2_3 \).

Forecasts may be biased due to a wrong model employed to forecast the predicted variable. We control for this by using the density forecast test of Berkowitz (2001). However, since Berkowitz’s (2001) test is a joint test of normality and autocorrelation, it is difficult to find why a statistical model fails the density forecast test. Alternatively, the test suggested by Diebold et. al. (1998) detects separately whether a model fails the distributional assumption or the assumption of no autocorrelation. Therefore, using the Diebold et. al. test, we can find why a model fails the density forecast test.

14Both these tests are based on dividing the range of variable into \( k \) mutually exclusive cells and comparing the observed relative frequencies with the probabilities of outcomes falling in these cells given by the forecast densities; see Wallis (2003). However, if the distribution is continuous, consideration of only the cell frequencies does not fully reflect the information available in the observations of each cell (see also Wallis (2003)).

15The density forecast is constructed as follows. We assume that disturbances are i.i.d. Gaussian. Then if \( \hat{\mu}_{t+1} \) is the one-step-ahead forecast of \( y_{t+1} \) made at time \( t \) and \( \hat{\sigma}_{t+1} \) is the standard deviation of \( \hat{\mu}_{t+1} \), then the Gaussian density forecast is \( F(y_{t+1}) = N(\hat{\mu}_{t+1}, \hat{\sigma}_{t+1}) \). Then the probability integral transform values are given by \( \{z_{t+1}\} = \{\Phi(\frac{y_{t+1} - \hat{\mu}_{t+1}}{\hat{\sigma}_{t+1}})\} \) where \( \Phi \) is the normal cdf. \( \{z^*_t\} = \{\frac{y_{t+1} - \hat{\mu}_{t+1}}{\hat{\sigma}_{t+1}}\} \) are the standardised forecast errors that are distributed \( N(0, 1) \) under the null. Here to test for normality, we employ the Doornik-Hansen (1994) test. To test for independence of \( z^*_t \) we use the Ljung-Box for autocorrelation see Harvey et al. (1989) p. 259; we consider up to the third moment.
4 Data and Empirical Results

Data were taken from the International Financial Statistics (IFS) database. Monthly average of money market rates were used in the empirical analysis, defined as the short-term borrowing rates between financial institution over the period January 1979 to April 1998. Industrial production was also selected from the same source as above (line 66). Empirical results are based on one-step-ahead forecasts computed recursively from April 1988 to April 1998. Let us consider the case of a hypothetical forecaster, who at time $t$ needs to forecast the probability of devaluation at $t + 1$, with $t$ going from March 1988 to March 1998, and starting date of the sample being constant. At each point of time, the forecaster uses the estimated coefficients at time $t$ to forecast time $t + 1$. Then, when a new observation becomes available, the forecaster re-estimates the model using data up to time $t + 1$ to forecast time $t + 2$. The process is iterated recursively until the last in-sample period March 1998.

We focus in the periods before currency crises. This is to evaluate the forecasting abilities of both linear and non-linear models to forecast the speculative attack on the Italian lira in September 1992 and the French franc in July 1993. We employ a first order autoregressive AR(1) model, a bivariate vector autoregressive (BVAR) model, and a bivariate linear regression model (LIN BVAR) as linear counterparts of the bivariate MRS model presented above. We use as a proxy for the fundamentals, the growth rate of industrial production ($DIP$). We have experimented with other fundamentals, such as the real exchange rate of the Italian lira and the French franc against the DM, but the results remain qualitatively similar.

Table 1 shows that neither the lags of the interest rate differential (IRD) affect the growth rate of industrial production, nor the lags of the latter affect the former. Granger causality tests also show that there is no causality between IRD and real exchange rate. On the basis of these results, we employ a multivariate MRS model where every individual variable is affected by its own lag:\(^\text{10}\)

\[
\begin{pmatrix}
q_t - \mu_{st} \\
x_t - \gamma_{st}
\end{pmatrix} =
\begin{pmatrix}
\phi_{11} & 0 \\
0 & \phi_{22}
\end{pmatrix}
\begin{pmatrix}
q_{t-1} - \mu_{st-1} \\
x_{t-1} - \gamma_{st-1}
\end{pmatrix}
+ 
\begin{pmatrix}
\epsilon_1 \\
\epsilon_2
\end{pmatrix}
\]  

(19)

\(^\text{10}\)Money market rates were selected from line 60 b. The starting date almost coincides with the inception of the EMS.

\(^\text{17}\)The linear bivariate model includes the first lagged value of $q_t = r_t^p - r_t^f$ and the growth rate of industrial production as explanatory variables. The LIN BVAR is given by:

$$q_t = \phi_{q_{t-1}} + \beta x_t + u_t$$

where $x_t = DIP_t$, and $u_t$ is the stochastic disturbance.

\(^\text{18}\)Table 1 presents results from a second order vector autoregressive model VAR(2). Although the optimal number of lags, based on AIC criterion is 1, we have selected 2 lags. This is so because, for a short period of time before the crisis of 1992, the AIC shows that the optimal number of lags is two.

\(^\text{19}\)We have also experimented with a model including two lags but we faced computational problems concerning convergence.
Table 1: Estimation of Linear BYAR Model

<table>
<thead>
<tr>
<th>Variables</th>
<th>IRD</th>
<th>t-stat</th>
<th>DIP</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IRD(-1)</td>
<td>1.151</td>
<td>-17.689</td>
<td>-0.003</td>
<td>-0.978</td>
</tr>
<tr>
<td>IRD(-2)</td>
<td>-0.163</td>
<td>-2.495</td>
<td>0.003</td>
<td>-1.033</td>
</tr>
<tr>
<td>DIP(-1)</td>
<td>-0.777</td>
<td>-0.593</td>
<td>0.464</td>
<td>-7.43</td>
</tr>
<tr>
<td>DIP(-2)</td>
<td>0.144</td>
<td>-0.11</td>
<td>0.329</td>
<td>-5.277</td>
</tr>
<tr>
<td>C</td>
<td>-0.047</td>
<td>-0.487</td>
<td>0.004</td>
<td>-0.97</td>
</tr>
<tr>
<td>France</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IRD(-1)</td>
<td>1.002</td>
<td>-14.974</td>
<td>0.001</td>
<td>-0.650</td>
</tr>
<tr>
<td>IRD(-2)</td>
<td>-0.097</td>
<td>-1.448</td>
<td>0.000</td>
<td>-0.190</td>
</tr>
<tr>
<td>DIP(-1)</td>
<td>-0.646</td>
<td>-0.175</td>
<td>0.555</td>
<td>-8.696</td>
</tr>
<tr>
<td>DIP(-2)</td>
<td>2.293</td>
<td>-0.617</td>
<td>0.320</td>
<td>-4.987</td>
</tr>
<tr>
<td>C</td>
<td>-0.271</td>
<td>-2.564</td>
<td>0.003</td>
<td>-1.653</td>
</tr>
</tbody>
</table>

where the vector \((\epsilon_1, \epsilon_2)'\) follows an i.i.d. bivariate Gaussian distribution:

\[
\begin{pmatrix}
\epsilon_1 \\
\epsilon_2
\end{pmatrix}
\sim N
\left[
\begin{pmatrix}
0 \\
0
\end{pmatrix},
\begin{pmatrix}
\sigma_{11} & \sigma_{12} \\
\sigma_{21} & \sigma_{22}
\end{pmatrix}
\right].
\tag{20}
\]

We assess the nature of a currency crisis in two steps. First, we compare the maximised likelihood values of the four MRS models. That is, we focus on currency crisis with multiple equilibria, assessing whether it is driven by a sunspot (self-fulfilling currency crisis model) or by a combination of fundamentals and market expectations (the escape-clause model). However, an in-sample fit doesn’t not necessary imply a good forecasting performance of the relevant model. Thus, in the second step, we compare the forecast performance of linear and non-linear models. The ML values and measures of forecast performance have been used as complementary indicators to evaluate the various models in our study. Under such circumstances, we assess the nature of currency crisis by comparing the forecast performance of all linear and non-linear models. If the linear model outperforms MRS models, then the currency crisis is driven only by fundamentals in line with Krugman’s first generation model.

For each sample presented in the first column of Table 2, we have computed the average log-likelihood values of all MRS models. Table 2 shows that in the case of Italy model \(D\) has the highest maximised log-likelihood values (ML) among the four MRS models. This is especially the case for the period after the speculative attack in 1992. Model \(D\) implies that the current state of IRD is affected by expectations about the future state of DIP. This is consistent with the view that currency crisis is driven by both fundamentals and market expectations. Thus, the Italian currency crisis in (1992) was not unrelated to fundamentals. This is consistent with De Grauwe (1994), who argued that there was a widespread agreement that Spain and Italy experienced a higher inflation rate than the EMS average during 1987-1992. During this period, without any realignment, tensions had been building up for these two countries in the form of
a growing loss of competitiveness. De Grauwe (1997) reports that the currencies of the two countries were overvalued in 1992 by between 25 and 30% and the choice for both countries was either to deflate their economies or to use a large realignment once the large capital inflows, that had financed the current account deficit until then, came to an end.

<table>
<thead>
<tr>
<th>Italy</th>
<th>MRS A</th>
<th>MRS B</th>
<th>MRS C</th>
<th>MRS D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>-411.279</td>
<td>-408.603</td>
<td>-405.637</td>
<td>-404.137</td>
</tr>
<tr>
<td>Before Crisis</td>
<td>-509.281</td>
<td>-503.854</td>
<td>-502.878</td>
<td>-497.73</td>
</tr>
<tr>
<td>After Crisis</td>
<td>-593.714</td>
<td>-585.916</td>
<td>-586.654</td>
<td>-578.365</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>France</th>
<th>MRS A</th>
<th>MRS B</th>
<th>MRS C</th>
<th>MRS D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>-556.41</td>
<td>-528.77</td>
<td>-533.735</td>
<td>-511.048</td>
</tr>
<tr>
<td>Before Crisis</td>
<td>-442.397</td>
<td>-420.027</td>
<td>-422.019</td>
<td>-397.2</td>
</tr>
<tr>
<td>After Crisis</td>
<td>-643.427</td>
<td>-633.288</td>
<td>-627.058</td>
<td>-605.843</td>
</tr>
</tbody>
</table>

We test for density forecast using the procedures suggested by Berkowitz (2001) and Diebold et. al. (1998). Empirical results show that both linear and non-linear models fail Berkowitz’s test. Figure 1 and 2 show recursive estimates of Diebold et. al. (1998) density forecast test (i.e. tests for normality and autocorrelation) concerning the four MRS models. The key finding of these figures is that some models fail the normality test and some the autocorrelation test. It is also worth noting that for some models, the failure of any of the tests happens for the period before the crisis and for some models for the period after the crisis. Figure 3 and 4 present recursive tests for non-normality and autocorrelation for the standardised forecast errors of linear models. Figure 3 indicates that models fail the non-normality test few months before the speculative attack in September 1992. Figure 4 shows that all linear models pass the test for autocorrelation.

Evidence that all models fail Berkowitz’s test implies that optimal forecasts depend on forecaster’s loss function. However, we know that within each regime the forecaster has a quadratic loss function. Therefore, we compare the forecast performance of linear and non-linear models on the basis of the RMSFE criterion.

Figure 5 presents estimates of RMSFE computed recursively for the two best performing models. More concretely, Figure 5 indicates the RMSFE of MRS
for model C and the RMSFE of AR(1).\textsuperscript{20} Results from RMSFE imply that the currency in 1992 was driven by a sunspot. Although this is not consistent with the implication of Table 2, it is in line with our argument that an in-sample fit does not necessary imply that the model has a good-forecasting performance. However, for the period before the currency crisis in 1992, the Diebold and Mariano (1995) (DM) test rejects the null that Model C is significantly different from Model D. Alternatively, the DM test does not reject the null for the period after the crisis of 1992.\textsuperscript{21} The DM test and results from ML values imply that fundamentals play a role in the crisis of 1992, but that was not the case in the speculative attacks of 1993 and 1995.

Figure 6 provides further evidence in favour of non-linear models. Figure 6 presents the p-values of the null hypothesis that forecast errors are on average zero. In our set up where we know both the model and the forecaster’s loss function,\textsuperscript{22} a p-value above 0.05 indicates that a forecast is unbiased. All non-linear models are found unbiased at least for the period before the currency crisis in September 1992. Unlike MRS models, linear models are found biased.\textsuperscript{23} This is so because, given the assumption that forecasters have a quadratic loss function, if the true data generating process is non-linear, then a linear model will systematically over-predict or under-predict the predicted variable. However, if regimes have different duration then over- and under-prediction will not cancel each other out and the mean of forecast errors will be significantly different from zero. Under such circumstances, a nonlinear model will be optimal in terms of using efficiently all current available information. More concretely, linear models, ignoring information concerning future states of expectation of devaluation, fail to adjust their forecast ex-ante leading to over-prediction or under-prediction of the forecasted variable.

The difference between the crisis in 1992 and the crises in 1993 and 1995 was that, in the latter crises, the currencies were not overvalued unlike certain currencies (sterling, lira, peseta, and Swedish Krona) in 1992. The correction of the over-valuation, after 1992, proved that the new level of the exchange rate was resilient in the long run. In 1995, the peseta and the lira were under strong pressure as an increase in US interest rates raised doubts about the capability of these countries to service their high domestic deficits without using an inflationary policy. In the last ‘storm’ of 1995, the peseta used the wide margins (10-12 %) and devaluated by 7 %, while the lira went more than 60 % above the pre-1992 parity.

In addition to this argument, since 1983, French monetary authorities have followed a competitive disinflation policy aiming at reducing inflation to a lower level than inflation in Germany, thereby attempting to improve competitiveness and therefore growth. However, this policy of competitiveness through disinflation has been successful only at bringing down inflation and not at cre-

\textsuperscript{20}Estimates of all other linear and non-linear models are available upon request.

\textsuperscript{21}The p-values of DM test for the periods before and after the crisis of 1992 were 0.771 and 0.045 respectively.

\textsuperscript{22}The use of the ML method indicates that within each regime the forecaster has a QLF.

\textsuperscript{23}Exception to this is the AR(1) model which becomes unbiased after the currency crisis.
ating a higher level of employment, which was lower after than before 1983 (see Blanchard and Muet (1993)).

Unlike the case of Italy, results from France show that speculative attacks on the French franc were driven by a sunspot. All models, linear and non-linear, fail the density forecast test of Diebold et. al. (1998). However, Figure 7 shows that the MRS model D passes Berkowitz’s density forecast test for the period before the currency crisis in 1992. Table 2 also shows that the MRS model D has the highest maximum likelihood value among all MRS models. Although both density forecast tests and the maximum likelihood values favour MRS model D, Figure 8 shows that MRS model A has the lowest RMSFE among all MRS models. Figure 9 provides clear evidence that the best performing MRS model A outperforms the forecast performance of best linear AR(1) model. The DM test rejects the null that MRS model A is significantly different from MRS model D but does not reject the null when we compare MRS model A and the AR(1) model.

Model A and Model D have different implications about the nature of currency crises. Under such circumstances, we rely on empirical fact which provides support to the former model. This is so because, in the crisis of 1993 and the stormy period of 1995, the situation was different from the crisis of 1992 as the over-valuations of the ‘peripheral currencies’ were corrected. The currency markets attacked currencies such as the Belgian franc, the French franc and the Danish korone that appeared not to be systematically overvalued. According to Eichengreen and Wyplosz (1993) and Kenen (1995), the crisis of 1993 was the result of market expectations about future changes in the French (and some small countries’) monetary policy because of the belief that the governments of these countries were in different cyclical position to Germany and would like to follow a different policy. In addition to this argument, Gross and Thygesen (1998) show that, by 1991, the French franc had depreciated in real effective terms, by about 10%, since 1980. When this is viewed with current account and the policy of competitiveness through disinflation, followed by France after 1983, one can be confident that the French franc was not overvalued. However, the negative results of the Danish referendum in 1992, and the French presidential election (1993), made the French franc suspect to speculative attack. Thus, the crisis of 1993 is consistent with the analysis of a self-fulfilling speculative attack.
Figure 1: Italy: Test for Normality: The Case of Non-linear Models.

Figure 2: Italy: Test for Autocorrelation: The case of Non-linear Models
5 Conclusion

The aim of this study is to provide an empirical framework to analyze the nature of currency crises. We do so by extending earlier work of Jeanne and Masson (2000) and Mouratidis (2008). Jeanne and Masson (2000) suggest that a currency crisis model with multiple equilibria can be estimated by a MRS model. However, Jeanne and Masson (2000) assume that the transition probabilities across equilibria are constant and independent of fundamentals. In this setup, currency crisis is driven by a sunspot unrelated to fundamentals. Mouratidis (2008) shows that the transition probabilities may be time-varying, based on fundamentals behavior. Under such circumstances, currency crisis is in line with the escape-clause model and is driven by both fundamentals and sunspots. More concretely, fundamentals firstly put a currency into the crisis zone and then a sunspot determines the timing of speculative attack.

Although a MRS model with time-varying transition probabilities captures the impact of observed dynamics of fundamentals on the probability of devaluation, it still disregards the impact of the unobserved dynamics of fundamentals on the probability of devaluation. Here, in line with recent literature in macroeconomics, we assume that not only the probability of devaluation, but also fundamentals follow Markov processes. If this is the case then we can adopt the multivariate MRS model suggested by Phillips (1991) to examine the interaction between the unobserved states of fundamentals and the unobserved state of the probability of devaluation.

Phillips (1991) shows that the unobserved states of fundamentals, included in a multivariate MRS model, may be either independent or perfectly correlated or being in a different phase. In this set up, if the unobserved state of fundamentals is independent of the unobserved state of probability of devaluation, then currency crisis is driven by a sunspot. Alternatively, if the unobserved state of fundamentals is perfectly correlated with the unobserved state of probability of devaluation, then currency crisis is consistent with the escape-clause model of Jeanne (1997, 2000). Finally, if unobserved state of fundamentals is in a different phase of that of the unobserved state of probability of devaluation, and the unobserved state of fundamentals leads the unobserved state of the probability of devaluation, then fundamentals not only can explain currency crisis but they can also predict it at least one period ahead. If unobserved states are in a different phase, but the unobserved state of fundamentals is led by the unobserved state of the probability of devaluation, then currency crisis is driven by a sunspot.

Empirical evidence was mixed. In the case of Italy, although the maximised likelihood values support model D, where expectation of devaluation is affected by expectation about fundamentals, the out-of-sample forecast comparison shows that the speculative attacks in 1993 and 1995 were driven by market expectations unrelated to fundamentals. This is so because Model C outperforms both linear and non-linear models after the crisis in 1992. This implies that fundamentals not only affect expectation of devaluation, but they can be used to forecast currency crisis one step ahead. On the other hand, in
the case of France, there is clear evidence that the best performing model was the MRS model $A$, which justifies that the speculative attacks on the French franc were driven by a sunspot.

An extension of our suggested framework could be to allow both the observed and the unobserved dynamics of fundamentals to affect the probability of devaluation. We can do so by making the transition probabilities of (17) time-varying. Thus, in such a set up, we could distinguish between the different channels that fundamentals influence the probability of devaluation. This potential research approach is left for the future.

References


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24In a different context, Sola et al. (2007) have employed a multivariate MRS model with time varying transition probabilities and independent unobserved states.


