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TRANSFERABILITY OF CAR-FOLLOWING MODELS BETWEEN DRIVING SIMULATOR AND FIELD TRAFFIC

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ABSTRACT

Over the last few decades, there have been two parallel streams of driving behaviour research: models using trajectory data collected from the field (using video recordings, GPS, etc.) and models using data from the driving simulators (where the behaviours of the drivers are recorded in controlled laboratory conditions). While the former source of data is more realistic, it lacks information about the driver and is typically not suitable for testing effects of future vehicle technologies and traffic scenarios. On the other hand, driving behaviour models developed using driving simulator data may lack behavioral realism. However, there has not been any previous study which compares these two different streams of mathematical models and investigates the transferability of the models developed using driving simulator data to real field conditions in a rigorous manner. The present paper aims to fill in this research gap by investigating the transferability of the car-following models between a driving simulator and two comparable real-life traffic motorway scenarios, one from the UK and the other one from the US. In this regard, stimulus-response based car-following models have been developed using three different microscopic data sources: (i) experimental data collected from the University of Leeds Driving Simulator (UoLDS), (ii) detailed trajectory data collected from Motorway 1 (M1), UK and (iii) detailed trajectory data collected from Interstate 80 (I-80), CA, USA. The parameters of these car-following models are estimated using the Maximum Likelihood Estimation technique and transferability of the models are investigated using statistical tests of parameter equivalence and Transferability Test Statistics. Estimation results indicate transferability in the model level but not fully in the parameter level for both pairs of scenarios.

Keywords: Car-following, NGSIM, Driving Simulator, Validation, Transferability

1. BACKGROUND

Road safety continues to be an important issue with road crashes among the leading causes of death - accounting for more than 1.2 million fatalities and 50 million injuries globally each year *(1)*. Driver behaviour is a factor in over 90 percent of crashes, with speeding as one of the major contributors *(1)*. Driving behaviour models, which provide mathematical representations of how the drivers make decisions involving acceleration-deceleration, lane-changing, overtaking, etc., are increasingly being used for evaluation and prediction of road safety parameters and formulating remedial measures. Reliable driving behaviour models are also critical for accurate prediction of congestion levels in microscopic traffic simulation tools *(2)* and analyses of emissions *(3)*. Moreover, driving behavior models have significantly contributed to the development and deployment of different Intelligent Transportation Systems (ITS) measures *(4), (5)*.

Microscopic driving behaviour models are typically developed using two types of data, (a) driving simulator data (where participant drivers drive an instrumented vehicle in a simulated roadway) and (b) real traffic data. Data from the driving simulators are collected following standardized procedures and are more controllable and reproducible. Further, driving simulators allow researchers to manipulate the surrounding conditions (e.g. geometric layout of the road, number and type of vehicles, weather and pavement conditions and so on) as well as driver specific conditions (e.g. level of distraction and fatigue) and run various hypothetical scenarios. However, there is scepticism regarding the simulator fidelity (physical and behavioural) of how well the driver's behaviour in simulator match with his/her behaviour in real roads *(6)*. On the other hand, the real traffic data best represent the true driving behaviour, but have several limitations: measurement errors, complex confounding of influencing factors, less control on the external factors, and absence of driver characteristics to name a few. Given the difference in the nature of the two data sources, it is of paramount importance to investigate the transferability of the model parameters between the driving simulator and the real traffic. It may be noted that in addition to these two sources, naturalistic driving data collected using instrumented vehicles (e.g UDRIVE *(7)*, SHRP2 *(8)* etc.) have also been used in research, but given the high costs involved, the availability of these data are still very limited. Moreover, similar to the driving simulator data, the naturalistic data are likely to be prone to behavioural incongruence; and similar to the field data, the external variables are often not fully controllable and it is not possible to test the effects of hypothetical scenarios.

There have been several previous researches on validation of driving behaviour observed in the driving simulator using isolated measures (e.g. speed, acceleration, reaction time, etc.). For instance, Törnros *(9)* attempted to validate driver's behaviour in terms of speed and lateral position in a simulated road tunnel. The results have shown that (i) behavioural validity in absolute terms is not satisfactory, especially referring to speed choice, while (ii) relative validity is achieved in both parameters. Godley et al. *(10)* conducted two different experiments in an instrumented car and a driving simulator comparing speed measurements. In both cases, the test roadway contained transverse rumble stripes at three locations and three additional control points (without rumble strips). The results again have demonstrated relative validity indicating that drivers decelerate in a similar pattern in the rumble strips, but have shown that drivers tend to drive faster in the real-life case than in the simulator and have hence failed to demonstrate absolute validity. Bella et al. *(11)* conducted a simulator validation study comparing drivers' speeds, in a deceleration lane and also found though relative validity was satisfactory in all scenarios, the differences in the mean speeds were significantly high in the simulator in non-demanding configurations (e.g. in presence of curves with large radius). This was assumed to stem from the different risk perception on the

simulated road as opposed to the real road. A similar study conducted from Yan et al. *(12)*, simulated a real signalized intersection, comparing drivers' speed behaviours in both cases. Comparisons of the surrogate safety measures from the simulator with the crash analyses for the field data have demonstrated relative validity. Further, the results illustrated that both the observed and simulated speed data followed normal distributions with equal means for each intersection approach - which validated the driving simulator in absolute terms as well. In addition, McGehee et al. *(13)* investigated the validity of the simulator using drivers' reactions and performances in an intersection scenario by conducting a series of experiments in Iowa Driving Simulator and a similar field scenario and showed the statistical equivalence of the driver's reaction times between the real-life and simulation cases. A validation study by Engen *(14)* used three different datasets from (a) driving simulator, (b) an instrumented vehicle and (c) and road side monitoring and compared data in terms of reaction times, speeds and lateral positions and time gaps. The findings have shown that the outputs from the simulator have less variance compared to field traffic. A recent study of Risto and Martens *(15)* compared drivers' choices in terms of headway while driving a driving simulator and an instrumented vehicle. Both experiments were conducted on same participants and the results have demonstrated significant similarity in speeds and headways in two cases. Another validation study conducted by Lee *(6)* analysed the behaviour of older drivers in driving simulator and on-road experiments and found significant positive association between the two driving performance indices (developed using principal component analyses) and after adjustment for age and gender of the drivers, was able to explain over two-thirds of the variability of the on-road driving performance indices.

Majority of the validation studies thus confirm relative validity, though the findings regarding the absolute validity are mixed. This raises the question whether or not the mathematical models of driving behaviour are transferable between the two settings. To the best of our knowledge, there has not been any previous research that examines the transferability of the parameters of a mathematical model of driving behaviour between a driving simulator and a similar filed traffic condition. This paper aims to fill in this critical research gap.

The objective of this paper is to investigate the transferability of the car-following models between a driving simulator and real traffic scenarios. In this regard, a state-of-the-art car-following model is re-estimated using two sets of microscopic traffic data extracted from video recordings of real traffic: one from the UK and the other one from the US (referred as Field Model UK and Field Model US respectively) and trajectory data from a comparable scenario in the driving simulator (referred as Sim Model). The performances of the models have been examined independently by using informal tests (i.e. signs and values of the parameter estimates) and formal tests of statistical differences (e.g. t-tests of parameter equality *(16)* and Transferability Test Statistic *(17)*).

The remainder of this paper is organized as follows: Section 2 focuses on the datasets, Section 3 outlines the details of the model structure, Section 4 presents the estimation results of the proposed models and Section 5 focuses on the transferability analyses. Finally, Section 6 summarizes the conclusions and suggests future research actions.

2. DATA

The following three secondary datasets have been used in this research:

- Driving simulator data of a UK Motorway scenario
- Video data from a segment of Motorway 1 (M1), UK
- Video data are from a segment of US101, USA

The descriptions of the study areas and the datasets are presented below followed by a critical comparison which highlights their similarities and differences.

2.1 Study Area

2.1.1 Driving Simulator Data

The simulator data has been derived from an experimental study in the University of Leeds Driving Simulator (UoLDS), one of the most advanced driving simulators in Europe. UoLDS has a Jaguar S-type vehicle cab (see Figure 1a) with all driver controls fully operational. The vehicle's internal Control Area Network (CAN) is used to transmit driver control information between the Jaguar and a network of nine high performance computers that manages the complete simulation. Control feedback is generated so that the driver seated in the cab feels (steering and pedal loading), sees (dashboard instrumentation) and hears (engine, transmission and environmental noise) an appropriate simulation of the driving environment (Figure 1b). The simulator incorporates an eight degree-of-freedom motion system. A hexapod motion platform, carrying the 2.5t payload of the dome and vehicle cab combination allows limited motion in all six orthogonal degrees-of-freedom of the Cartesian inertial frame. Additionally, the platform is mounted on a railed sled that allows a further 5m of effective travel in sway and surge. These aim to reduce, while not fully eliminate, differences between real world and simulator driving behaviour *(18)*.

a. Simulator Experience b. Motorway

FIGURE 1: Sample of driving Scenarios in UoLDS *(18)*

The experiment included forty participants (20 females, 20 males) aged between 19 to 83 years. Each participant had to drive approximately 30-40 minutes on a road section of 2,000 meters (see following Fig. 2). The participants were instructed to drive the simulator vehicle cab as if it were in a real-life vehicle, obeying the specific speed limits and all legal regulations. The full simulator study, performed as part of the Smart Motorway Project of the UK Highways Agency *(19)*, included driving decisions in presence of lane closures and roadwork, but in order to preserve similarity with the field scenario (described in 2.1.2), only part of the data that had no disruption, similar network topography and traffic flow levels as the field data has been used in this study. Since, the focus of the study is on car-following behavior, the lanes affected by road closure (which are likely to have more lane changes) are excluded and only data from the two rightmost lanes have been retained for the model development. The full road network of the driving simulator study and the selected segment are presented in Figure 2.

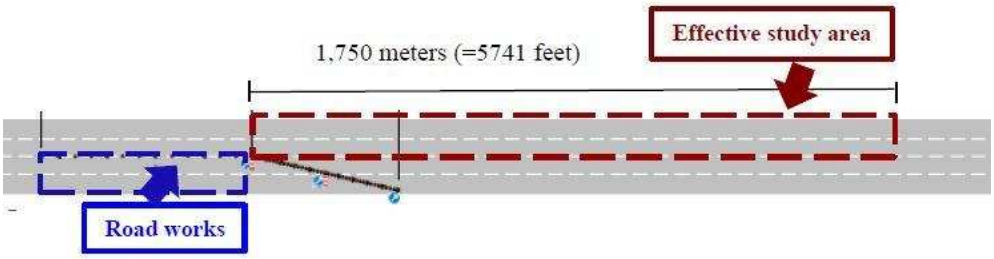

FIGURE 2: Schematic of road section, which was used in UoLDS experimental study (Source: figure adopted from Highways Agency, UK *(19)*)

2.1.2 Field Data UK

The disggregate vehicle trajectory data collected between J42-J43 of the M1 motorway network in the UK has been used as the first source of field data. The data was collected in May 2013 from an over-bridge located 620m downstream from J42 and the trajectory data was extracted using a semi-automated vehicle trajectory extractor application by Lee et al. *(20)*. Due to the camera angle and features of the trajectory extraction software, only data from the first 320m was found to be usable. The details of the data collection and processing have been reported by Kusuma et al. *(21)*. The road section constitutes of five traffic lanes (Figure 3). Since, the focus of this research is on car-following behaviour, the lanes that have the lowest number of lane changes (lanes 4 and 5) have been used in this research.

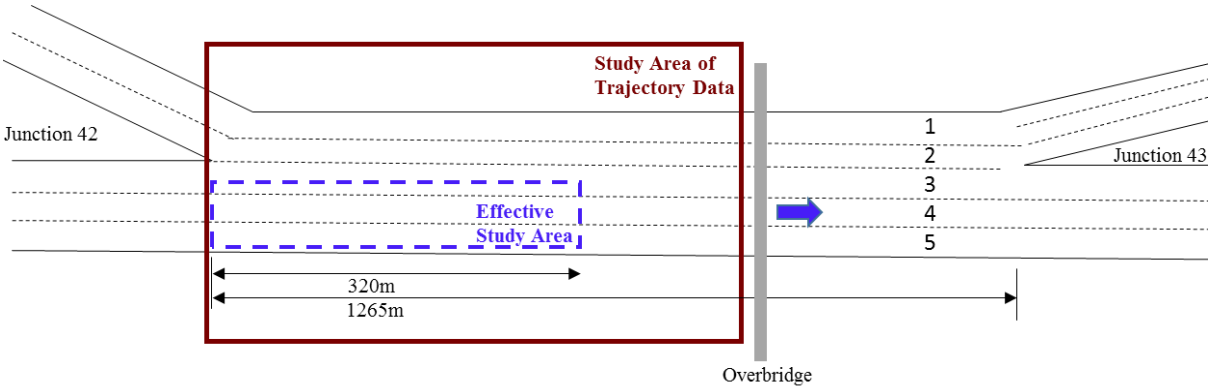

FIGURE 3: Estimation data collection site at the M1 motorway (Source: figure adopted from Kusuma et al. *(21)*)

2.1.3 Field Data US

The diaggregate vehicle trajectory data collected from the eastbound direction of Interstate 80 (I-80) in Emeryville, California, USA as part of the Next Generation Simulation (NGSIM) program has been used as the field data. The data was collected in April 2005 using seven synchronized digital video cameras set on top of tall buildings and covering a road length of approximately 500 meters (1640 feet). Detailed coordinates of individual vehicles have been extracted using the customized software NG-VIDEO at a 0.1 second time resolution *(20)*. The original road section constitutes of six traffic lanes including a high-occupancy vehicle (HOV) lane (Figure 4). Since, the focus of this research is on car-following behaviour, the lanes that have the lowest number of lane changes (lanes 3 and 4) have been used in this research. Further, since

maximum similarity with the driving simulator scenario is a critical element of this research, data between 5:15-5:30 pm, when the average flow rates were similar, have been used in the study. The explanatory variables used in the model estimation are created by VISUAL BASIC scripts (full details available in Papadimitriou2015) *(23).*

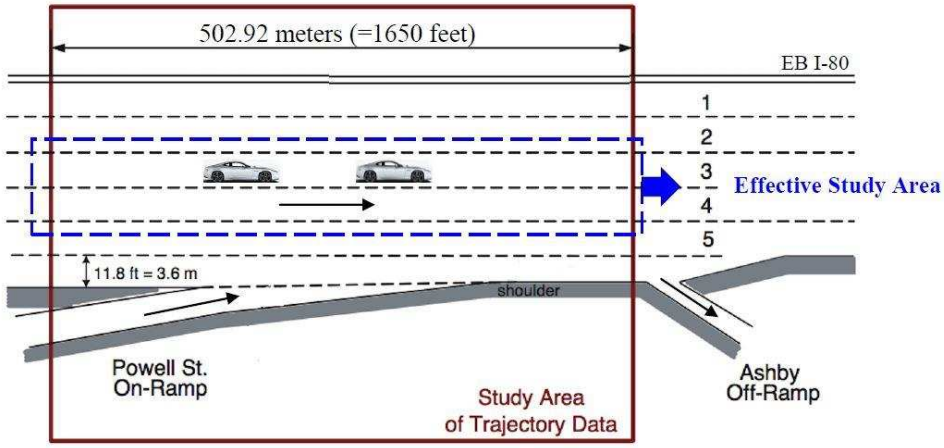

FIGURE 4: Estimation data collection site at the I-80 motorway
(Source: figure adopted from Choudhury et al. *(24)*)

2.2 Comparison

Though a conscious effort has been made to maximize the similarity between the simulator and the field datasets, since they are all secondary datasets, there are some differences. It may be noted that even in a primary data collection using the driving simulator, even though the traffic flow rates and the behavior of the ambient traffic are controllable, since the speeds and accelerations are driven by the decisions taken by the participant, the observed speed and acceleration distributions are not fully controllable.

The key aggregate characteristics of the three datasets are presented in Table 1. Figure 5 presents the comparison between the distributions of the key variables in the three datasets.

As seen in Table 1 and Figure 5, the average speeds of the drivers are higher in the UoLDS data compared to the Field Data US. This is not unusual since in the simulator environment, though the participants have been instructed to drive as they would do in real roads, there is no actual risk of potential injury. This may encourage the driver to drive aggressively. It may be noted though that the difference was found to be much smaller between the UoLDS data and the Field Data UK.

Moreover, though the mean acceleration/deceleration levels in all three datasets had similarity, the driving simulator data revealed a larger clustering around the mean value. This is not unexpected given the much smaller number of drivers in the Sim Data. In all three datasets, accelerations close to 0 was found to have higher proportions (see Table 1 and Figure 5).

Concerning the distribution of the time headway, the UoLDS and the Field Data UK had flatter distributions compared to the Field Data US, but values from 2 sec to 4 sec appeared to be the most frequent in all three data sets (see Figure 5). It may be noted that in all three datasets, time headway values have been restricted to the upper bound (=5 sec) due to adopted definition of the car-following regime. Overall, though the three datasets had significant similarities in terms of geometry and flow levels, the speeds, accelerations and headways had significant variations.

TABLE 1: Summary of aggregate characteristics

	Sim Data					Field Data UK					Field Data US				
Survey year	2014					2013					2005				
Source	University of Leeds					University of Leeds					NGSIM, FHWA				
Road category	Motorway					Motorway					Motorway				
Survey duration (min)	≈ 30-35 minutes of driving for each participant					15 minutes (17:15-17:30)					15 minutes (17:00-17:15)				
Luminance conditions	Daytime					Daytime					Daytime				
Pavement conditions	Dry					Dry					Dry				
Number of vehicles	40					527					598				
Number of observations	116,644					6,836					379,397				
Time resolution (sec)	0.17					1.00					0.10				
Length of road section (m)	1,750					320					502.92				
Traffic flow (veh/h/lane)	1,700					1,500					1,600				
Comments	2 out of 4 traffic lanes seperated roadwork parts seperated and removed (see Fig. 2)					2 out of 5 traffic lanes seperated (see Fig. 3)					2 out of 6 traffic lanes seperated (see Fig. 4)				
Variables	**Maximum**	**Minimum**	**Mean**	**Median**	**Std. Dev.**	**Maximum**	**Minimum**	**Mean**	**Median**	**Std. Dev.**	**Maximum**	**Minimum**	**Mean**	**Median**	**Std. Dev.**
Acceleration (m/s/s)	2.04	-5.52	0.00	-0.01	0.29	6.96	-6.97	-0.70	-0.55	1.40	3.41	-3.41	-0.03	0.00	1.34
Subject speed (m/s)	39.18	18.99	29.72	30.36	3.08	50.36	11.35	26.93	26.08	5.65	29.05	0.13	5.00	4.58	2.08
Front vehicle speed (m/s)	40.57	18.71	29.67	30.10	3.95	50.36	11.35	25.87	68.48	4.36	29.05	0.02	4.84	4.57	2.21
Time headway (sec)	5.00	0.00	2.69	2.64	1.14	5.00	0.02	2.30	2.22	0.03	5.00	0.00	3.07	3.02	0.96
Space headway (m)	190.97	0.00	80.18	77.71	35.18	129.91	3.24	51.25	43.43	29.04	60.00	0.00	14.22	13.10	5.61

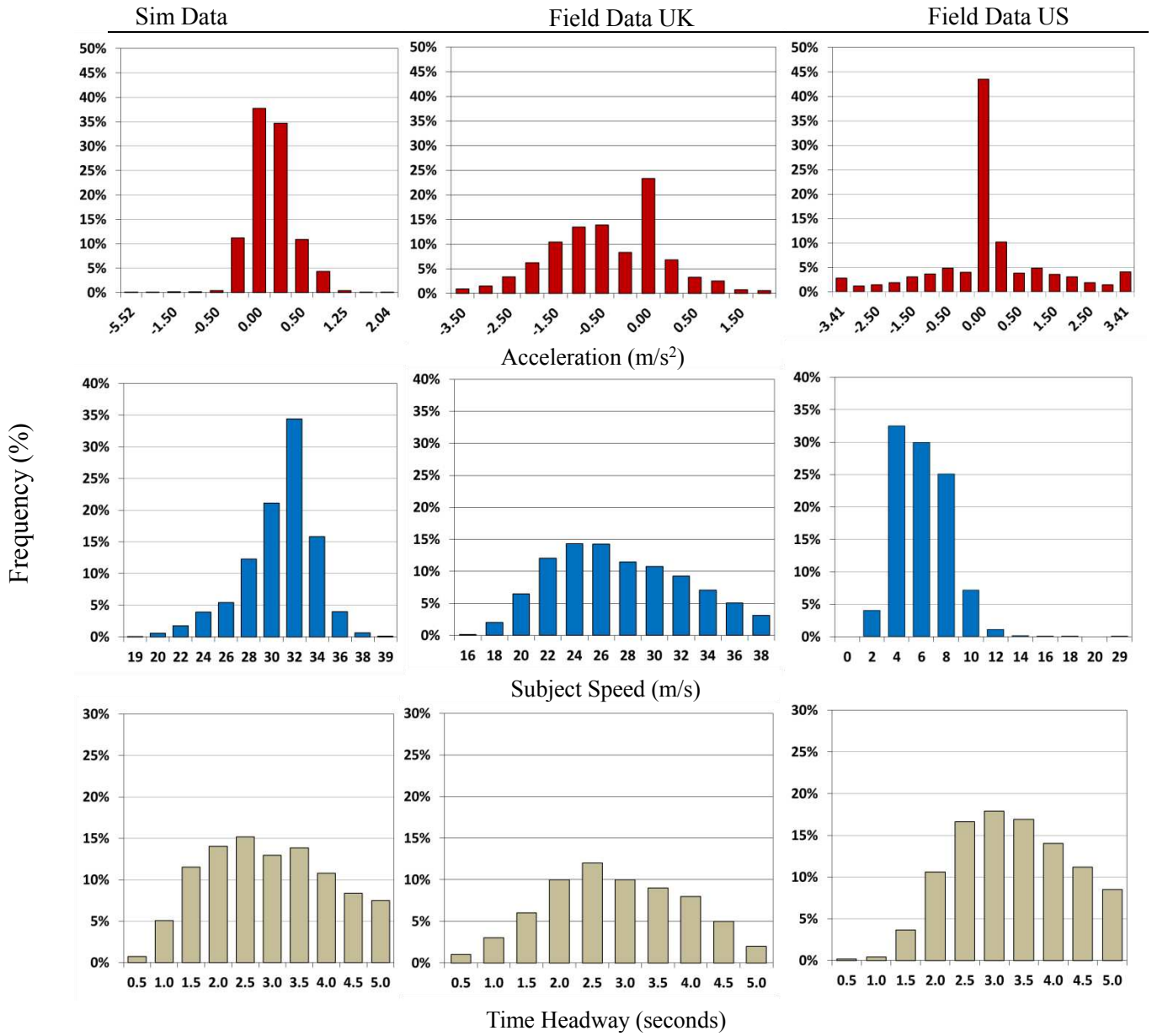

FIGURE 5: Distribution of key variables in three datasets

3. METHODOLOGY

This section covers the details of the model structure, the likelihood function used for estimation and the transferability tests used in this study.

3.1 Model Structure

The longitudinal movement decisions of the driver tend to vary significantly depending on the headway with the front vehicle. The acceleration models therefore typically have 2 states:

(a) car-following regime (constrained driving conditions)
(b) free-flow regime (unconstrained driving conditions)

The model structure used in this study is derived from Ahmed's study *(25)*, which was practically an extension of the earlier stimulus-response studies such as Subramanian's *(26)* and Gazis et al. *(27)*. In the stimulus-reponse framework, the subject driver accelerates/decelerates in response to the stimulus (generally the speed difference) of the vehicle in the front (leader). The sensitivity towards this stimulus can vary among the drivers and for the same driver on different situations. Due to reaction time (typically between 0.5 to 3 seconds), there is a time lag between the stimulus and the observed actions (accelerations/ decelerations). Ahmed's *(25)* model extended this basic model by making it more flexible by assuming non-linear functions for the sensitivity function. The stimulus part is typically a function related to the leader's relative speed (see Equation 1):

$$\Delta V_n(t) = V_n^{Leader}(t) - V_n(t) \tag{1}$$

Where,

$\Delta V_n(t)$ = Relative speed of driver n with respect to the leader at time t;

$V_n^{Leader}(t)$ = Speed of the lead vehicle of driver n at the time t;

$V_n(t)$ = Speed of driver n at the time t.

Further, the stimulus component can have two variations (i) when the speed difference between the leader and the subject driver is positive, which essentially means that the leader drives faster than the follower and (ii) when the difference is negative, which means that the follower drives faster than the leader. The first case refers to acceleration conditions, whereas the second to deceleration.

The overall car-following acceleration is given by:

$$a_n(t) = \begin{cases} a_n^{cf,acc}(t), \text{ if } V_n(t-\tau_n) \geq 0 \\ a_n^{cf,dec}(t), \text{ otherwise} \end{cases} \tag{2}$$

However, it should be clarified that the correspondence to positive and negative stimuli may be different for the simple reason that these two situations are fundamentally different and expected to trigger different behavioural responses. The acceleration decisions, which are triggered by a positive leader relative speed, is likely to be caused by speed advantage reasons and by 'herd-effect' (i.e. when people tend to adopt their behaviors or actions according to the others). Deceleration decisions on the other hand are likely to be prompted by safety considerations. In order to capture the aforementioned situations, the coefficients of explanatory variables of the model can be positive or negative according to each stimuli.

The sensitivity part is a function related to the explanatory variables such as subject vehicle speed, space and time headway, relative speed, etc.
The general formulation can be expressed as follows:

$$a_n^j(t) = s^j\left[X_n^j(t)\right]k^j\left[\Delta V_n(t-\tau_n)\right] + \varepsilon_n^j(t) \quad j \in \{acc, dec\} \tag{3}$$

Where,

$s^j[.]$ = sensitivity function;

$X_n^j(t)$ = explanatory variables affecting the sensitivity of the driver *n* at observed time *t*;

$k^j[.]$ = stimulus function;

$\varepsilon_n^j(t)$ = random error term of driver *n* at time *t*;

$\varepsilon_n^j(t) \sim N(0, \sigma_j^2)$, that is, the random error is assumed to be distributed normally.

The model assumes that the correlations between acceleration decisions from the same driver over time are captured only by the reaction time. The observations of the same driver are therefore independent conditional on the reaction time. Under this assumption, the probability density function of the car-following acceleration and car-following deceleration are given by:

$$f\left(a_n^j(t) \mid \tau_n\right) = \frac{1}{\sigma_j}\phi\left(\frac{a_n^j(t) - s^j\left[X_n^j(t)\right]k^j\left[\Delta V_n(t-\tau_n)\right]}{\sigma_j}\right) \tag{4}$$

The distribution of the combined car-following model is given by:

$$f\left(a_n(t) \mid \tau_n\right) = f\left(a_n^{acc}(t) \mid \tau_n\right)^{V_n(t-\tau_n)\geq 0} f\left(a_n^{dec}(t) \mid \tau_n\right)^{V_n(t-\tau_n)<0} \tag{5}$$

The trajectory data constitutes as series of acceleration decisions of the same driver. The acceleration profile of the driver can be expressed as follows:

$$f(a_n(1), a_n(2), a_n(3)...., a_n(\tau_n) \mid \tau_n) = \prod_1^{T_n} f(a_n(t) \mid \tau_n) \tag{6}$$

Where, T_n is the number of observations of driver *n*. Assuming the observations of different drivers are independent, the log-likelihood function (conditional on reaction time) is presented by Equation (5).

$$LL = \int_{\tau_n} \sum_{n=1}^{N} \ln\left[f(a_n(1), a_n(2), a_n(3)...., a_n(\tau_n))\right] \tag{7}$$

The unconditional likellihoods can be derived by integrating the function over the reaction time distribution and the model parameters can be derived by maximizing this likelihood function.

3.2 Evaluating Models Performance and Transferability

Review of literature revealed several formal statistical tests of transferability *(28)* among which the *t-tests of individual parameter equality* and *Transferability Test Statistic* (TTS) have been found to be most widely used and selected for this study.

The t-tests of individual parameter equality compares the individual pairs of coefficients by testing the t-stat for absolute difference between the parameter estimates of equivalent variables between the two models (e.g. of Galbraith and Hensher's study, *(16)*). The t-stat differences can be expressed as follows:

$$t_{diff,k} = \frac{\hat{\beta}_{trans,k} - \hat{\beta}_{appl,k}}{\sqrt{(\frac{\hat{\beta}_{trans,k}}{t_{trans,k}})^2 + (\frac{\hat{\beta}_{appl,k}}{t_{appl,k}})^2}} \quad (8)$$

Where,

$\hat{\beta}_{trans,k}$ and $\hat{\beta}_{appl,k}$ = estimates for the k-th parameter in the transferred and application models;

$t_{trans,k}$ and $t_{appl,k}$ = the respective t-stat. ratios of the parameter estimates;

$t_{diff,k}$ = t-stat. ratio for the difference between parameters. At 95% level of confidence, the model parameters are classified to be statistically different (i.e. non-equal) if $t_{diff,k} > 1.96$.

The Transferability Test Statistic (TTS) (Atherton and Ben-Akiva's study, *(17)*) refers when the transferred model is statistically equivalent to the applicable (estimated) model in the application context, is rejected or not. The respective formula is presented below:

$$TTS_{appl}(\hat{\beta}_{trans}) = -2(LL_{appl}(\hat{\beta}_{trans}) - LL_{appl}(\hat{\beta}_{appl})) \quad (9)$$

Where,

$LL_{appl}(\hat{\beta}_{trans})$ = log-likelihood on the application context data using transferred context parameters;

$LL_{appl}(\hat{\beta}_{appl})$ = log-likelihood on the application context data using application context parameters;

$TTS_{appl}(\hat{\beta}_{trans})$ = transferability test statistic of the transferred model in application context.

The TTS value follows a chi-squared (χ^2) distribution with degrees of freedom (dof) equal to the number of parameters. At 95% level of confidence, the models are classified to be statistically different (i.e. non-transferable) if $\chi^2 > \chi^2_{critical}$ *(17)*.

4. RESULTS

The development of the models aimed to achieve three key objectives,

(i) A logical structure

(ii) Rational signs of model parameters according to the stimulus-response concept and

(iii) Best goodness-of-fit and statistically significant model parameters.

The OX Metrics econometric package *(29)* was used for the estimation of the acceleration model parameters. The explanatory variables were statistically checked, both individually (e.g. standard errors and t-stat values) and according to its correlation with the other explanatory variables (e.g. covariance matrices). The critical time headway (differentiating between car-following and free-flow) has been selected to be 5 seconds in line with previous studies on motorway scenarios *(25)*.

According to the literature, reaction time ranges from a minimum value of 0.5 seconds to a maximum value of 2.5 seconds *(25-40)*. Models have been estimated using different reaction times within this range and reaction time equal to 0.5 seconds has been selected for both cases based on goodness of fit values.

Adjusted Rho Square ($\overline{\rho^2}$) measures the fraction of an initial log-likelihood value explained by the final model taking into account the model complexity (i.e. discounting for the additional parameters). The formula is defined as follows:

$$\overline{\rho^2} = 1 - \frac{\mathrm{LL}(\beta^*) - \mathrm{k}}{\mathrm{LL}(0)} \tag{10}$$

Where, $\mathrm{LL}(\beta^*)$ is the maximum log-likelihood value $\mathrm{LL}(0)$ is the maximum log-likelihood value and k is the number of estimated parameters.

The final estimation results are presented in Table 2.

TABLE 2: Estimation Results

Car-following acceleration

Variable	**Sim Model**		**Field Model UK**		**Field Model US**		**t-stat. diff* (absolute values)**	
	Parameter	t-stat.	Parameter	t-stat.	Parameter	t-stat.	Sim Model & Field Model UK	Sim Model & Field Model US
Constant	0.247	10.46	0.039	10.77	0.154	5.80	8.70	2.62
Relative speed (m/s)	0.226	3.86	3.266	20.35	3.989	28.54	17.79	24.83
Time headway (sec)	0.012	0.29	0.028	1.17	1.348	6.18	0.36	6.03
Sigma	0.152	38.56	0.024	24.79	0.233	24.60	31.62	7.84

Car-following deceleration

Variable	**Sim Model**		**Field Model UK**		**Field Model US**		**t-stat. diff* (absolute values)**	
	Parameter	t-stat.	Parameter	t-stat.	Parameter	t-stat.	Sim Model & Field Model UK	Sim Model & Field Model US
Constant	-0.218	-12.39	-0.145	-11.85	-3.722	-1.65	3.41	1.55
Relative speed (m/s)	0.327	4.50	1.984	48.42	11.098	28.54	19.86	27.23
Time headway (sec)	0.054	2.04	0.089	13.82	1.463	6.41	1.28	6.13
Sigma	0.127	44.36	0.024	25.38	0.227	54.60	34.07	19.95

*t-tests of individual parameter equivalence (Galbraith and Hensher, 1982)

It may be noted that a couple of additional variables have also been tested but not included in the final models. These include the current and lagged speed and acceleration of the driver, the type of the front vehicle, the type of the subject vehicle type and traffic density. The coefficients of these variables were found to be non-intuitive and insignificant.

4.1 Estimation Results of the Sim Model (Driving simulator data)

The car-following acceleration model estimated using the driving simulator data is defined as follows:

$$a_n^{cf,acc}(t) = 0.247\frac{1}{\Delta T_n(t)^{0.012}}\left|\Delta V(t-\tau_n)\right|^{0.226} + \varepsilon_n^{cf,acc}(t) \qquad (11)$$

$$\varepsilon_n^{cf,acc}(t) \approx N(0,(0.152)^2)$$

Where,
t = current time period;
τ_n = reaction time for driver *n*;
$a_n(t)$ = acceleration for driver *n* at time *t*;
$\Delta T_n(t)$ = time headway at time *t* (sec);
$|\Delta V(t-\tau_n)|$ = absolute value of relative speed between subject and leader vehicle at time *t*

Whereas, the estimated car-following deceleration model is:

$$a_n^{cf,dec}(t) = -0.218\frac{1}{\Delta T_n(t)^{0.054}}\left|\Delta V(t-\tau_n)\right|^{0.327} + \varepsilon_n^{cf,dec}(t) \qquad (12)$$

$$\varepsilon_n^{cf,dec}(t) \approx N(0,(0.127)^2)$$

As can be seen in Table 2 and equations (11) and (12), all estimated coefficients of the Sim model have the expected signs and magnitudes. Concerning the stimulus term of the car-following regime, it increases with relative speed hence as expected and the sign is positive in both cases of acceleration and deceleration models (referring to a positive correlation between the relative speed and the magnitude of acceleration that the driver applies).
In the sensitivity component, the signs of the constant parameters are positive for acceleration and negative for deceleration as expected. Apart from this, both constant parameters have similar magnitudes showing similarity in acceleration and deceleration ranges if all else are equal. Regarding, the time headway, as expected, the drivers tend to be less sensitive to the same stimulus as the time headway gets larger. On the other hand, in deceleration model, it can be concluded that drivers tend to decelerate more to the same stimulus when the time headway reduces. That could be logically justified for the reason that drivers' safety concerns grow when the time headway decreases, by making them to decelerate so as to obtain again a safe headway from leader vehicle. Most of the parameters are significant at 95% level of confidence, except for time headway. This t-stat. value of this variable is equal to 0.26 and this may be related to low time headway variability in driving simulator data (see Figure 5). This has however been retained for comparison purposes since the time headway was found to be a statistically significant variable in the Field Model US.

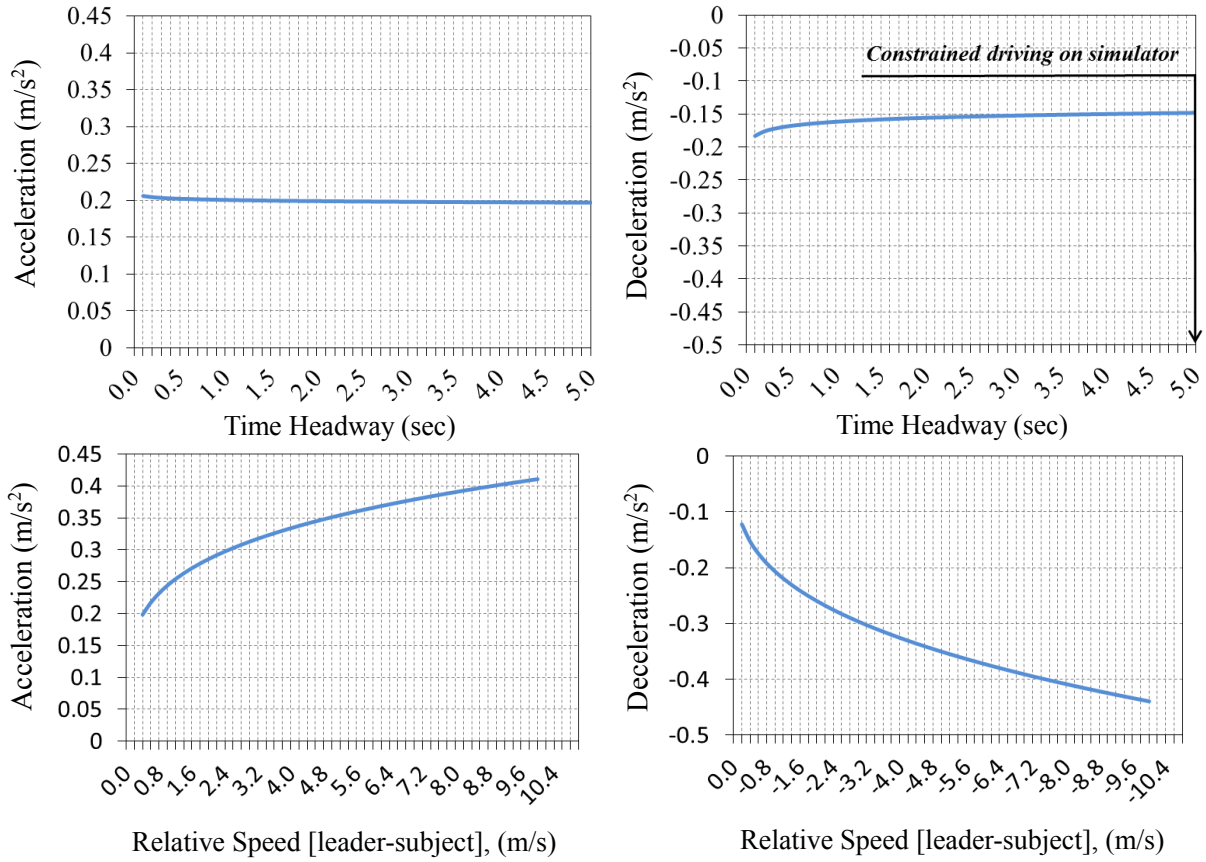

FIGURE 6: Sensitivity of different variables in car-following acceleration and deceleration models according to the Sim Model

Figure 6 presents the sensitivity analysis showing the performance of variables in both acceleration and deceleration models. The default values set assumed to be: the relative speed equal to 0.5 or (-0.5) m/s and time headway equal to 2.7 sec. These numbers have been derived from mean values of driving simulator sample.

According to the sensitivity analysis it is apparent that the mean acceleration (deceleration) is not substantially affected in magnitude by time headway. On the contrary, relative speeds have larger impacts on acceleration-deceleration magnitudes.

4.2 Estimation Results of the Field Model UK (Real Traffic Data from M1, UK)

The car-following acceleration model estimated using field data from the UK is as follows:

$$a_n^{cf,acc}(t) = 0.039\frac{1}{\Delta T_n(t)^{0.028}}\left|\Delta V(t-\tau_n)\right|^{3.266} + \varepsilon_n^{cf,acc}(t) \tag{13}$$

$$\varepsilon_n^{cf,acc}(t) \approx N(0,(0.024)^2)$$

Where,

t = current time period;

τ_n = reaction time for driver *n*;

$a_n(t)$ = acceleration for driver *n* at time *t*;

$\Delta T_n(t)$ = time headway at time *t* (sec);

$|\Delta V(t-\tau_n)|$ = absolute value of relative speed between subject and leader vehicle at time *t*

Whereas, the estimated car-following deceleration model is as follows:

$$a_n^{cf,dec}(t) = -0.145\frac{1}{\Delta T_n(t)^{0.089}}\left|\Delta V(t-\tau_n)\right|^{1.984} + \varepsilon_n^{cf,dec}(t) \quad (14)$$

$$\varepsilon_n^{cf,dec}(t) \approx N(0,(0.024)^2)$$

Similar to the Sim Model, signs of all parameters of the Field Model UK have the expected logical signs. Further, all model parameters are statistically significant at 95% level of confidence apart from the coefficient of time headway which was insignificant also in the Sim Model. However, the parameter has been retained as the sign is intuitive and the corresponding parameter has been found to be statisitically significant in the Field Model US.

Regarding the stimulus part of the model, both car-following acceleration and decelerations increase with relative speeds. It may be noted that these parameters are much larger in magnitude compared to the Sim Model parameters. The behavioural incongruence while driving the driving simulator can be a potential reason for this.

In the sensitivity component, the time headway parameters have expected positive signs. This referes that the drivers tend to accelerate less as the time headway with the leader increases. On the other hand, in the deceleration model, it can be concluded that as the time headway decreases, the drivers tend to decelerate more to the same stimulus, due to safety concerns in order to avoid a potential collision and finally obtain again a safe headway from the leader vehicle. The time headway parameter is slightly larger for deceleration compared to acceleration as expected since acceleration only leads to speed advantage whereas, deceleration is prompted by collision avoidance (safety). It may be noted that the similar trend has been observed for Sim Model as well.

The following Figure 6 shows the sensitivity analysis, presenting the performance of the models. The default values set assumed to be: relative speed = 0.7 or (-0.7) m/s and time headway equal to 2.3 sec. These numbers derived from the corresponding mean values in the sample dataset. An attempt has been made to keep the sensitivity analysis in low values of acceleration-deceleration, so as to highlight the drivers' behaviours under these crucial and sensitive constrained traffic conditions.

According to Figure 7. the drivers are extremely sensitive to the changes in the relative speed, both for the acceleration and for deceleration compared to the Sim Model. The effect of the time headway variable however is more similar to that of the driving simulator.

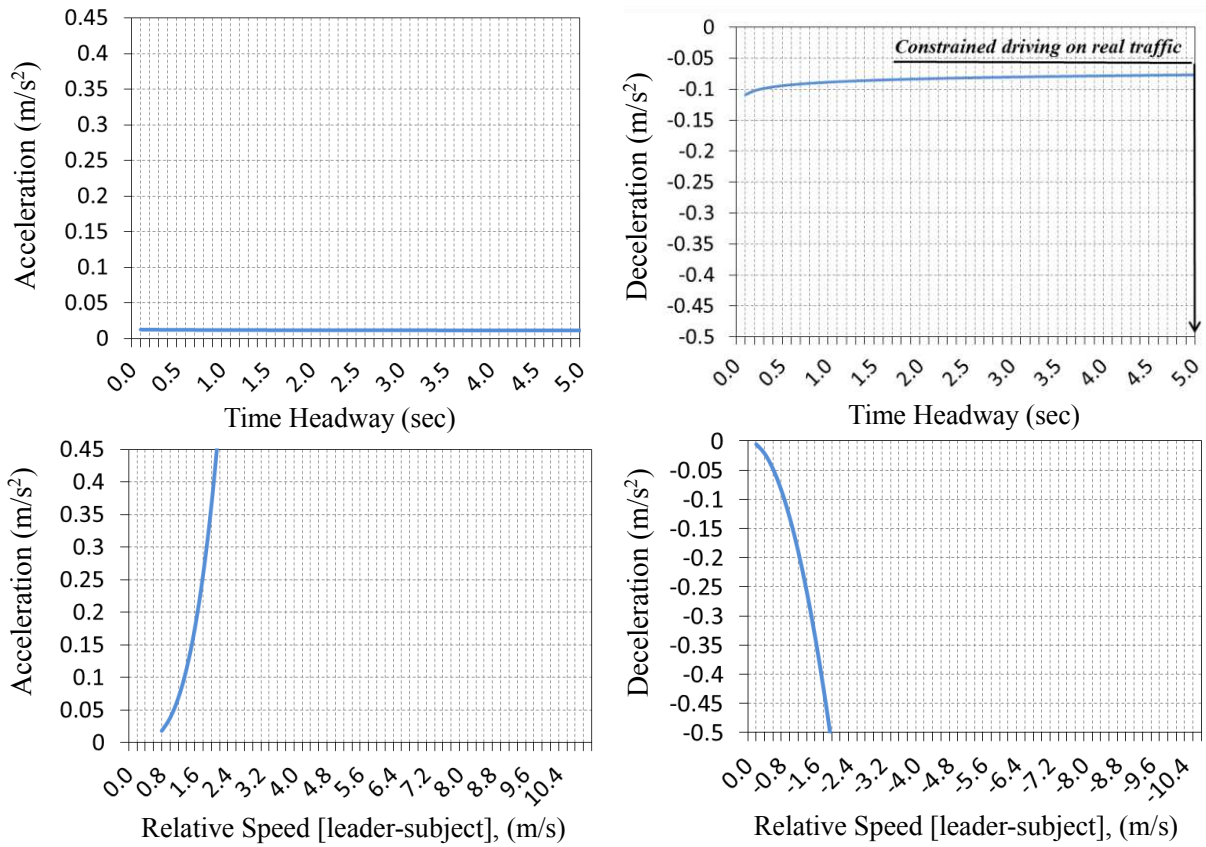

FIGURE 7: Sensitivity of different variables in car-following acceleration and deceleration models according to the Field Model UK

4.2 Estimation Results of the Field Model US (Real Traffic Data from US101, USA)

The estimated car-following acceleration usung field data is as follows:

$$a_n^{cf,acc}(t) = 0.154\frac{1}{\Delta T_n(t)^{1.348}}\left|\Delta V(t-\tau_n)\right|^{3.989} + \varepsilon_n^{cf,acc}(t) \qquad (15)$$

(15)

$$\varepsilon_n^{cf,acc}(t) \approx N(0,(0.233)^2)$$

Where,

t = current time period;

τ_n = reaction time for driver *n*;

$a_n(t)$ = acceleration for driver *n* at time *t*;

$\Delta T_n(t)$ = time headway at time *t* (sec);

$|\Delta V(t-\tau_n)|$ = absolute value of relative speed between subject and leader vehicle at time *t*

Whereas, the estimated car-following deceleration model is as follows:

$$a_n^{cf,dec}(t) = -3.722\frac{1}{\Delta T_n(t)^{1.463}}\left|\Delta V(t-\tau_n)\right|^{11.098} + \varepsilon_n^{cf,dec}(t) \quad (16)$$

$$\varepsilon_n^{cf,dec}(t) \approx N(0,(0.227)^2)$$

Similar to the Sim model, signs of all parameters of the Field model have the expected logical signs. Further, all model parameters are statistically significant at 95% level of confidence apart from the constant for acceleration (which is statistically significant at 90% level of confidence). Regarding the stimulus part of the model, both car-following acceleration and decelerations increase with relative speeds. It may be noted that these parameters are much larger in magnitude compared to the Sim model parameters (particularly for the decleration model). The behavioural incongruence while driving the driving simulator can be a potential reason for this.

In the sensitivity component, the time headway parameters have expected positive signs. This referes that the drivers tend to accelerate less as the time headway with the leader increases. On the other hand, in the deceleration model, it can be concluded that as the time headway decreases, the drivers tend to decelerate more to the same stimulus, due to safety concerns in order to avoid a potential collision and finally obtain again a safe headway from the leader vehicle. The time headway parameter is slightly larger for deceleration compared to acceleration as expected since acceleration only leads to speed advantage whereas, deceleration is prompted by collision avoidance (safety).

The following Figure 8 shows the sensitivity analysis, presenting the performance of the models. The default values set assumed to be: relative speed = 0.7 or (-0.7) m/s and time headway equal to 3 sec. These numbers derived from the mean values of the sample. An attempt has been made to keep the sensitivity analysis in low values of acceleration-deceleration, so as to highlight the drivers' behaviours under these crucial and sensitive constrained traffic conditions.

According to Figure 7 the mean acceleration values are exteemely sensitive in low time headway values showing a noticeable aggressive way of driving compared to the Sim Data and the Field Data UK. As the time headway increases, drivers tend not to accelerate at all and maintain stable speed. On the other hand, deceleration values show that the drivers decelerate at a slower rate as the time headway grows. For the relative speed, for acceleration, the drivers are extremely sensitive to the changes in the relative speed as in the Field Model UK (but not in the Sim Model).

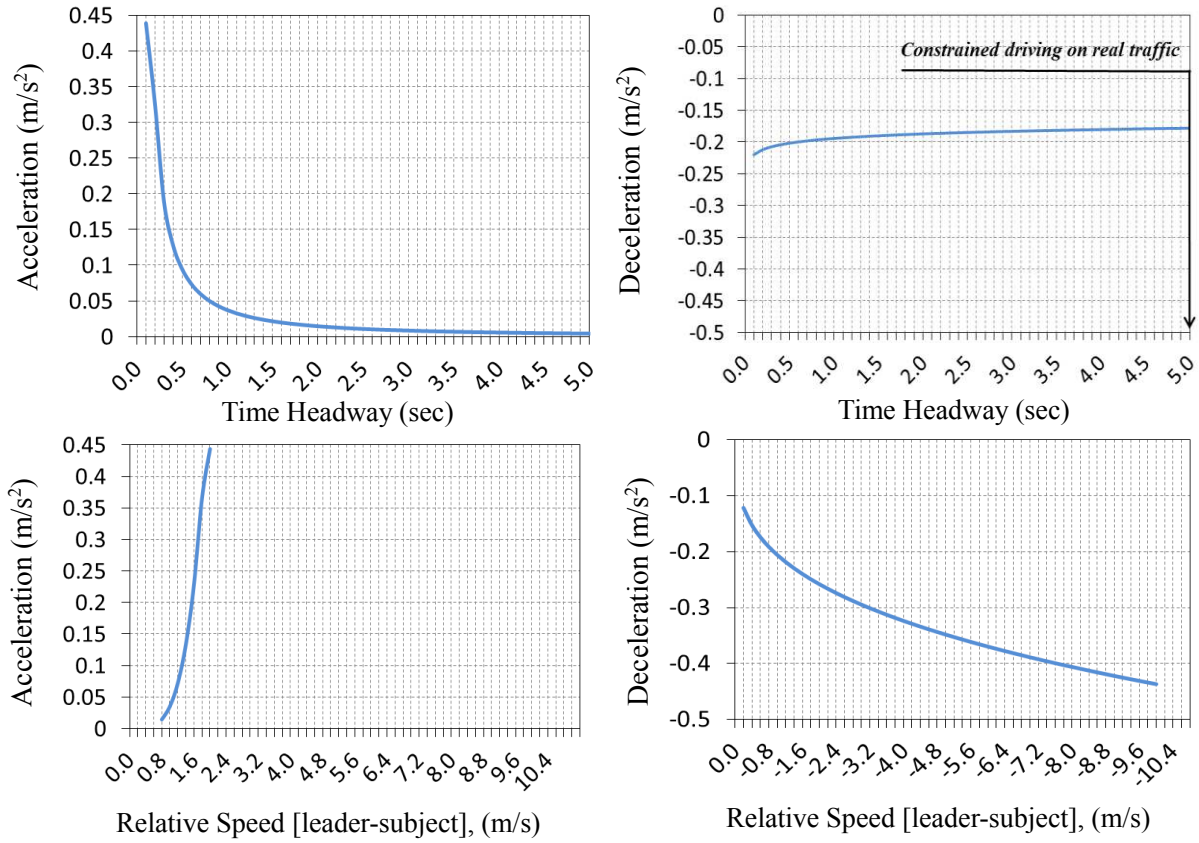

FIGURE 8: Sensitivity of different variables in car-following acceleration and deceleration models according to the Field Model US

5. MODEL COMPARISON AND TRANSFERABILITY

As mentioned, two forms of formal transferability tests have been conducted: the t-tests of individual parameter equality (parameter level transferability) and TTS (model level transferability).

Comparison of the model parameters and t-stat differences (as presented in the last two columns of Table 2) reveal the following:

- There are statistically significant differences in magnitudes of the relative speed (stimulus) parameters between the Sim Model and both Field Models. This reflects the behavioural incongruence of drivers in the driving simulator which is in line with the findings of the driving simulator validation studies (detailed in Section 2).
- The differences in the magnitudes of the time headway (sensitivity) parameters are however not statistically different only between the Sim Model and Field Model US. It may be noted though for acceleration componenets, the time headway parameter was not found to be statistically significant in the Sim Model and Field Model UK in the first place. However, the results provide some indication that some parameters may be more transferable between the driving simulator and the field if the driver population is similar (i.e. both from the same location).

- Most parameters (apart from the deceleration constant) are not directly transferable between the driving simulator and the Field Data US.

The Transferability Test Statistic (TTS) results are summarized in Table 3.

TABLE 3: Transferability Test Statistic (TTS) Results

	Case 1		**Case 2**	
Summary statistics	Sim to Field 1	Sim to Field 2	Field 1 to Sim	Field 2 to Sim
Degrees of freedom (df)	2	2	2	2
LLapplic(βtransf)	4578.24	59.6819	12.1245	10.2592
LLapplic(βapplic)	4579.63	60.7376	13.2316	13.2316
-2[LLapplic(βtransf) -LLapplic(βapplic)]	**2.78**	**2.11**	**2.21**	**5.94**

As presented in Table 3, two different scenarios have been tested so as to validate if driving simulator behaviour can be transferred to real-life world and vice versa. Comparison with the critical $\chi^2_{critical}$ value at 95% level of significance (which is 5.99) indicates that the models are transferable for both directions (i.e. driving simulator to field and vice versa). The former is of more practical importance.

6. CONCLUSIONS

In this paper, we have investigated the transferability of the parameters of a car-following model between a driving simulator and two real world scenario using secondary datasets from the UK and USA. According to the results of the t-tests of individual parameter equality between the Sim Model and the Field Model USA, most of the model parameters have been found to be significantly different in parameter level at 95% level of confidence (except the constant parameter of the car-following deceleration model). The parameter level transferability has been found to be slightly better between the Sim Model and the Field Model UK where the sensitivity parameter (coefficient of the time headway variable) has also been found to be transferable.
The transferability statistical test (TTS) results however indicate bi-directional transferability on the model level in both cases. This means that as a package, the models are transferable from the driving simulator to the field. This holds even if there are differences in geographical locaitons. For instance, for the second case study, the Sim Data was collected in the UK while the Field Data was collected from the USA, but the model level transferability still holds. On a practical term, this indicates that the predictions of acceleration/deceleration values generated from models (as in microsimulation packages) using Simulator or Field data will result insignificant differences. This is an important endorsement for use of driving simulator data for development of driving behaviour models for application in microsimulation tools.
However, if an analyst is interested about effects of a specific variable in isolation (e.g. effect of headway on acceleration or effect of relative speed on acceleration, etc.), the results may not be directly transferable. This finding is expected to have immense practical importance while applying the driving behaviour models estimated using driving simulator data for quantifying the relative benefits of alternative safety improvement measures in the field. In this particular research, the discrepancy appears to be less if the field location is geographically/spatially closer.

However, it may be noted that the results need to be used with caution since the two field datasets have been collected and processed by different teams using different hardware and software. Though, the state-of-the-art technologies have been used in both cases, given the state of image processing technology, both datasets are likely to have some measurement errors and in the absence of ground truth data, it is not possible to precisely determine the extent of such errors. This initial finding therefore needs to be investigated further with additional field datasets from geographically closer and further locations preferably cross-verified with more accurate datasources (e.g. high precision GPS data).

The results also have some other limitations - mainly due to the nature of the secondary data used in the two different environments (real roads and simulation). For instance, though care has been taken to maximize the similarity between the Sim and the Field datasets, differences in speeds, accelerations and headways (partially arising from differences in congestion levels) have been observed. This is in line with previous studies on driving simulator validation though, where the drivers have been observed to drive at higher speeds in the driving simulator in similar speed limits and congestion levels. Primary data collection in the driving simulator, can however help to minimize these differences.

Further, the models developed as part of the study ignores the heterogeneity among the drivers in reaction times, desired headways and headway thresholds demarakating car-following and other acceleration regimes *(41)*. In this regard, an interesting direction can be to incorporate the effect of driver charcteristics (e.g. age, gender, experience) in the combined models which are also expected to improve the predictive capabilities of the models (for instance better capture the heterogeneity in the reaction times and headways) .

Moreover, the scope of this study is limited to car-following models based on stimulus-response framework. Further research is required to test if the similar findings hold for other acceleration regimes (e.g. free-flow, emergency, etc.), different model frameworks (e.g. the Intelligent-Driver Model, Psycho-physical Models, etc.) as well as other choice dimensions (e.g. lane changing). It will be also interesting to validate the findings of this study by examining differences in traffic predictions of Sim Models and Field Models in microscopic traffic simulation tools.

Based on the findings of this study, we are currently investigating methods to make the models more transferable using updating mechanisms (e.g. Bayesian Updating *(42)*), Combined Transfer Estimation Techniques *(43)* as well as estimating the models jointly with both data sources using data combination techniques (e.g. *(44)*). On a parrallel study we are investigating the spatial and temporal transferability of driving behavior models (which also includes investigating the effects of different congestion levels) in further detail which may provide further insights to the results of this research.

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