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POSITIVELY GAMMA DISCOUNTING: COMBINING THE OPINIONS OF EXPERTS ON THE SOCIAL DISCOUNT RATE*

Mark C. Freeman and Ben Groom

The aggregated term structure of social discount rates that results from Weitzman’s (2001) survey of expert opinion is shown to be highly sensitive to the nature of the responses. If variation reflects irreducible differences in ethical judgements, the term structure can decline rapidly. If variation occurred because respondents were forecasting future rates under uncertainty, the term structure is much flatter because additional experts provide new information. The former approach triples the social cost of carbon when compared to the latter. The distinction between heterogeneity and uncertainty illustrates the need for a nuanced treatment of survey data in intergenerational policy making.

The issue of social discounting has long been a major source of disagreement amongst economists and philosophers, with some perspectives being described as not simply myopic but ‘ethically indefensible’, ‘rapacious’ and ‘defective’. Such strong sentiments arise from the fact that the estimated present values of very long-term projects are generally highly sensitive to the choice of discount rates that are deployed. For example, the present value of £1 in 100 years is fifty times higher when discounted at 1% than at 5%, and this ratio increases exponentially with the time horizon. As a consequence, the policy prescriptions on intergenerational projects are often determined by the rate at which the costs and benefits are discounted. Indeed, some have argued that the immediate and dramatic action on climate change recommended by the Stern Review on the Economics of Climate Change (Stern, 2007) resulted solely from the inappropriately low social discount rate (SDR) that

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was used, a position which evoked accusations of ‘perhaps stoking the dying embers of the British Empire’ (Nordhaus, 2007, p.691).

In recent years, however, something resembling a consensus has emerged in the field of social discounting. A recent *Policy Forum* article in *Science* summarises the case for using lower rates for discounting long-term costs and benefits than their short-term equivalent counterparts (Arrow *et al.*, 2013). Reviews of the relevant academic literature that supports this approach have also recently been provided by Gollier (2012) and Arrow *et al.* (2012). The UK, French, Norwegian and Danish governments all now recommend schedules of declining discount rates (DDRs) as the time horizon increases.1 Similar policy recommendations are currently being considered by the authorities in the US.

In this paper we return to a study that has been highly influential in shaping the policy landscape for declining discount rates. Weitzman (2001) sought the opinion of a large number of economists on the appropriate discount rate that should be used for calculating the present value of future global warming damages. The responses to the survey were widely dispersed, with the sample frequency distribution closely resembling the probability density function of a gamma distribution. When these responses were aggregated according to Professor Weitzman’s preferred method, known as ‘gamma discounting’, the resulting SDR schedule declined sharply.

We focus here on the interpretation of the expert responses and the method by which they were combined to calculate the social discount rate within gamma discounting. Specifically, the question that Professor Weitzman asked contained a significant ambiguity. As a consequence, experts might have interpreted the survey in one of two distinct ways. Our central point is that the appropriate method of aggregation depends critically on which of these interpretations each expert had in mind when responding.

Under the first interpretation, experts might have revealed their individual ethical views.

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1 Within the UK, the Treasury-recommended DDR schedule forms the basis for the economic evaluation of the High Speed 2 rail link by government. It is also used for capital budgeting purposes by the Nuclear Decommissioning Authority.
concerning intergenerational justice. Differences in such subjective opinions are essentially irreducible. In this case, sampling additional experts only serves to characterise better the extent of disagreement and does nothing to diminish the variation in responses. This may reasonably lead the social planner to construct the same schedule of SDRs as reported by Weitzman (2001).

Under the second interpretation, the variation in survey responses might instead have reflected forecasting errors about some objective ‘true’ value. These responses then reveal the nature of our uncertainty about the future rather than the extent of heterogeneity in ethical positions. In this case, increasing the sample of experts provides additional information to the social planner, improving the quality of the aggregated forecast. We show that this generates a term structure of SDRs which declines slowly and, in some cases, is essentially flat.

This point has important implications for economic valuations made across a wide range of key policy areas. We demonstrate this through four examples; the social cost of carbon, the costs of teenage obesity, nuclear decommissioning costs and the economic benefits of the High Speed 2 (HS2) rail link.

1. The Survey Question

The basis of the gamma discounting framework of Weitzman (2001) was an emailed survey to PhD-level economists that generated \( n = 2160 \) responses, \( r_i \) for \( i \in [1, n] \), to the question ‘Taking all relevant considerations into account, what real interest rate do you think should be used to discount over time the (expected) benefits and (expected) costs of projects being proposed to mitigate the possible effects of global climate change?’

The individual responses were widely dispersed with a range from -3% to +27%. The problem that then faces the social planner is how to aggregate this range of values into a single social discount rate, \( R(H) \), to apply when discounting a certainty-equivalent cash
flow that will arrive at time $H$. Without fully explaining his rationale, Weitzman (2001) proposed taking the simple average of individual discount factors:

$$\exp(-HR(H)) = \frac{1}{n} \sum_{i=1}^{n} \exp(-Hr_i)$$

(1)

Our analysis centres on equation 1. For now we note that $R(H)$ when defined this way has the following properties: $\lim_{H \to 0} R(H) = r_i$, where $r_i$ is the mean of the individual responses, $\lim_{H \to \infty} R(H) = \min \{r_i\}$, and $dR(H)/dH < 0$. The decline in the term structure occurs since ‘from today’s perspective, the only relevant limiting scenario is the one with the lowest interest rate — all of the other states at that far distant time, by comparison, are relatively much less important now because their present value has been reduced by the power of compound discounting at a higher rate’ (Weitzman, 1998, p.205).

The reason for the declining discount rate also has a simple mathematical explanation. Exponential functions are convex, so by Jensen’s inequality $n^{-1} \sum_{i=1}^{n} \exp(-Hr_i) > \exp(-Hr_i)$. The greater $H$, the more curved the exponential function, explaining why the appropriate SDR declines with the maturity of the project. Further, the more dispersed the responses, the greater the magnitude of the Jensen’s inequality. These points can be simply demonstrated by example. Suppose that there were only two responses; $r_1 = 3\%$ and $r_2 = 5\%$. Then $R(H) = [4.0\%, 3.9\%, 3.6\%, 3.2\%]$ for $H = [1, 30, 100, 400]$. By contrast, if we preserve the mean but increase the spread of the responses by setting $r_1 = 1\%$ and $r_2 = 7\%$, $R(H) = [4.0\%, 2.8\%, 1.7\%, 1.2\%]$ for the same values of $H$.

Rather than applying equation 1 directly, Weitzman (2001) took the following elegant analytical approximation. He noticed that the sample frequency of responses, $\phi(r_i)$, closely resembled the probability density function of a gamma distribution with shape parameter $\alpha$ and rate parameter $\beta$. Using this continuous distribution to describe the individual
responses, from equation 1:

\[
R(H) = -\frac{1}{H} \ln \left( \int_0^\infty e^{-Hr} \frac{\beta^\alpha}{\Gamma(\alpha)} r^{\alpha-1} e^{-\beta r} dr \right) = -\frac{\alpha}{H} \ln \left( \frac{\beta}{\beta + H} \right) \tag{2}
\]

thus giving a simple closed-form solution for the \(H\)-period discount rate.

To determine the term structure of the SDR that arises from gamma discounting, we estimate \(\alpha, \beta\) using a maximum likelihood method based on the strictly positive responses that Professor Weitzman received, giving \(\hat{\alpha} = 2.54\) and \(\hat{\beta} = 63.08\). This results in a sharply declining term structure of social discount rates; \(R(H) = 4.00\%, 3.29\%, 2.41\%\) and \(1.27\%\) for \(H = [1, 30, 100, 400]\) years respectively.

We now consider how experts might have interpreted the survey question that Professor Weitzman posed. The most prominent distinction here, as highlighted by Arrow et al. (1996), is between those who view long-term discounting as a fundamentally ethical issue, (e.g. Stern, 2008), and others who prefer to calibrate the SDR to reflect ‘market and policy factors as they currently exist’ (Nordhaus, 2007, p 692). Respectively, these are commonly referred to as normative and positive (or prescriptive and descriptive) positions on social discounting. This dichotomy was central to the controversial aftermath of the Stern Review as the low ethically derived rates applied by Stern prescribe a more urgent response to climate change than many markets-based discount rate schedules (e.g. Nordhaus, 2007; Stern, 2008).

Our contention in this paper is that the wording of the survey leaves sufficient scope to be interpreted within either of these paradigms.\(^2\) The use of the term ‘interest rate’ points towards a positivist framework. Alternatively, it seems equally likely that the reference to climate change may have evoked ethical considerations, leading to normative responses.

We do not take a stance on which was the appropriate reading of the survey question.

\(\text{\footnotesize{\(^2\)}There are, of course, a range of other possible interpretations between these two extremes. The supplementary wording of the survey included such terms as ‘gut feeling’, ‘back-of-the-envelope guesstimate’ and ‘off the top of your head’, making it clear that a variety of rationales could have underpinned any given expert’s response. Additionally, the sample may mix some purely normative responses with others that were purely positive. We dichotomize the debate for reasons of simplicity and clarity and note that this distinction is well understood in the literature.\)}}
Instead, we demonstrate that the term structure of SDRs that emerges from this survey depends crucially on whether the exercise elicited ethical responses reflecting heterogeneity, or market-based responses reflecting uncertainty.

2. Ethical Responses

Assume first that each expert took an ethical position concerning intergenerational justice when determining his or her response. Combining these heterogeneous preferences is essentially a social choice problem for which there is no uncontroversial solution and not all approaches lead to a declining social discount rate. For instance, Heal (2012) reminds us that the median value will be the outcome of a number of plausible social choice rules, including majority voting.

That said, a number of different theoretical frameworks exist that might provide justification for the use of equation 1 when determining the social discount rate in a normative context. We briefly note one example here and engage in a fuller discussion of this point in our online Appendix A.

Following Gollier and Zeckhauser (2005), Jouini et al. (2010) imagine a pure exchange economy with different agents who all have logarithmic utility. These individuals differ in three respects: their beliefs about future consumption growth, their initial endowment levels, $w_i$, and their rates of pure time preference (utility discount rate), $\delta_i > 0$. Acting atomistically, each agent would choose a different path of consumption, so disagreement arises over how to share the exogenous consumption stream. Within the framework proposed by Jouini et al. (2010) the agents are experts who disagree on consumption growth and $\delta_i$. The social planner resolves this disagreement by invoking an “as-if” market between the experts. The resulting intertemporal Arrow-Debreu equilibrium, and hence socially efficient, discount rate is given by:

$$ R(H) = -\frac{1}{H} \ln \left( \sum_{i=1}^{n} \frac{w_i \delta_i}{\sum_{j=1}^{n} w_j \delta_j} \exp \{-H r_i\} \right) $$

(3)
Equation 1 follows from here provided that \( w_i \delta_i \) is constant across experts. The efficient term structure is declining since intertemporal trade between experts leads those with a high utility discount rate to prefer paths with more consumption early on, leaving experts with low \( \delta_i \) as the chief determinants of the long-run discount rate.

This framework neatly captures the idea that variation in ethical judgements reflects fundamental and irreducible disagreements. This follows by virtue of \( R(H) \) being independent of the number of experts surveyed, \( n \), above the minimum threshold required for the sample to be representative of the population. Therefore, with ethical responses, gamma discounting can be theoretically defended, although its usage remains contentious.

3. The Positivist Interpretation

Suppose instead that the survey responses resulted from within a purely positivist framework, where experts were requested to ‘look carefully at the returns on alternative investments — at the real real interest rate—as the benchmarks for climatic investments’ (Nordhaus, 2007, p 692). In this setting, the standard model for determining the term structure of social discount rates, which has been highly influential in determining international governmental policy in this area, is the expected net present value (ENPV) condition:

\[
R(H) = -\frac{1}{H} \ln \left( E \{ \exp \left( -H\tau_H \right) \} \right)
\]

(4)

where \( \tau_H = H^{-1} \sum_{t=0}^{H-1} r_{ft} \) and \( r_{ft} \) is the yield on a Treasury bond at time \( t \). \( \tau_H \) can be interpreted as the average future risk-free rate over the horizon of interest. Empirical schedules of the SDRs using equation 4 have been provided by Newell and Pizer (2003), Groom et al. (2007) and Freeman et al. (2013) amongst others.\(^3\)

\(^3\)The theoretical case for using ENPVs within environmental economics was presented by Weitzman (1998) and subsequently discussed in detail by Traeger (2012), Gollier and Weitzman (2010), Freeman (2010) and Gollier (2009), amongst others. In our online Appendix B we note that equation 4 also has a long tradition in financial economics. In particular, the Local Expectations Hypothesis of Cox et al. (1981) is equivalent to the ENPV condition but the underlying assumptions concerning the stochastic nature of \( \tau_H \) and the
Within this setting, the most natural interpretation of each individual’s survey response is that it reflects his or her own personal estimate of the future realised value of $\bar{r}_H$; $r_i = E_i[\bar{r}_H]$. This is fundamentally different from the information content of a survey eliciting ethical opinions; $r_i$ is now a forecast of $\bar{r}_H$. Variations in response arise from asymmetric information or differences in professional judgement over how best this forecasting process might be undertaken. These different choices do not reflect fundamentally different ethical stances and are not irreducible in this sense. At time $H-1$ the true value of $\bar{r}_H$ will be revealed and, with the benefit of hindsight, we will all agree on which respondents gave the most accurate forecast.

The social planner must now decide how to combine these different forecasts. The simplest method is to take a statistical approach. Let $e_i = r_i - \bar{r}_H$ denote the forecast error of expert $i$. First assume that all experts are unbiased, $E[e_i] = 0$, that the forecast error variance of each is identical, $Var[e_i] = \sigma^2$, and that experts are independent. As Weitzman provides us with such a large sample, irrespective of the sample frequency distribution, $\phi(r_i)$, under weak regularity conditions the central limit theorem tells us that the probability density function of $\bar{r}_H$, $f(\bar{r}_H) = N(\bar{r}_i, \sigma^2/n)$. The $H-$period discount rate from equation 4 is now:

$$ R(H) = \bar{r}_i - 0.5\sigma^2 \frac{H}{n} $$

(5)

In contrast to equation 1, the appropriate measure of uncertainty is the standard error, not the standard deviation, of the sample distribution. More experts provide additional information to the social planner. This reduces her uncertainty over the “true” value of $\bar{r}_H$, which in turn lessens the Jensen’s inequality effects that drive declining schedules of social discount rates.

Of course, expert opinions are not independent; see, for example, Clemen and Winkler (1985) and Graham (1996). To account for this correlation, we generalise the statistical resolution of uncertainty are much less stylised than in the original thought experiment of Weitzman (1998).
approach by turning to the substantial literature on combining probability distributions (e.g. Genest and Zidek, 1986). To capture the fact that each additional expert now brings less fresh information than the previous one, we follow Clemen and Winkler (1985) by mapping the total number surveyed, $n$, onto $N$, the effective number of independent experts. We discuss in detail in our online Appendix C how this relatively technical exercise can be undertaken and how this then influences the social planner’s probability density function of $\bar{r}_H$, $f(\bar{r}_H)$, for use in equation 4.

Unfortunately, Weitzman’s survey does not ask experts how they arrived at their responses, and therefore it is not possible to empirically estimate the correlation between different expert forecasts. We therefore take a range of possible values; $N \in \{1000, 100, 50, 25, 12\}$.\footnote{We concentrate on exponentially correlated forecasts, which have been used in the context of gamma distributions by Kotz and Adams (1964). Forecasts are ranked in ascending order and the correlation between the $i^{th}$ and $j^{th}$ expert is assumed to be $\rho^{|i-j|}$ for constant $\rho$. The cases $N \in \{1000, 100, 50, 25, 12\}$ correspond to $\rho \in \{35.71\%, 90.95\%, 95.37\%, 97.66\%, 98.87\%\}$. See online Appendix C for further details.} For the final case, this means that the information content of Weitzman’s survey of 2,000+ economists is the same as could be found in a sample of 12 truly independent experts. The derived term structures of social discount rates are presented in Figure 1:

\[\text{[Insert Figure 1 about here]}\]

These results differ markedly from those reported by Weitzman (2001). The 400-year (30-year) discount rate is only 32\% (82\%) of the short-term rate under gamma discounting, but this increases to 72\% (97\%) when placing a positivist interpretation on the survey with $N = 12$. Increasing $N$ to 1000 leads to a term structure that is essentially flat. This reflects the fact that, with more information, forecast errors are reducible in a way that ethical opinions are not.
4. Applications

To demonstrate the importance of this point for decision making across a range of key policy areas, we consider four examples of long-term cash flows. First, we use the profile of damages from Newell and Pizer (2003) associated with each marginal ton of carbon emitted in order to estimate the social cost of carbon (SCC). Next, we take the schedule of estimated costs from decommissioning 19 now non-operational nuclear power stations in the UK as given in the Nuclear Decommissioning Authority (NDA) Report and Accounts 2012/13. Third, we use estimates from Wang et al. (2003) of the incremental costs that arise for a woman between the ages of 41–65 conditional on her being obese at the age of 14. Finally, we use official estimates of the benefits that are expected to arise between 2026–2085 from Phase 1 (London to Birmingham) of the HS2 rail link. A detailed description of each of these schedules of cash flows is given in our online Appendix D and their profile is illustrated in Figure 2:

[Insert Figure 2 about here]

We estimate present values for each of these examples using all of the term structures of the SDR presented in Figure 1. We also use the schedule recommended by the Green Book (UK Treasury, 2003) as well as a non-declining 4% rate (which is very close to $\bar{r}_i$). Results are reported in Table 1:

[Insert Table 1 about here]

The policy decisions taken by the social planner will often be significantly influenced by the schedule of discount rates selected. The greatest sensitivities are for the longest horizon cash flows, as reflected by the social cost of carbon. Here gamma discounting gives a present value (PV) that is three times as great as the $N = 12$ case. For both HS2 and teenage obesity, the gamma discounting PV is more than half as much again as the $N = 12$ PV. The PV of decommissioning costs for the previous generation of nuclear power stations is most
robust to different possible choices. However, if these cash flows are delayed by 50 years, to broadly capture the costs of decommissioning the next generation of power stations, the present value estimated from gamma discounting is 175% greater than the PV calculated from the $N = 12$ case. By contrast, in no case does the $N = 12$ PV differ by more than 10% from that calculated using a non-declining discount rate of 4% at all horizons. For larger $N$, the results become even closer to the non-declining discount rate values.

5. Conclusion

We have shown that the term structure of discount rates that results from Weitzman’s (2001) survey is highly dependent on whether the responses reflect forecasts of future risk-free interest rates or the ethics of intergenerational equity. In the former case, very long-term present value calculations barely differ from those calculated using a flat term structure. In the latter case, the term structure can decline rapidly. We have demonstrated that this has important implications across a wide range of policy areas for those making decisions with intergenerational consequences.

In our online Appendix E we make a further point. Even in a purely normative world, standard approaches potentially exaggerate the decline. This is because such responses frequently contain ex-post verifiable elements, such as the growth rate of per-capita consumption. A mixed normative-positivist approach is therefore recommended in this case, which again flattens the term structure.

Gamma discounting has been highly influential in shaping the international policy landscape on declining discount rates. This paper shows that closer scrutiny on both the motives behind individual responses and the empirics of aggregation is required before any further policy changes can be justified on the basis of such surveys of expert opinion.

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Fig. 1. The $H$-Period Social Discount Rate, $R(H)$
Fig. 2. The Proportion of the Total Cash Flow that is Estimated to Arise in Each Year
Table 1

The Present Value of Intergenerational Projects

<table>
<thead>
<tr>
<th>N</th>
<th>SCC ($/tC)</th>
<th>Current NDA (£bn)</th>
<th>Delayed NDA (£bn)</th>
<th>Teenage Obesity ($)</th>
<th>HS2 Benefits (£bn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N=1000</td>
<td>5.692</td>
<td>39.06</td>
<td>5.27</td>
<td>14,753</td>
<td>13.91</td>
</tr>
<tr>
<td>N=100</td>
<td>5.738</td>
<td>39.08</td>
<td>5.32</td>
<td>14,780</td>
<td>13.95</td>
</tr>
<tr>
<td>N=50</td>
<td>5.706</td>
<td>38.93</td>
<td>5.28</td>
<td>14,660</td>
<td>13.84</td>
</tr>
<tr>
<td>N=25</td>
<td>5.543</td>
<td>38.42</td>
<td>5.10</td>
<td>14,238</td>
<td>13.46</td>
</tr>
<tr>
<td>N=12</td>
<td>5.346</td>
<td>37.56</td>
<td>4.88</td>
<td>13,560</td>
<td>12.86</td>
</tr>
<tr>
<td>Gamma</td>
<td>15.928</td>
<td>44.97</td>
<td>13.41</td>
<td>20,767</td>
<td>21.01</td>
</tr>
<tr>
<td>Green Book</td>
<td>10.154</td>
<td>43.84</td>
<td>9.57</td>
<td>19,181</td>
<td>18.68</td>
</tr>
<tr>
<td>Flat 4%</td>
<td>5.713</td>
<td>39.11</td>
<td>5.29</td>
<td>14,796</td>
<td>13.95</td>
</tr>
</tbody>
</table>

Notes: This table presents net present values for five different cost schedules. “SCC” is the social cost of carbon in $2000 per ton of carbon. “Current NDA” costs are taken from the Nuclear Decommissioning Authority annual report and accounts 2012/13. The “Delayed NDA” costs are the same as those for current decommissioning, but delayed by 50 years to reflect the process of running down a new generation of nuclear power plants. Teenage Obesity costs monetise the estimated impacts that are realised between ages 41 and 65 from being obese at the age of 14. These values are based upon the calibration of Wang et. al. (2003). The HS2 Benefits are taken from the estimates of Net Transport Benefits provided at the hs2.org.uk website. N denotes the equivalent number of independent observers, “Gamma” refers to the gamma discounting schedule provided by Weitzman (2001). “Green Book” applies the UK Treasury’s current recommended schedule of discount rates. “Flat 4%” applies a non-declining discount rate of 4% at all horizons.