**Proceedings Paper:**

Zhang, B., Liu, W. orcid.org/0000-0003-2968-2888 and Gou, X. (2016) Sparse antenna array design for directional modulation. In: Sensor Array and Multichannel Signal Processing Workshop (SAM). 2016 9th IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM 2016), July 10-13, 2016, Rio de Janeiro, Brazil. IEEE . ISBN 9781509021031

https://doi.org/10.1109/SAM.2016.7569671

# Sparse Antenna Array Design for Directional Modulation

Bo Zhang†, Wei Liu† and Xiaoming Gou*
† Communications Research Group, Department of Electronic and Electrical Engineering
University of Sheffield, Sheffield S1 3JD, United Kingdom
* School of Information and Electronics, Beijing Institute of Technology, Beijing 100081, China

***Abstract*—Directional modulation (DM) can be achieved based on uniform linear arrays (ULAs), where the maximum half wavelength spacing is needed to avoid spatial aliasing. To exploit the degrees of freedom (DOFs) in the spatial domain, sparse arrays can be employed for more effective DM design. In this paper, the problem of antenna location optimisation for sparse arrays in the context of DM is addressed for the first time, where compressive sensing based formulations are proposed employing the group sparsity concept. Design examples are provided to verify the effectiveness of the proposed designs.**

**Keywords:** Directional modulation, sparse array, compressive sensing, group sparsity.

## I. INTRODUCTION

In conventional wireless communication systems, since the same constellation mappings are used in all directions of the transmit antennas, it is possible for the signals to be captured and demodulated by highly sensitive eavesdroppers even if they are located in sidelobe regions of the antennas. To avoid this, the directional modulation (DM) technique has been developed to improve security by keeping known constellation mappings in a desired direction or directions, while scrambling them for the remaining ones [1, 2].

In [3], a reconfigurable array is designed by switching elements for each symbol to make their constellation points not scrambled in desired directions, but distorted in other directions. A method named dual beam DM was introduced in [4], where the I and Q signals are transmitted by different antennas. In [5], phased arrays are employed to show that DM can be implemented by phase shifting the transmitted antenna signals properly. The bit error rate (BER) performance of a system based on a two-antenna array was studied using the DM technique for eight phase shift keying modulation in [6]. A more systematic pattern synthesis approach was presented in [7], followed by an energy-constrained design in [8]. Recently, in [9], the time modulation technique was introduced to DM to form a four-dimensional (4-D) antenna array.

However, most existing research in DM is based on uniform linear arrays (ULAs) with a maximum half wavelength spacing to avoid grating lobes. To have a larger aperture and a higher spatial resolution given a fixed number of antennas, sparse arrays are normally employed in traditional array signal processing [10, 11]. The increased degrees of freedom (DOFs) in the spatial domain allow the system to incorporate more constraints into the design of various beamformers. Many methods have been proposed to design such a sparse array, including the genetic algorithm (GA) [12–14], simulated annealing (SA) [15], and compressive sensing (CS) [16–21].

In this work, we extend the CS-based sparse array design to the area of DM and try to optimise the antenna locations for a given set of modulation symbols and desired transmission directions. The key is to realise that we can not perform this optimsation individually for each symbol; otherwise we would end up with different antenna locations for different transmission symbols. Instead we need to find a common set of optimised antenna locations for all required transmission symbols with the desired directions. As a result, the traditional CS-based narrowband sparse array design methods will not work and group sparsity based approach has to be adopted, and a class of CS-based design methods is proposed for the design of sparse arrays for direction modulation.

The remaining part of this paper is structured as follows. A review of the DM technique based on phased arrays is given in Sec. II. The class of CS-based design methods is presented in Sec. III, including $l_1$ norm minimisation and reweighted $l_1$ norm minimisation. In Sec. IV, design examples are provided, with conclusions drawn in Sec. V.

## II. REVIEW OF DIRECTIONAL MODULATION

### *A. Narrowband beamforming based on ULAs*

A narrowband linear array for transmit beamforming is shown in Fig. 1, consisting of $N$ equally spaced omni-directional antennas with the spacing from the first antenna to its subsequent antennas represented by $d_n$ for $n = 1, \ldots, N-1$, where the transmission angle $\theta \in [0^\circ, 180^\circ]$. The output signal and weight coefficient for each antenna are respectively denoted by $x_n$ and $w_n$ for $n = 1, \ldots, N$. The steering vector of the array is a function of angular frequency $\omega$ and transmission angle $\theta$, given by

$$\mathbf{s}(\omega, \theta) = [1, e^{j\omega d_1 \cos\theta/c}, \ldots, e^{j\omega d_{N-1}\cos\theta/c}]^T, \quad (1)$$

where $\{\cdot\}^T$ is the transpose operation, and $c$ is the speed of propagation. For a ULA with a half-wavelength spacing ($d_n - d_{n-1} = \lambda/2$), the steering vector is simplified to

$$\mathbf{s}(\omega, \theta) = [1, e^{j\pi\cos\theta}, \ldots, e^{j\pi(N-1)\cos\theta}]^T. \quad (2)$$

Then the beam response of the array is given by

$$\mathrm{p}(\theta) = \mathbf{w}^H \mathbf{s}(\omega, \theta), \quad (3)$$

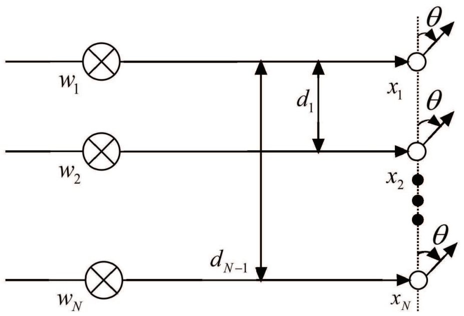


Fig. 1. A narrowband transmit beamforming structure.

where $\{\cdot\}^H$ represents the Hermitian transpose, and $\mathbf{w}$ is the weight vector including all corresponding coefficients

$$\mathbf{w} = [w_1, w_2, \ldots, w_N]^T. \tag{4}$$

*B. DM design for a given array geometry*

The objective of DM design for a given array geometry is to find the set of weight coefficients giving the desired constellation values in the directions of interest while scrambling the values and simultaneously maintaining a magnitude response as low as possible in other directions. For $M$-ary signaling, such as multiple phase shift keying (MPSK), there are $M$ sets of desired array responses $p_m(\theta)$, with a corresponding weight vector $\mathbf{w}_m = [w_{m,1}, \ldots, w_{m,N}]^T$, $m = 1, , \ldots, M$. Each desired response $p_m(\theta)$ as a function of $\theta$ is split into two regions: the mainlobe and the sidelobe. We sample each region and put the sampled desired responses into two vectors $\mathbf{p}_{m,ML}$ and $\mathbf{p}_{m,SL}$, respectively. Without loss of generality, we consider only one point $\theta_{ML}$ in the mainlobe and $R-1$ points $\theta_1, \theta_2, \ldots, \theta_{R-1}$ in the sidelobe region. Therefore, we have

$$\begin{aligned} \mathbf{p}_{m,SL} &= [p_m(\theta_1), p_m(\theta_2), \ldots, p_m(\theta_{R-1})] \\ \mathbf{p}_{m,ML} &= p_m(\theta_{ML}) \ . \end{aligned} \tag{5}$$

All constellation points for a fixed $\theta$ share the same steering vector and we put all the $R-1$ steering vectors at the sidelobe region into an $N \times (R-1)$ matrix $\mathbf{S}_{SL}$, and the steering vector at the mainlobe direction $\theta_{ML}$ is denoted by $\mathbf{s}(\theta_{ML})$. For the $m$-th constellation point, its corresponding weight coefficients can be found by

$$\begin{aligned} \min \quad & ||\mathbf{p}_{m,SL} - \mathbf{w}_m^H \mathbf{S}_{SL}||_2 \\ \text{subject to} \quad & \mathbf{w}_m^H \mathbf{s}(\theta_{ML}) = p_{m,ML}, \end{aligned} \tag{6}$$

where $||\cdot||_2$ denotes the $l_2$ norm. The objective function and constraint in (6) ensure a minimum difference between desired and designed responses in the sidelobe, and a desired constellation value to the mainlobe or the direction of interest. To ensure that constellation is scrambled in the sidelobe regions, the phase of the desired response $\mathbf{w}_m^H \mathbf{S}_{SL}$ at different sidelobe directions can be randomly generated.

## III. Proposed design method

*A. Group sparsity based design*

For a standard sparse array design method [21], a given aperture is densely sampled with a large number of potential antennas. First, consider Fig. 1 as being a grid of potential active antenna locations. Then $d_{N-1}$ is the aperture of the array and the values of $d_n$, for $n = 1, 2, \ldots, N-1$, are selected to give a uniform grid, with $N$ being a very large number. Through selecting the minimum number of non-zero valued weight coefficients to generate a response close to the desired one, sparseness is introduced. In other words, if a weight coefficient is zero-valued, the corresponding antenna will be inactive and therefore can be removed, leading to a sparse result. Assume $\mathbf{p}$ is the vector holding the desired responses at the $R$ sampled angles (one point in the mainlobe and $R-1$ points in the sidelobe as described earlier), and $\mathbf{S}$ is the $N \times R$ matrix composed of the $R$ steering vectors. Then the design can be formulated as follows

$$\min \quad ||\mathbf{w}||_1 \quad \text{subject to} \quad ||\mathbf{p} - \mathbf{w}^H \mathbf{S}||_2 \leq \alpha, \tag{7}$$

where the $l_1$ norm $||\cdot||_1$ is used as an approximation to the $l_0$ norm $||\cdot||_0$, and $\alpha$ is the allowed difference between the desired and designed responses.

Now, in the context of sparse array design for DM, we could modify (6) and find the sparse set of weight coefficients $\mathbf{w}_m$ through the following formulation

$$\begin{aligned} \min \quad ||\mathbf{w}_m||_1 \quad \text{subject to} \quad & ||\mathbf{p}_{m,SL} - \mathbf{w}_m^H \mathbf{S}_{SL}||_2 \leq \alpha \\ & \mathbf{w}_m^H \mathbf{s}(\theta_{ML}) = p_{m,ML}. \end{aligned} \tag{8}$$

However, the solution to (8) cannot guarantee the same set of active antenna positions for all constellation points. If a weight coefficient is zero in an antenna position for one constellation point, but non-zero for others, the antenna still cannot be removed. To solve the problem, similar to [22], group sparsity is introduced here, which imposes zero-valued coefficients at the same antenna locations for all constellation points simultaneously. To achieve this, we first construct the following matrices

$$\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_M] \tag{9}$$

$$\mathbf{P}_{SL} = [\mathbf{p}_{1,SL}, \mathbf{p}_{2,SL}, \cdots, \mathbf{p}_{M,SL}]^T \ , \tag{10}$$

and the vector

$$\mathbf{p}_{ML} = [p_{1,ML}, p_{2,ML}, \ldots, p_{M,ML}]^T \ . \tag{11}$$

Each row of the $N \times M$ weight matrix $\mathbf{W}$ holds the weight coefficients at the same antenna location for different constellation points and it is denoted by $\tilde{\mathbf{w}}_n = [w_{n,1}, \ldots, w_{n,M}]$ for $n = 1, \ldots, N$. Now define $\hat{\mathbf{w}}$ as a vector of $l_2$ norm of $\tilde{\mathbf{w}}_n$, given by

$$\hat{\mathbf{w}} = [||\tilde{\mathbf{w}}_1||_2, ||\tilde{\mathbf{w}}_2||_2, \ldots, ||\tilde{\mathbf{w}}_N||_2]^T. \tag{12}$$

Then the group sparsity based sparse array design for DM can be formulated as

$$\begin{aligned} \min \quad ||\hat{\mathbf{w}}||_1 \quad \text{subject to} \quad & ||\mathbf{P}_{SL} - \mathbf{W}^H \mathbf{S}_{SL}||_2 \leq \alpha \\ & \mathbf{W}^H \mathbf{s}_{ML} = \mathbf{p}_{ML} \ . \end{aligned} \tag{13}$$

The problem in (13) can be solved using cvx, a package for specifying and solving convex problems [23, 24].

### B. Reweighted $l_1$ norm minimisation

Different from $l_0$ norm which uniformly penalises all non-zero valued coefficients, the $l_1$ norm penalises larger weight coefficients more heavily than smaller ones. To make the $l_1$ norm a closer approximation to the $l_0$ norm, a reweighted $l_1$ norm minimisation method can be adopted here [25–27], where a larger weighting term is introduced to those coefficients with smaller non-zero values and a smaller weighting term to those coefficients with larger non-zero values. This weighting term will change according to the resultant coefficients at each iteration. Applying this idea to the group sparsity problem in (13), for the $i$-th iteration, it is formulated as follows

$$
\begin{aligned}
\min \quad & \sum_{n=1}^{N} \delta_n^i ||\tilde{\mathbf{w}}_n^i||_2 \\
\text{subject to} \quad & ||\mathbf{P}_{SL} - (\mathbf{W}^i)^H \mathbf{S}_{SL}||_2 \le \alpha \\
& (\mathbf{W}^i)^H \mathbf{s}_{ML} = \mathbf{p}_{ML} \ ,
\end{aligned}
\tag{14}
$$

where the superscript $i$ indicates the value of the corresponding parameters at the $i$-th iteration, and $\delta_n$ is the reweighting term for the $n$-th row of coefficients, given by $\delta_n^i = (||\tilde{\mathbf{w}}_n^{i-1}||_2 + \gamma)^{-1}$. The iteration processes are described as follows:

1) For the first iteration ($i = 1$), calculate the initial value $||\tilde{\mathbf{w}}_n||_2$ by solving (13).
2) Set $i = i + 1$. Use the value of the last $||\tilde{\mathbf{w}}_n^{i-1}||_2$ to calculate $\delta_n^i$, and then find $\mathbf{W}^i$ and $||\tilde{\mathbf{w}}_n^i||_2$ by solving the problem in (14).
3) Repeat step 2 until the positions of non-zero values of the weight coefficients do not change any more for some number of iterations.

Here $\gamma > 0$ is required to provide numerical stability to prevent $\delta_n^i$ becoming infinity at the current iteration if the value of a weight coefficient is zero at the previous iteration, and it is chosen to be slightly less than the minimum weight coefficient that will be implemented in the final design (i.e. the value below which the associated antenna will be considered inactive and therefore removed from the obtained design result), where $\delta_n^i ||\tilde{\mathbf{w}}_n^i||_2 = \frac{||\tilde{\mathbf{w}}_n^i||_2}{||\tilde{\mathbf{w}}_n^i||_2 + \gamma}$.

## IV. Design examples

In this section, we provide several representative design examples to show the performance of the proposed formulations in comparison with a standard ULA. The mainlobe direction is $\theta_{ML} = 90°$ and the sidelobe regions are $\theta_{SL} \in [0°, 85°] \cup [95°, 180°]$, sampled every $1°$. The desired response is a value of one (magnitude) with $90°$ phase shift at the mainlobe (QPSK) and a value of $0.1$ (magnitude) with random phase shifts over the sidelobe regions.

To have a fair comparison, we first obtain the DM result using the method in (6) based on a 26-element ULA with half-wavelength spacing. Based on the design result, we then calculate the error norm between the designed and the desired responses of this ULA and this value is used as $\alpha$ in the sparse array design formulations in (13) and (14).

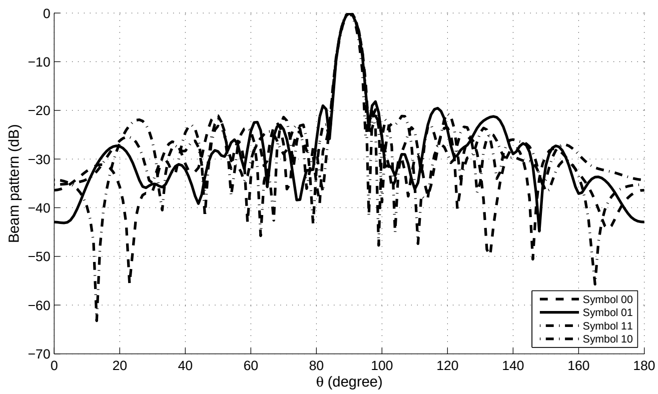


Fig. 2. Resultant beam responses based on the design in (6).

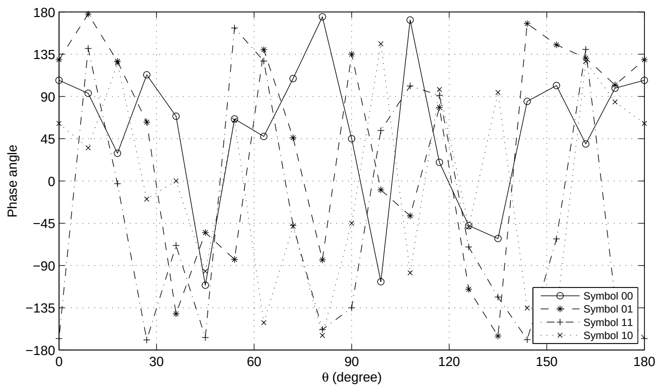


Fig. 3. Resultant phase patterns based on the design in (6).

### A. ULA design example

By using (6), the resultant beam pattern for each constellation point is shown in Fig. 2, where all main beams are exactly pointed to $90°$ with a reasonable sidelobe level. Moreover, the phase at the main beam direction is $90°$ spaced and random in the sidelobe directions, as shown in Fig. 3.

### B. Sparse array design examples

With the above ULA design, we obtain $\alpha = 2.5017$. Since the resultant sparse array may have a larger aperture than the ULA, we have set the maximum aperture to be $17.5\lambda$, consisting of $500$ equally spaced potential antennas.

By the standard group-sparsity based formulation in (13), $29$ active antennas are obtained, with an average spacing of $0.625\lambda$. The resultant beam pattern for each constellation point is shown in Fig. 4, where all main beams are exactly pointed to $90°$ with a reasonable sidelobe level. The phase at the main beam direction is $90°$ spaced and random in the sidelobe directions, as shown in Fig. 5. As shown in Table I, although its resultant value for $||\mathbf{p} - \mathbf{w}^H\mathbf{S}||_2$ is a little better than the ULA, the number of antennas is larger than the ULA, which is not desirable.

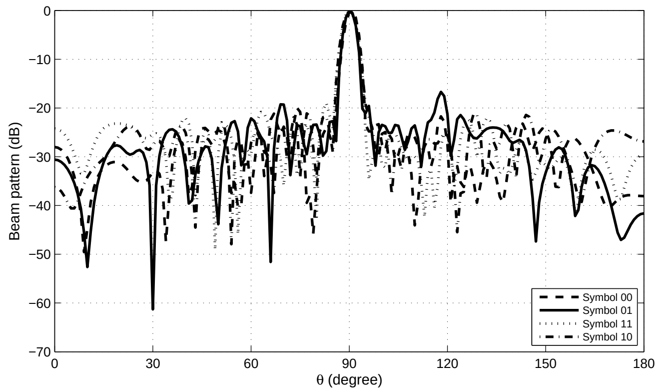


Fig. 4. Resultant beam responses based on the design in (13).

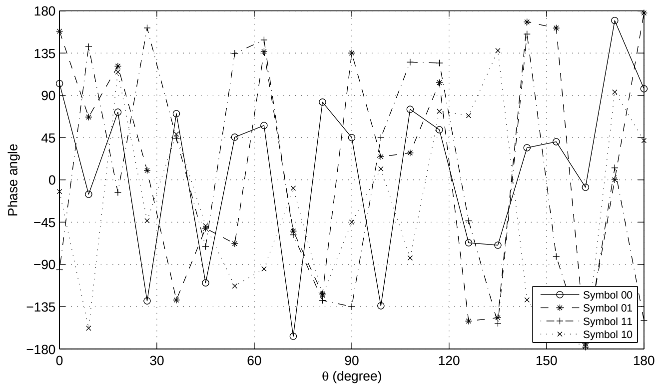


Fig. 5. Resultant phase patterns based on the design in (13).

Now we examine the performance of the reweighted method in (14). In this design, there is an additional parameter $\gamma$, and we have chosen $\gamma = 0.001$, which means that antennas associated with a weight value smaller than $0.001$ will be considered inactive. With the other parameters same as in previous examples, it results in $20$ active antennas with an average spacing of $0.653\lambda$. So as expected, a sparser solution has been obtained compared to the design in (13). The array

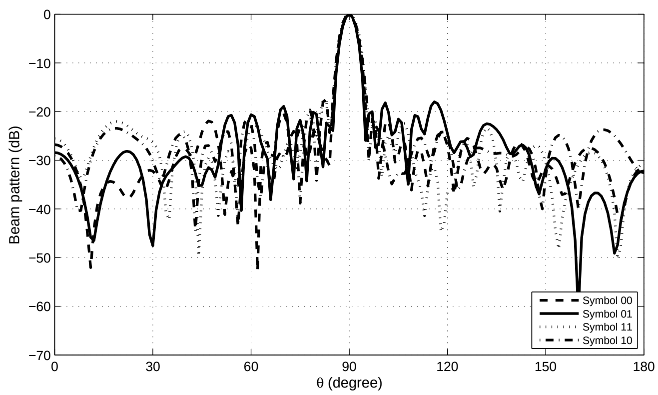


Fig. 6. Resultant beam responses based on the reweighted design in (14).

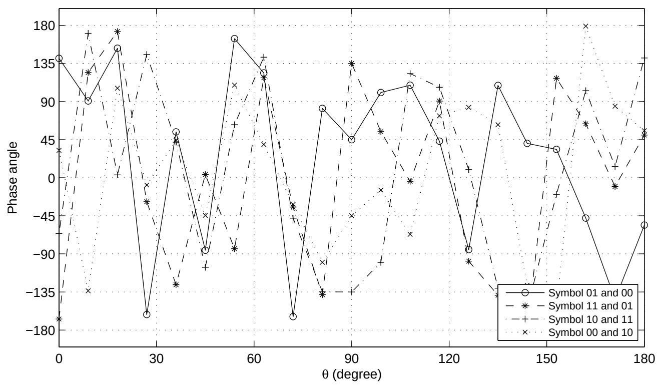


Fig. 7. Resultant phase patterns based on the reweighted design. (14).

TABLE I
SUMMARY OF THE DESIGN RESULTS.

| | ULA | Usual $l_1$ | Reweighted $l_1$ |
|---|---|---|---|
| Antenna number | 26 | 29 | 20 |
| Aperture/$\lambda$ | 12.5 | 17.5 | 12.4 |
| Average spacing/$\lambda$ | 0.5 | 0.625 | 0.653 |
| $\|\|\mathbf{p} - \mathbf{w}^H\mathbf{S}\|\|_2$ | 2.5017 | 2.3381 | 2.4925 |

response for each constellation point is shown in Fig. 6 and the phase pattern in Fig. 7, all indicating a satisfactory design. Their array responses are also closer to the desired ones than the ULA according to the value of $||\mathbf{p} - \mathbf{w}^H\mathbf{S}||_2$, as shown in Table I.

More importantly, this reweighted design is achieved with 6 less antennas compared to the ULA case, highlighting the advantage of employing sparse array instead of a standard ULA in directional modulation applications.

## V. Conclusions

The antenna location optimisation problem for directional modulation based on sparse antenna arrays has been studied and compressive sensing based design methods were proposed exploiting the group sparsity concept, including the usual $l_1$ norm minimisation and the reweighted $l_1$ norm minimisation. As shown in the provided design examples, all sparse designs have achieved a main lobe pointing to the desired direction with scrambled phases in other directions. In particular, the reweighted $l_1$ norm minimisation method can provide a sparser solution as expected, achieving a similar performance as the ULA but with less number of antennas. One note about the the directional modulation technique is that it is based on the assumption that there is no multipath effect between the transmitter and the receiver; in the presence of multipath, this technique will struggle and further research is needed in this field.

## Reference

[1] A. Babakhani, D. B. Rutledge, and A. Hajimiri. Transmitter architectures based on near-field direct antenna modulation. *IEEE Journal of Solid-State Circuits*, 43(12):2674–2692, December 2008.

[2] A. Babakhani, D. B. Rutledge, and A. Hajimiri. Near-field direct antenna modulation. *IEEE Microwave Magazine*, 10(1):36–46, February 2009.

[3] M. P. Daly and J. T. Bernhard. Beamsteering in pattern reconfigurable arrays using directional modulation. *IEEE Transactions on Antennas and Propagation*, 58(7):2259–2265, March 2010.

[4] T. Hong, M. Z. Song, and Y. Liu. Dual-beam directional modulation technique for physical-layer secure communication. *IEEE Antennas and Wireless Propagation Letters*, 10:1417–1420, December 2011.

[5] M. P. Daly and J. T. Bernhard. Directional modulation technique for phased arrays. *IEEE Transactions on Antennas and Propagation*, 57(9):2633–2640, September 2009.

[6] H. Z. Shi and A. Tennant. Enhancing the security of communication via directly modulated antenna arrays. *IET Microwaves, Antennas & Propagation*, 7(8):606–611, June 2013.

[7] Y. Ding and V. Fusco. Directional modulation transmitter radiation pattern considerations. *IET Microwaves, Antennas & Propagation*, 7(15):1201–1206, December 2013.

[8] Y. Ding and V. Fusco. Constraining directional modulation transmitter radiation patterns. *IET Microwaves, Antennas & Propagation*, 8(15):1408–1415, September 2014.

[9] Q. J. Zhu, S. W. Yang, R. L. Yao, and Z. P. Nie. Directional modulation based on 4-D antenna arrays. *IEEE Transactions on Antennas and Propagation*, 62(2):621–628, February 2014.

[10] A. Moffet. Minimum-redundancy linear arrays. *IEEE Transactions on Antennas and Propagation*, 16(2):172–175, March 1968.

[11] H. L. Van Trees. *Optimum Array Processing, Part IV of Detection, Estimation, and Modulation Theory*. Wiley, New York, 2002.

[12] K. Chen, Z. He, and C. Han. Design of 2-dimensional sparse arrays using an improved genetic algorithm. In *Proc. IEEE Workshop on Sensor Array and Multichannel Signal Processing*, pages 209–213, July 2006.

[13] L. Cen, Z. L. Yu, W. Ser, and W. Cen. Linear aperiodic array synthesis using an improved genetic algorithm. *IEEE Transactions on Antennas and Propagation*, 60(2):895–902, February 2012.

[14] M. B. Hawes and W. Liu. Location optimisation of robust sparse antenna arrays with physical size constraint. *IEEE Antennas and Wireless Propagation Letters*, 11:1303–1306, November 2012.

[15] A. Trucco and V. Murino. Stochastic optimization of linear sparse arrays. *IEEE Journal of Oceanic Engineering*, 24(3):291–299, July 1999.

[16] G. Prisco and M. D'Urso. Exploiting compressive sensing theory in the design of sparse arrays. In *Proc. IEEE Radar Conference*, pages 865–867, May 2011.

[17] L. Carin. On the relationship between compressive sensing and random sensor arrays. *IEEE Antennas and Propagation Magazine*, 51(5):72–81, October 2009.

[18] L. Cen, W. Ser, W. Cen, and Z. L. Yu. Linear sparse array synthesis via convex optimization. In *Proc. IEEE International Symposium on Circuits and Systems*, pages 4233–4236, May 2010.

[19] G. Oliveri and A. Massa. Bayesian compressive sampling for pattern synthesis with maximally sparse non-uniform linear arrays. *IEEE Transactions on Antennas and Propagation*, 59(2):467–481, 2011.

[20] G. Oliveri, M. Carlin, and A. Massa. Complex-weight sparse linear array synthesis by bayesian compressive sampling. *IEEE Transactions on Antennas and Propagation*, 60(5):2309–2326, 2012.

[21] M. B. Hawes and W. Liu. Compressive sensing based approach to the design of linear robust sparse antenna arrays with physical size constraint. *IET Microwaves, Antennas & Propagation*, 8:736–746, July 2014.

[22] M. B. Hawes and W. Liu. Sparse array design for wideband beamforming with reduced complexity in tapped delay-lines. *IEEE Trans. Audio, Speech and Language Processing*, 22:1236–1247, August 2014.

[23] CVX Research. CVX: Matlab software for disciplined convex programming, version 2.0 beta. http://cvxr.com/cvx, September 2012.

[24] M. Grant and S. Boyd. Graph implementations for nonsmooth convex programs. In V. Blondel, S. Boyd, and H. Kimura, editors, *Recent Advances in Learning and Control*, Lecture Notes in Control and Information Sciences, pages 95–110. Springer-Verlag Limited, 2008. http://stanford.edu/~boyd/graph$_$dcp.html.

[25] E. J. Candès, M. B. Wakin, and S. P. Boyd. Enhancing sparsity by reweighted $l_1$ minimization. *Journal of Fourier Analysis and Applications*, 14:877–905, 2008.

[26] G. Prisco and M. D'Urso. Maximally sparse arrays via sequential convex optimizations. *IEEE Antennas and Wireless Propagation Letters*, 11:192–195, February 2012.

[27] B. Fuchs. Synthesis of sparse arrays with focused or shaped beampattern via sequential convex optimizations. *IEEE Transactions on Antennas and Propagation*, 60(7):3499–3503, May 2012.