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Knight, Marina Iuliana orcid.org/0000-0001-9926-6092 and Nunes, M.A. (2016) Discussion of 'Should we sample more frequently?: decision support via multirate spectrum estimation' by G.P. Nason, B. Powell, D. Elliott and P.A. Smith. Journal of the Royal Statistical Society: Series A (Statistics in Society). ISSN 1467-985X

https://doi.org/10.1111/rssa.12210

Discussion on the paper "Should we sample a time series more frequently?: decision support via multirate spectrum estimation"

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We congratulate the authors for a stimulating paper which challenges current statistical thinking and practice to consider time series sampled at different sampling rates– unfortunately, an often overlooked question. This work has the potential to help with principled decision-making in a range of scientific areas and, in this respect, we also commend the release of the `regspec` software accompanying the article to aid practitioners.

Whilst the examples in the paper focus on arguably 'well-behaved', stationary processes with short-memory, one could also consider the benefits of the proposed methodology in more complex settings. Specifically, consider processes with different spectral characteristics, such as spikes or exponential decay, present for long memory time series such as fractionally integrated processes or fractional Gaussian noise (see for example Beran et al. (2013)). In such contexts, the focus is often on the estimation of long-range dependence intensity in the series, quantified through a single quantity; the Hurst exponent H. Although several long-memory estimation techniques exist, both in the time and in the frequency domains, we are not aware of any that could indeed handle combining information captured through the process sampling at more than just one sampling rate.

Interestingly, this work could pave the way to achieving just that: more reliable long-memory estimation by means of corroborating information from subsampled series. As is intuitive, process sub-sampling results in poorer Hurst exponent estimation, for both time- and spectral domain-based methods. Classical spectral estimation techniques such as Lomb-Scargle or LSSA estimation (Vaníček, 1971; Lomb, 1976; Scargle, 1982) have been shown to be unreliable for long-memory processes, and particularly so if the series has been subsampled (Broersen et al., 2000; Broersen, 2007). We conjecture that the by-product of this work, i.e. the spectral estimate that incorporates all process information, could be conducive to a new, more reliable long-memory estimator constructed in the frequency domain.

An example of the proposed methodology on a long-memory process is shown in Figure 15. While only illustrative, this example shows that the `regspec` correction seems promising here also. Could the authors comment on the wider influence of their work on estimation of secondary quantities often derived from such spectra, e.g. the Hurst exponent, and the potential of any the-

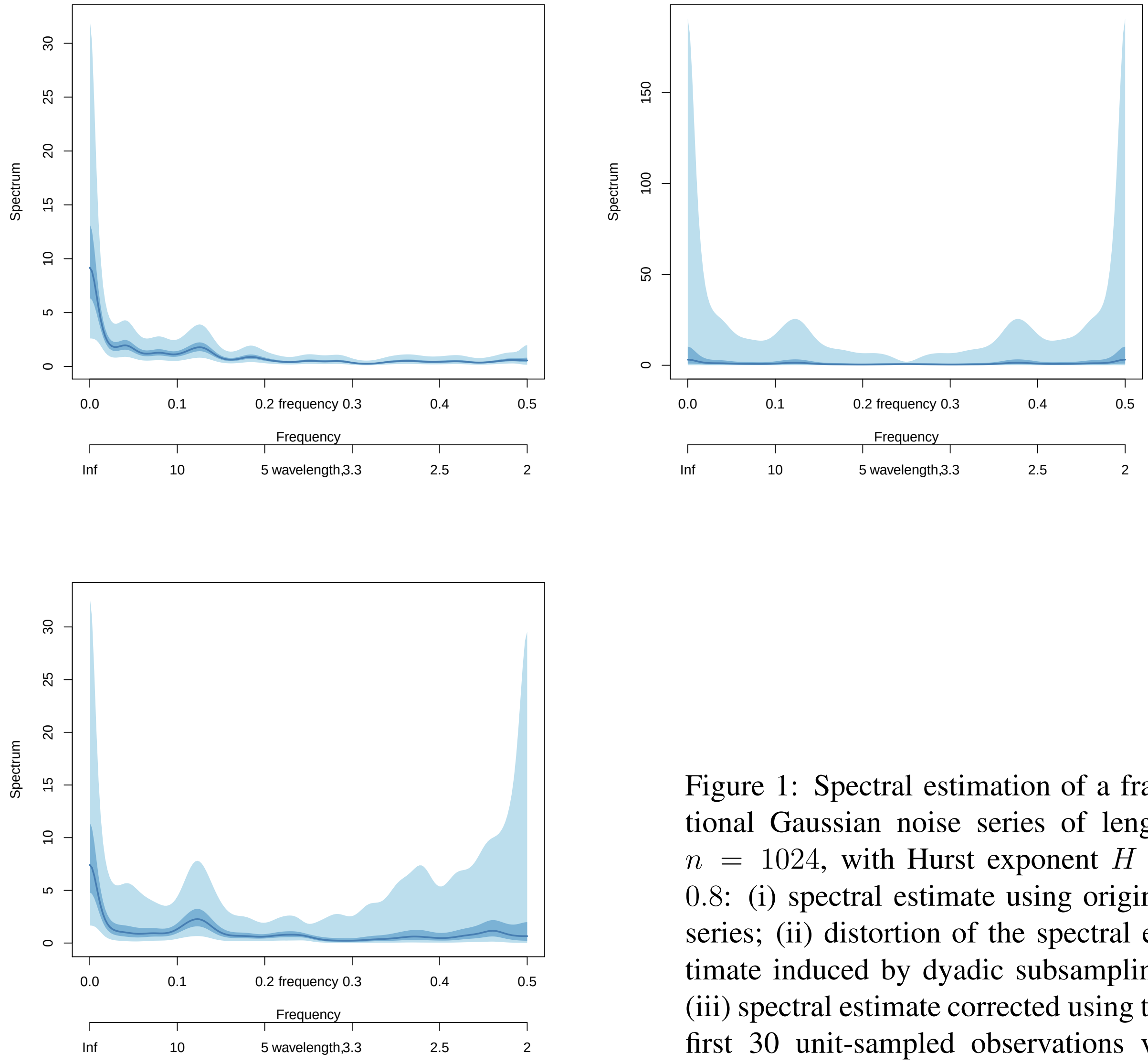

Figure 1: Spectral estimation of a fractional Gaussian noise series of length $n = 1024$, with Hurst exponent $H = 0.8$: (i) spectral estimate using original series; (ii) distortion of the spectral estimate induced by dyadic subsampling; (iii) spectral estimate corrected using the first 30 unit-sampled observations via `regspec`.

oretical development in this direction?

We would also like to hear the authors' insights on the possibility of extending the work to the analysis of *multivariate* (multirate) series for, for example, cross-spectral or polyspectral estimation.

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