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# An atlas of endohedral Sc<sub>2</sub>S cluster fullerenes

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## Abstract:

Structural identification is a difficult task in the study of metallofullerenes, but understanding of the mechanism of formation of these structures is a pre-requisite for new high-yield synthetic methods. Here, systematic density functional theory calculations demonstrate that metal sulfide fullerenes Sc<sub>2</sub>S@C<sub>n</sub> have similar cage geometries from C<sub>70</sub> to C<sub>84</sub> and form a close-knit family of structures related by Endo-Kroto insertion/extrusion of C<sub>2</sub> units and Stone-Wales isomerization transformations. The stabilities predicted for favored isomers by DFT calculations are in good agreement with available experimental observations, have implications for the formation of metallofullerenes, and will aid structural identification from within the combinatorially vast pool of conceivable isomers.

Keywords: metallic sulfide cluster fullerenes; electron transfer; Stone-Wales rotation; Endo-Kroto C<sub>2</sub> insertion

## 1. Introduction

Endohedral metallofullerenes (EMFs) are closed cage-shaped molecules with encaged metal atoms or clusters [1, 2]. Since their first synthesis in macroscopic amounts [3], EMFs have attracted extensive interest from chemists, physicists and materials scientists. To date, more than two hundred EMFs have been characterized, including mono-metallofullerenes, dimetallofullerenes, trimetallofullerenes, and fullerenes incorporating clusters various kinds [4]. Amongst the EMFs, those in which the encapsuland is a metallic cluster have attracted attention because of their novel geometries and properties [5, 6]. However, as far as we are aware, the mechanism of formation of EMFs has not yet been clarified, even though so many EMFs have been reported. For small fullerenes, a recent theoretical study shows that C<sub>2</sub> insertion (by the Endo-Kroto mechanism) can facilitate the formation of larger fullerenes without additional Stone-Wales (S-W) rotations [7]. Where the cage is larger than C<sub>70</sub>, direct C<sub>2</sub> insertion in classical isomers will form non-IPR (IPR = isolated pentagon rule [8]) isomers, which is not in agreement with experimental observations that all reported stable (neutral, bare) fullerenes are IPR-satisfying. Hence, C<sub>2</sub> insertion alone cannot account for the formation of fullerenes. Some post-insertion step, for which S-W rotation is the best candidate, is a necessary stage in the formation of fullerenes.

For endohedral metallic cluster fullerenes, formation processes may be similar to those for bare fullerenes, at least in terms of the cages involved, but they are likely to be affected by the presence of the additional participant. A pre-formed fullerene cage would almost certainly be broken by insertion of a metallic cluster into its interior, as the available windows, hexagonal, pentagonal or heptagonal faces, are small. It seems highly improbable that a metallic cluster will insert to form an EMF directly. Thus, reasonable formation paths would seem to require

growth or shrinkage of a cage with a cluster inside, or curling up to form fullerene cages with simultaneous encapsulation of a metal atom/cluster, as is consistent with molecular dynamics simulations [9].

Recent experiments show that non-classical fullerene cages with a heptagonal ring can also encapsulate metallic clusters [10], or can be captured as chlorofullerenes [11] from the carbon-arc plasma *in situ*. Theoretical studies show that heptagonal rings may play an important role in the formation of bare fullerenes [12, 13] and trimetallic nitride template fullerenes [14]. To predict the geometrical structures of some poorly characterized metallofullerenes and seek insight into their formation, it was decided to make a systematic study of classical and non-classical isomers  $Sc_2S@C_n$  ( $n = 70-84$ ). This was carried out with the help of an extended face-spiral algorithm for construction of cage candidates with small numbers of heptagonal faces. The results show that heptagon-including metallic sulfide fullerenes  $Sc_2S@C_n$  are not competitive with classical  $Sc_2S@C_n$  in terms of total energy. Interestingly, there are strong structural similarities amongst low-energy  $Sc_2S@C_n$  isomers of equal and adjacent cage sizes. These similarities are given concrete form in terms of S-W isomerization and  $C_2$  insertion/extrusion transformations. Our results provide clues to finding new metallofullerenes from within the tens of thousands of conceivable structural isomers. They also give potentially useful information on the formation mechanisms of EMFs.

## 2. Computational Details

The isomers to be considered are generated by an extension of the face-spiral algorithm to allow up to one heptagonal and 13 pentagonal faces [15]. This approach is known to be complete in the size range. The nomenclature and labelling of isomers follow the face-spiral algorithm. Briefly, classical isomers are labelled by their positions in the sequence of canonical spirals, and a superscript (1h) is used to indicate a non-classical isomer with one heptagon, with a number indicating its position in the spiral order for these cages. Topological coordinates [15] are used to provide initial cage structures, which are then optimized for charges 0, -2, -4 and -6, first at the semi-empirical PM3 level and, for a selection of the best cages at each charge, at the B3LYP/3-21G level. Based on the energy ranking for the optimized cages with charge -4, and some other favored isomers with different charges, the favored cages are used as parents to construct  $Sc_2S$ -based EMFs. The  $Sc_2S$  moiety is placed close to the geometric centre of the cage, and irrespective of the initial shape and atom ordering (Sc-S-Sc or S-Sc-Sc) optimizes to a structure where S lies between the two Sc centres; this is a simple consequence of the fact that the moiety donates electrons to the cage, generating electrostatic repulsion between the newly formed ionic Sc centres. The numbers of isomers considered by nuclearity are shown in S1. Geometrical optimizations are

performed at B3LYP/3-21G and then B3LYP/6-31G\* levels on  $\text{Sc}_2\text{S}@C_n$  with  $n$  from 70 to 84. All calculations are performed with Gaussian 09 software [16]. The results for the favored isomers are shown in Figure 1 and Table 1, and the optimized coordinates calculated for the isomers of lowest energy are provided in S2. In the optimized structures for the favoured isomers, the Sc-S-Sc angle varies between  $140^\circ$  and  $180^\circ$  in most cases, with  $\text{Sc}_2\text{S}@C_{76}:19151$  having a much smaller angle of  $108^\circ$ . The linear Sc-S-Sc moieties are found in  $\text{Sc}_2\text{S}@C_{74}:13333$  and  $\text{Sc}_2\text{S}@C_{84}:51591$  (see coordinates in S2).

### 3. Results and discussion

Table 1 Numbers of pentagon adjacencies ( $N_{55}$ ), relative energy ( $\Delta E$ , kcal/mol), HOMO-LUMO gap (eV) and Sc-Sc distance ( $\text{\AA}$ ) for the lowest-energy isomers of  $\text{Sc}_2\text{S}@C_n$ .

Cage	$N_{55}$	$\Delta E$	Gap	Sc-Sc	Cage	$N_{55}$	$\Delta E$	Gap	Sc-Sc	Cage	$N_{55}$	$\Delta E$	Gap	Sc-Sc
$C_{70}:7892-C_2$	2	0.0	1.84	3.597	$C_{76}:19151-T_d$	0	0.0	1.45	3.801	$C_{82}:39715-Cs$	0	0.0	1.72	4.073
$C_{70}:7957-C_2$	2	20.8	1.54	4.584	$C_{76}:17490-C_3$	2	11.3	1.32	4.546	$C_{82}:39717-C3v$	0	0.3	2.08	4.008
$C_{70}:7924-C_1$	2	20.9	1.35	4.241	$C_{76}:19138-C_1$	1	12.0	0.76	4.599	$C_{82}^{1h}:155199(C1)$	1	12.8	1.81	4.129
$C_{72}:10528-C_5$	2	0.0	1.85	4.225	$C_{78}:24088-C_{2v}$	2	0.0	1.76	4.207	$C_{82}:39718-C2v$	0	14.3	1.20	4.363
$C_{72}:10616-C_5$	2	13.9	1.93	4.425	$C_{78}:24109-D_{3h}$	0	1.7	0.97	4.485	$C_{84}:51591-D2d$	0	0.0	1.68	4.621
$C_{72}:10530-C_1$	2	19.1	1.66	3.979	$C_{78}:24107-C_{2v}$	0	3.7	1.53	4.168	$C_{84}:51383-C1$	1	10.4	1.90	4.447
$C_{74}:13333-C_2$	2	0.0	1.68	4.651	$C_{80}:31923-D_{5h}$	0	0.0	1.12	4.108	$C_{84}:51580-C1$	0	12.3	1.73	4.163
$C_{74}:14246-D_{3h}$	0	11.8	1.14	3.815	$C_{80}:31922-C_{2v}$	0	9.0	1.67	3.840					
$C_{74}:14239-C_{2v}$	2	11.9	1.21	4.157	$C_{80}:31924-I_h$	0	9.2	0.84	4.196					

Figure 1 shows the structures for the three most favored isomers for each of the eight formulas  $\text{Sc}_2\text{S}@C_n$  with  $n = 70-84$ .

It is shown in Table 1 that the isomer of lowest energy isomer for  $\text{Sc}_2\text{S}@C_{70}$  has a large HOMO-LUMO gap (1.84 eV) and parent cage, the non-IPR  $C_{70}:7892$ , with two pentagon adjacencies. The stability of this isomer is in good agreement with experimental results [17]. In fact, this cage has recently been shown to be the parent of a geometrically and electronically similar cluster  $\text{Sc}_2\text{O}$  [18]. The two isomers next in energy order are also classical, but lie more than  $20 \text{ kcal mol}^{-1}$  higher.

It is shown in Table 1 that  $\text{Sc}_2\text{S}@C_{72}:10528$  is the first favored isomer in energy terms and has a large gap of 1.85 eV. The next most favored isomer is higher in energy by  $13.9 \text{ kcal mol}^{-1}$  than the first, which corresponds to an equilibrium fractional population of only 3% at 2000 K. This isomer and others of higher energy are unlikely to be isolated in significant yield. The isomer predicted to be most favored corresponds to experimental observations [19a] and recent theoretical results [19b]. The same parent cage is shared by the electronically

similar cluster  $\text{Sc}_2\text{C}_2$  in the metallic carbide fullerene  $\text{Sc}_2\text{C}_2@C_{72}$  [20], and the electronically and geometrically similar cluster  $\text{Sc}_2\text{O}$  in the metallic oxide fullerene  $\text{Sc}_2\text{O}@C_{72}$  [21].

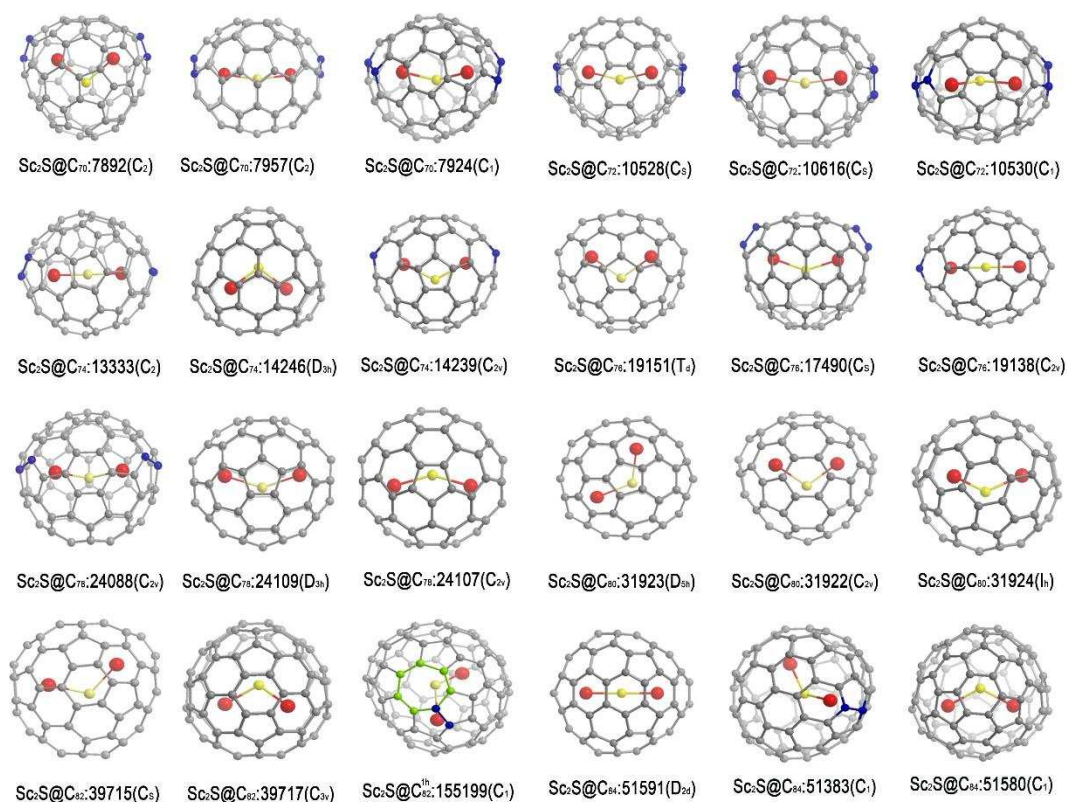


Fig. 1 Optimized structures of the three isomers of lowest energy for each  $\text{Sc}_2\text{S}@C_n$  with  $n$  from 70 to 84 as predicted at the B3LYP/6-31G\* level

We have also studied  $\text{Sc}_2\text{S}@C_{74}$  [22]. The results show the most favored isomer to be IPR-violating and  $\text{Sc}_2\text{S}$  cluster to be linear inside cage  $C_{74}$ :13333. The next two isomers in order of energy lie 11.8 and 11.9 kcal mol<sup>-1</sup> higher in energy than the first. The cage of the second favored isomer is the only IPR isomer of  $C_{74}$ , which itself is open-shell and hence reactive as a neutral cage, but is apparently passivated in the form of  $\text{Sc}_2\text{S}@C_{74}$ .

A molecule of stoichiometry  $\text{Sc}_2\text{S}@C_{76}$  has been detected by mass spectrometry, but without further experimental characterization [19]. Our calculations indicate that the isomer of lowest energy is  $\text{Sc}_2\text{S}@C_{76}$ :19151, followed by  $\text{Sc}_2\text{S}@C_{76}$ :17490, in agreement with recent theoretical results [23]. The third isomer in order of energy is  $\text{Sc}_2\text{S}@C_{76}$ :19138. XRD characterization has shown that the  $C_{76}$ :19151 cage also encapsulates the  $\text{Sc}_2\text{O}$  cluster to form  $\text{Sc}_2\text{O}@C_{76}$  [24]

For  $\text{Sc}_2\text{S}@C_{78}$ , the isomer predicted to have lowest energy has cage  $C_{78}$ :24088, a non-IPR fullerene with two pentagon pairs, instead of the well-known IPR cage,  $C_{78}$ :24109, which encapsulates two La atoms in the form of  $\text{La}_2@C_{78}$ [25] in relatively high yield. Cage  $C_{78}$ :24088 would be only ninth best in terms of the energy of the

tetra-anion  $C_{78}^{4-}$ , reminding us that a simple electron-transfer model does not account for the stability of  $Sc_2S@C_n$  in every case.

The cage of the most favored  $Sc_2S@C_{80}$  is IPR-satisfying  $C_{80}$ :31923, which gives the  $C_{80}$  tetra-anion of second lowest energy (See S2). The tetra-anion with lowest calculated energy is based on  $I_h$ -symmetrical  $C_{80}$ :31924, but the triply degenerate LUMO of this cage would tend to indicate acceptance six additional electrons, rather than four that  $Sc_2S$  can offer, and it turns out that the isomer of  $Sc_2S@C_{80}$  based on the  $I_h$  cage has only the third best predicted energy. The cage of the second favored  $Sc_2S@C_{80}$  is IPR-satisfying  $C_{80}$ :31922, which has recently been shown to be the parent of a geometrically and electronically similar cluster  $Sc_2O@C_{80}$  [26] This case demonstrates some of the complexities of the interaction between cage and encapsuland, where predictions of stability based on electronic factors may be in conflict with simple steric matching of size and shape.

In terms of calculated relative energies of the tetra-anion (See S2),  $C_{82}$ :39717 should be the best cage for encapsulating a  $Sc_2S$  cluster and, in fact, this metallic sulfide fullerene was the first to be reported for any nuclearity [27]. Our calculations predict that  $Sc_2S@C_{82}$ :39717 and the isomer of lowest calculated energy,  $Sc_2S@C_{82}$ :39715, are essentially iso-energetic. Although evidently smaller than that of  $Sc_2S@C_{82}$ :39717, the HOMO-LUMO gap of  $Sc_2S@C_{82}$ :39715 (1.71 eV) is still relatively large, and this isomer has recently been reported in experiment [28]. Interestingly, non-classical  $C_{82}^{1h}$ :155199, the cage of the third best isomer of  $Sc_2S@C_{82}$  in our calculations, could serve as a structural bridge between the cages of two experimental isomers of  $Sc_2S@C_{82}$ , as shown in Fig.2.

For  $Sc_2S@C_{84}$ , the calculations show that the favored isomer is IPR- $Sc_2S@C_{84}$ :51591, sharing a parent cage with  $Sc_2C_2@C_{84}$  [29]; the second lowest isomer is non-IPR  $Sc_2S@C_{84}$ :51383, with predicted high kinetic stability. Our recently reported calculations show that  $Sc_2S@C_{84}$ :51575 is favored at high temperature ( $\sim 2800$  K); this last isomer can transform to the most favorable isomer  $Sc_2S@C_{84}$ :51591 via S-W rotation [30].

Geometry optimizations on  $Sc_2S@C_n$  with non-classical candidate cages show none that are competitive with classical isomers. The best of the non-classical  $Sc_2S@C_n$  isomers with  $n = 70$  to  $82$  lie respectively 28.5, 32.5, 34.9, 27.8, 34.3, 17.9 and 28.2 kcal mol<sup>-1</sup> higher than their classical competitors. Thus, isolation of non-classical isomers of these compounds is unlikely.

### Cluster effect

$I_h-C_{80}$  is the commonest parent cage for encapsulation of TNT clusters, as demonstrated by numerous experimental observations [4-6]. However, it is not favored for encaging  $Sc_2S$ . There are other indications of differences in stability for  $Sc_2S$  and TNT clusters. For example, as mentioned above,  $C_{82}$ :39718, the cage of the

most favored isomer of  $\text{Sc}_3\text{N}@C_{82}$  [31] is a structural bridge between the cages of two favored candidates, i.e.,  $C_{82}:39717$  and  $C_{82}:39715$  for encaging TNT clusters. However, when the encaged cluster is  $\text{Sc}_2\text{S}$ , the bridging isomer is the non-classical isomer  $C_{82}^{\text{th}}:155199$ . Significantly,  $C_2$  addition to  $C_{82}:39715$ , the favored cage for encaging  $\text{Sc}_2\text{S}$ , can form  $C_{84}:51383$ , the parent cage of the second  $\text{Sc}_2\text{S}@C_{84}$ ; however,  $C_2$  addition to  $C_{82}:39718$ , the parent cage of the favored  $\text{Sc}_3\text{N}@C_{82}$  can form  $C_{84}:51365$ , the parent cage of EMFs  $\text{M}_3\text{N}@C_{84}$  ( $\text{M} = \text{Tb}, \text{Tm}$  and  $\text{Gd}$ ) [32, 33]. These results suggest that the encaged cluster mediates, or even controls, the formation mechanism; this important role could arise from differences in electron-donating capacity (six electrons for TNT cluster and four for the  $\text{Sc}_2\text{S}$  or  $\text{Sc}_2\text{O}$  cluster) and/or different degrees of geometrical match to the cages.

Our recent calculations have shown that the lower symmetry and local deformations associated with introduction of a heptagonal ring favor encapsulation of mixed (and intrinsically less symmetrical) metal nitride clusters [14]. In the present case, however, no non-classical  $\text{Sc}_2\text{S}$ -based EMF is predicted to be competitive with those based on classical cages. Non-classical cages that include one heptagonal face will also tend to have more pentagon adjacencies, almost universally [34] destabilizing in the neutral. As there are two metallic atoms rather than three, it may be easier for them to find suitable internal positions, whether the cage is symmetrical or not, especially as  $\text{Sc}_2\text{S}$  has a low bending force constant [35, 36] and hence is intrinsically more flexible than an  $\text{M}_3\text{N}$  cluster. This may account for the lesser role of non-classical cages in encapsulation of the  $\text{Sc}_2\text{S}$  cluster.

### Structural interdependence

The calculations here give a set of favored structures for  $\text{Sc}_2\text{S}$ -based EMFs with eight different cage sizes. As noted above, all predictions for isomers of lowest energy are in agreement with available experimental and theoretical data. Interestingly, there is an evident structural dependence among the parent cages of the low-energy isomers  $\text{Sc}_2\text{S}@C_n$ . As shown in Fig. 2, a  $C_2$  unit added to  $C_{70}:7982$  can form  $C_{72}:10528$ , and a further  $C_2$  addition forms  $C_{74}:13333$ . The corresponding  $\text{Sc}_2\text{S}$ -based EMFs are either those experimentally reported or theoretically predicted to be of lowest energy. A further addition leads to  $C_{76}:17490$ , the parent cage of the second favored isomer of  $\text{Sc}_2\text{S}@C_{76}$ . Further  $C_2$  addition to  $C_{76}:17490$  can form  $C_{78}:22010$ , the parent cage of many TNT fullerenes [37, 38]; another  $C_2$  addition can form  $C_{80}^{\text{th}}:112912$ , one Stone-Wales step away from the well-known  $I_h$ -symmetric  $C_{80}:31924$ , the favored cage for hexavalent ( $\text{M}_3\text{N}$ ) clusters. Carrying on the process of  $C_2$  addition can lead to the parent cage ( $C_{82}:39717$ ) of the first experimentally reported isomer of  $\text{Sc}_2\text{S}@C_{82}$ , and the parent cage of the recently reported, and essentially iso-energetic isomer,  $\text{Sc}_2@C_{82}:39715$  with consecutive S-W rotations.  $C_2$  addition on  $C_{82}:39718$  can form  $C_{84}:51365$ , the parent of  $\text{M}_3\text{N}@C_{84}$  ( $\text{M}=\text{Tb}, \text{Tm}$  and  $\text{Gd}$ ) [32, 33].

In another part of the map, the unique IPR isomer of  $C_{70}$ , i.e.,  $C_{70}:8149$ , has a tetra-anion energy that makes it

the favored candidate for encaging a cluster that would donate four electrons.  $C_2$  addition to  $C_{70}$ :8149 can lead to  $C_{72}$ :11188, fifth best in terms of its tetra-anion energy. In turn,  $C_{72}$ :11188 can isomerize into a non-classical isomer  $C_{72}^{1h}$ :29907 with two pairs of fused pentagons, which can form  $C_{74}$ :14246 via directly  $C_2$  addition.  $C_{74}$ :14246 can form  $C_{76}$ :19138 and  $C_{78}$ :24088 via successive  $C_2$  addition. Interestingly,  $C_2$  addition on the parent cage of most favored  $Sc_2S@C_{80}$ :31923 can lead to  $C_{82}$ :39663, which has recently been proved to be the parent of  $Gd_3N@C_{82}$ [39, 40], this cage is an S-W rotation away from the parent cage of the most favored isomer  $Sc_2S@C_{82}$ :39715.

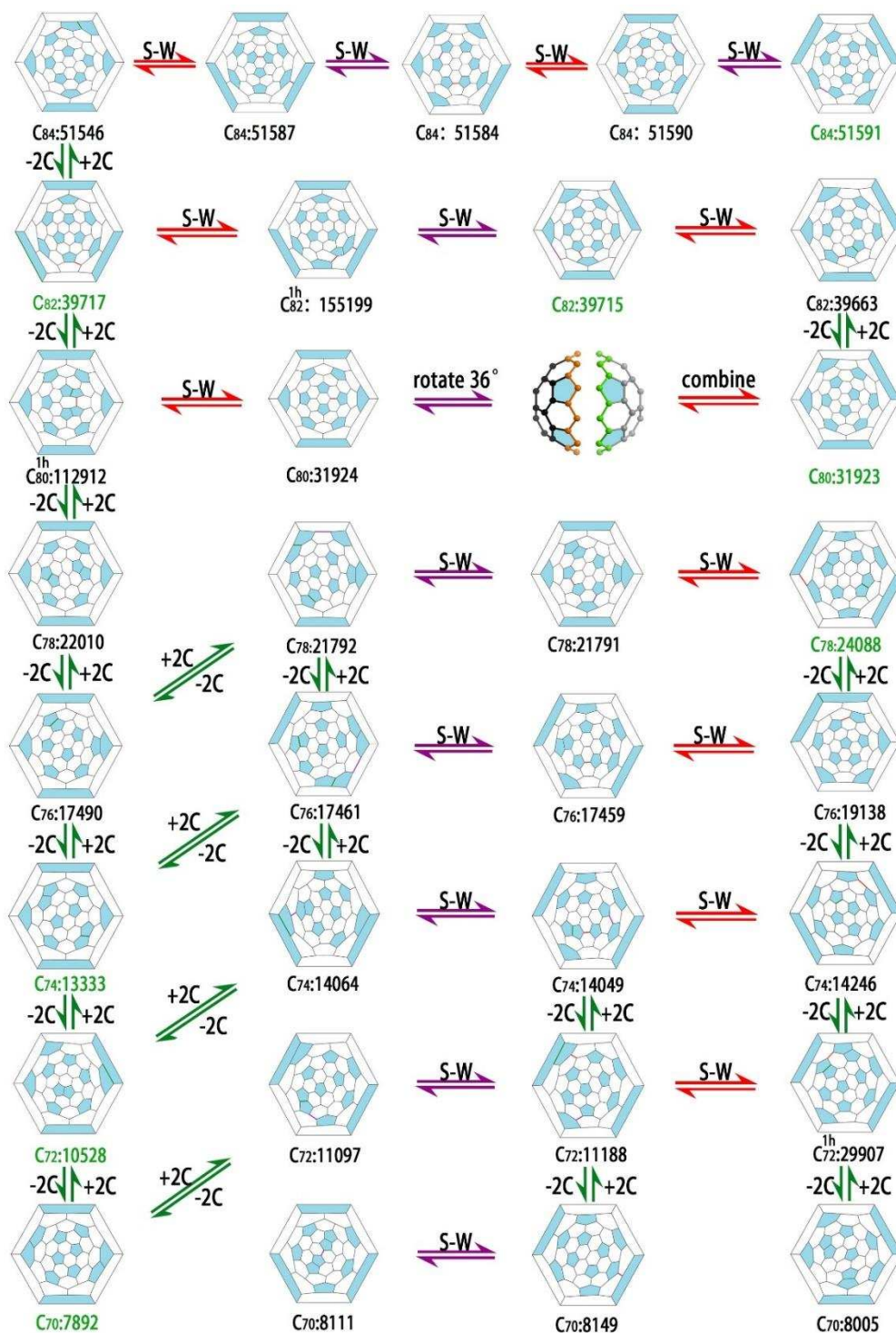




Fig. 2 The structural web (atlas) of  $Sc_2S@C_n$  isomers. Arrows in red and purple indicate that a Stone-Wales isomerization can interconvert two isomers; colored edges indicate the atoms undergoing a Stone-Wales rotation.

Exploration of the details encoded in the figure show that the vertical and diagonal connections in the map provide a route for expansion/shrinkage of all eight parent cages of low-energy isomers from  $Sc_2S@C_{70}$  to  $Sc_2S@C_{84}$ .

$C_{70}$ :7892, the parent of the most favored isomer of  $Sc_2S@C_{70}$  can transform into  $C_{70}$ :8111 via a  $C_2$  extrusion ( $C_{68}$ :6094) and  $C_2$  insertion, and then isomerize into  $C_{70}$ :8149;  $C_{68}$ :6094 has been captured as  $C_{68}Cl_8$  by in situ chlorination in the gas phase during radio-frequency synthesis [41] and theoretically predicted to be the parent of  $Sc_2O_2$  [42].  $C_{72}$ :10528, the parent of the most favored isomer of  $Sc_2S@C_{72}$ , has a transformation to  $C_{72}$ :11188, one of the molecules of lowest energy for this stoichiometry. The isomer of lowest energy  $Sc_2S@C_{74}$ :13333, can transform into the second-best  $Sc_2S@C_{74}$ :14246 as shown in Fig.2.  $C_{76}$ :14790 can transform into  $C_{76}$ :19138, the second best cage for both tetra-valent and di-valent clusters, according to the relative energies of the anions. In fact,  $C_{76}$ :19138 has been shown to encage divalent Sm [43].  $C_{76}$ :19142 can transform into  $C_{76}$ :19151 via an S-W rotation (not shown), the parent cage of lowest energy for  $Sc_2S@C_{76}$ .  $C_{78}$ :24099 can transform into  $C_{78}$ :24109 via an S-W rotation, the isomer most favored for encaging  $Sc_2O$  [44].  $C_{78}$ :22010, the most favored cage for the large metallic cluster  $Gd_3N$  [37] can transform into  $C_{78}$ :24088, the most favored cage for  $Sc_2S$ . Interestingly,  $C_{70}$ :7892,  $C_{72}$ :10528,  $C_{74}$ :13333,  $C_{76}$ :17490 and  $C_{78}$ :22010 all have similar transformations (extrusion/insertion and then isomerization) to their low energy counterparts. The  $I_h-C_{80}$  cage can transfer into  $D_{5h}-C_{80}$  via three kinds of path; the first is by growth, shrinkage and isomerization, the second is by shrinkage, growth and isomerization; the third path (at least in a formal mathematical sense) is by a  $36^\circ$  rotation around a  $C_5$  axis of one half of  $I_h-C_{80}$  against the other.  $C_{82}$ :39717 can isomerize into  $C_{82}$ :39715, the parent cage of a recently experimentally reported isomer  $Sc_2S@C_{82}$  [28] via a non-classical bridge cage,  $C_{82}^{hh}$ :155199. Successive S-W rotations can transfer  $C_{84}$ :51546 into the most favored neutral cage  $C_{84}$ :51590, and the parent cage of lowest energy  $Sc_2S@C_{84}$ ,  $C_{84}$ :51591.

In summary, favored cages can grow or shrink in the vertical direction on the map, or isomerize in the horizontal direction into other favored cages. Horizontal connections in the map typically provide a possible route from one favored isomer to another of  $Sc_2S@C_n$ . These include the global rotation of one hemisphere against another, which would have a high energy barrier for a pre-formed cluster, but for a mechanism involving combination of fragments would be a low-energy process. The parent fullerene cages and the low-energy  $Sc_2S$ -based EMFs form a web of complex genealogical relationships. Many of the cages in the near region of this map are favored for encapsulation of other types of mono-metallic, di-metallic or metallic clusters, all

demonstrating common stabilizing substructures and motifs.

### Formation mechanisms

The calculations of optimal structures demonstrate that  $C_2$  insertion/extrusion is not only a topological requirement but also a possible bridge for growth of favored  $Sc_2S$ -based EMFs, connecting isomers of low energy. As the most common metallic atom(s) or clusters would have extremely high barriers to insertion through a pentagon or hexagon of fullerene, it seems likely that the  $Sc_2S$  cluster would be encaged early in the process inside a fullerene (or fullerene-like) cage that could continue to grow or shrink via  $C_2$  insertion or extrusion, eventually forming a stable EMF.  $D_{3h}-C_{78}$ ,  $I_h-C_{80}$ ,  $D_{5h}-C_{80}$ ,  $C_{2v}-C_{82}$ ,  $C_{3v}-C_{82}$  and  $D_{2d}-C_{84}$  are all IPR-satisfying and comprise the set of parent cages of most EMFs, but they cannot be formed by direct  $C_2$  insertion into a lower IPR fullerene, and so at least one S-W isomerization step is necessary. This remains true in models of growth from smaller clusters, since even if an IPR-satisfying cage of  $C_{n+2}$  can be formed directly via  $C_2$  insertion into the heptagon of a non-classical isomer, an S-W isomerization would be needed for initial transformation from a classical isomer.

Molecular dynamics simulations have shown that hot giant fullerenes can lose or gain carbon in high temperature conditions [45]. This existence of a structural web such as shown in Fig. 2 suggests that, irrespective of the formation mechanism of  $Sc_2S@C_n$ , there are generally many  $Sc_2S@C_n$  species with different sizes, since any as-formed  $Sc_2S@C_n$  can produce other metallic sulfide fullerenes via top-down [46] or bottom-up [47] paths and S-W isomerization. Experiment shows that there are always many  $Sc_2S@C_n$  species in the soot [17, 19a, 48], and multiple species with different metallic atoms or clusters encaged inside different fullerene cages are produced under the same reaction conditions. For example, a mixture of several  $Sc_2O@C_n$ , multiple  $Sc_2C_2@C_n$ , and even  $Sc_3N@C_{80}$  species are found in soot produced under conditions designed to yield  $Sc_2O@C_n$  by introducing  $CO_2$  as the oxygen source during the arcing process [21]; a mixture of  $Sc_2S@C_n$  and  $Sc_2C_2@C_{80}$  isomers is found in soot in a conventional Krätschmer-Huffman reactor for producing metallic sulfide fullerenes under an atmosphere of  $SO_2$  [19]. The difficulty in isolating a particular EMF is mainly a result of the diversity of EMFs in reaction mixtures.

Recently, Wang et al. identified four key structural motifs which govern the relative energies of anions of classical fullerene cages and thus can be used to search for suitable cages in endohedral metallofullerenes [49]. This is a significant finding, but more rules would be desirable since these motifs are compatible with many structural isomers. Our study helps to refine the predictive picture by setting the mass of experimental results scattered in the literature into the context of a relatively small family of low-energy isomers.

A further remark can be made. We have considered fullerenes encapsulating metallic sulfide clusters from the point of view of their structural interdependence. However, our main findings are likely to be valid for  $\text{Sc}_2\text{C}_2$  and  $\text{Sc}_2\text{O}$  in fullerene cages. The carbide  $\text{Sc}_2\text{C}_2$  has similar electronic properties to  $\text{Sc}_2\text{S}$ , and  $\text{Sc}_2\text{O}$  and  $\text{Sc}_2\text{S}$  are similar in both electronic and geometrical requirements. Thus, the present calculations may have implications for a larger family of EMFs beyond compounds of  $\text{Sc}_2\text{S}$ .

#### 4. Conclusion

Systematic DFT calculations demonstrate that there is a close structural interdependence amongst the favored isomers of  $\text{Sc}_2\text{S}@C_n$  with size from  $C_{70}$  to  $C_{84}$ , and that one-heptagon non-classical  $\text{Sc}_2\text{S}@C_n$  are generally much less competitive relative to classical cages in terms of total energetics. This is quite unlike the situation that we found for tri-metallic nitride fullerenes. These results indicate that formation of  $\text{Sc}_2\text{S}@C_n$  and other EMFs is guided by the encaged clusters, and suggests a similar formation route in that cages featuring as preferred have closely related structures.

As a whole, the stability of EMFs is determined by two main factors: one is electron transfer and the other is effect of size. Calculations show that many of the lowest energy isomers of  $\text{Sc}_2\text{O}@C_n$  share the same cages with  $\text{Sc}_2\text{S}@C_n$  apart from the cases with  $n=74, 78$  and  $84$ . Since both  $\text{Sc}_2\text{O}$  and  $\text{Sc}_2\text{S}$  tend to donate 4 electrons to a fullerene cage, they tend to select the same isomer as host cage and these results indicate that electron transfer interactions play a vital role. Of course, the cluster sizes are different, and rankings are evidently different when comparing structures involving a series of cages of size  $C_n$  (when the sizes and shapes of both cages and the encaged clusters play an important role).

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