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Shakedown of layered pavements under repeated moving loads

Shu Liu[1], Juan Wang[2], Hai-Sui Yu[3], FREng and Dariusz Wanatowski [4], M.ASCE

[1] Ph.D. student, Nottingham Centre of Geomechanics, The University of Nottingham, Nottingham, NG7 2RD, UK; isxsl1@nottingham.ac.uk
[2]Assistant Professor, Department of Civil Engineering, The University of Nottingham Ningbo, 315100, China; juan.wang@nottingham.edu.cn
[3]Professor, Nottingham Centre of Geomechanics, The University of Nottingham, Nottingham, NG7 2RD, UK; hai-sui.yu@nottingham.ac.uk
[4]Associate Professor, Department of Civil Engineering, The University of Nottingham Ningbo, 315100, China; d.wanatowski@nottingham.edu.cn

ABSTRACT: In recent years, shakedown theory has been suggested as a more rational theoretical foundation for pavement structural design. This paper suggests a numerical approach to find shakedown load limit of layered pavements based on an investigation of residual stress field which play an important role in helping the structure to reach the shakedown status. A finite element model is established for pavement structures under repeated moving surface loads, where the Mohr-Coulomb yield criterion with associated plastic flow is assumed to capture the plastic behaviour of pavement materials. A criterion based on static shakedown theorem is suggested to distinguish shakedown and non-shakedown status of pavement structures subjected to different magnitudes of loads, thereby achieving a numerical shakedown limit. Comparisons between the numerical shakedown limits and theoretical shakedown limits of Wang and Yu (2013a) show good agreements. Investigation of the development of residual stresses in layered pavements also provides deep insight to the application of shakedown theory. In addition, the proposed approach can be easily extended to pavement materials following non-associated plastic flow rule.

INTRODUCTION

Pavement structural design is a process intended to find the most economical combination of layer thicknesses and material types under designed loads during pavement service life. Rutting, one of the major distress forms in asphalt pavement, is mainly caused by the accumulation of permanent deformation under repeated traffic loads. Shakedown analysis, based on elastic-plastic theory, is aimed at obtaining the maximum load against excessive accumulated deformation in pavement structures; therefore, it has been recognized as a more rational criterion for road pavement design compared to the existing pavement design methods based on elastic theory (Yu 2011).

Shakedown is concerned with the responses of an elastic-plastic structure subjected to cyclic or repeated loads. According to Yu (2006), when the applied cyclic load is above the yield limit but lower than a critical load limit, termed as "shakedown limit", the structure may perform some initial plastic deformation; however, after a number of load cycles, the structure ceases to experience any further plastic strain and respond purely elastically to the subsequent load. This phenomenon is called "shakedown". Otherwise, if the load is higher than the shakedown limit, the structure will continue to exhibit plastic strains for however long the load cycles are applied. In the application of pavement engineering, the pavement shakedown limit can be used as the design load to ensure the pavement structure is in shakedown status so that the excessive rutting due to accumulated plastic strain can be prevented.

The shakedown limit can be determined by either numerical elastic-plastic analysis (Wang 2011; Wang and Yu 2013a) or two fundamental shakedown theorems: Melan's static (lower-bound) shakedown theorem and Koiter's kinematic (upper-bound) shakedown theorem. In the past few decades, theoretical solutions for pavement shakedown limits have been developed by means of two fundamental shakedown theorems (Koiter 1960; Sharp and Booker 1984; Boulbibane and Ponter 2005; Li and Yu 2006; Yu 2005; Wang 2011; Yu and Wang 2012; Wang and Yu 2013). Some validation experiments for shakedown theory applied to pavement systems were also carried out (Brown et al. 2012). However, there is very limited information on the development of plastic strains and residual stresses in the layered pavement structure. And the comparison between the theoretical solutions and numerical simulation results is urgently needed for layered pavements.

In this paper, a numerical elastic-plastic approach is developed to capture the shakedown limits of layered pavements in a visible way. A comparison is given between the numerical shakedown solution and the theoretical shakedown solution calculated by Wang and Yu (2013a). The development of plastic strains and residual stresses in pavement structures are also examined in detail for different load levels around the shakedown limit.

PROBLEM DEFINITION

An idealised two-dimensional plane strain pavement problem subjected to repeated contact loads is considered in this paper. It is assumed that the contact loads are applied by an infinitely long cylinder, as shown in Fig. 1 (Johnson 1985; Yu and Hossain 1998, Wang and Yu 2013a). The contact loads under the moving cylinder consist of a normal load P and a shear load Q, and a linear relationship is assumed between them, which gives a frictional coefficient, μ:

$$\mu = \frac{Q}{P} \tag{1}$$

And the normal load distribution (p) and shear load distribution (q) in the contact area can be described as:

$$p = \frac{p_0}{\mathrm{a}}\sqrt{(\mathrm{a}^2 - x^2)} \;\;\&\;\; q = \frac{\mu p_0}{\mathrm{a}}\sqrt{(\mathrm{a}^2 - x^2)} \qquad (-\mathrm{a} \le x \le \mathrm{a}) \tag{2}$$

where a is the half-length of contact area, and p_0 (=2P/πa) is the maximum vertical stress located at $x = z = 0$ as shown in Fig. 1.

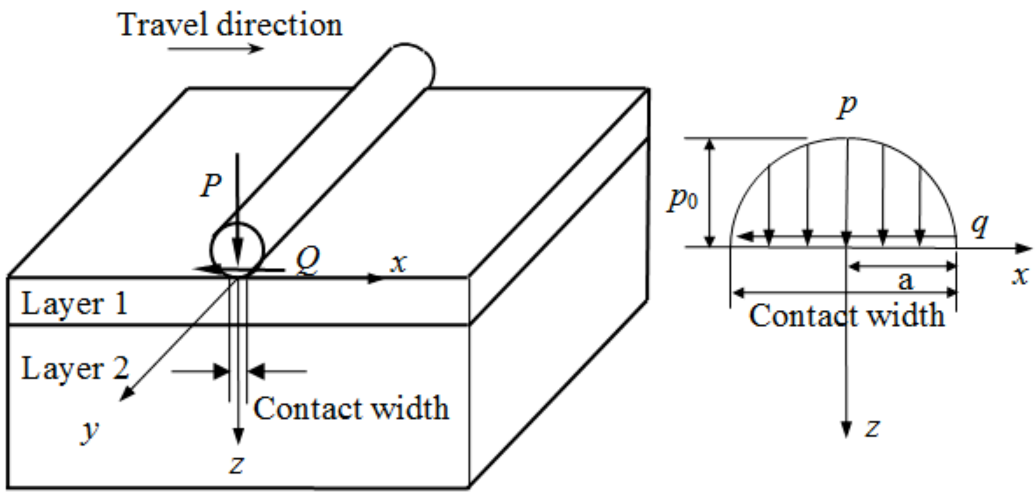


Fig. 1. Problem definition: plane strain layered pavement and loading.

In this paper, an example of a two–layered pavement is considered and the materials in both layers are considered as cohesive-frictional media. Surface traction is not taken into account, i.e. Q = 0 and also μ=0.

METHOD DESCRIPTION

The present numerical approach for elastic-plastic analysis makes use of the finite element (FE) method. A pavement model is established using the commercial FE software ABAQUS and a user subroutine DLOAD is developed to control the load applied on the pavement surface. Fig. 2 shows the FE model of a two-layered pavement. The horizontal movement is restrained at two vertical boundaries and a vertical restraint is applied on the bottom boundary. According to sensitive studies, the minimum dimension of the region under moving loads is 40a (length) × 25a (height), where a = 0.1 m in this paper. In order to reduce the influence of two vertical boundaries to the numerical results, two unloading areas (10a length for each) are provided on both sides of the loading area. Eight-noded, reduced-integrated, quadrilateral elements (CPE8R) are applied here to obtain more accurate results (ABAQUS 6.10 user's manual and Wang (2011)). High mesh density is arranged in the first layer and near the interface between two layers due to high stress and strain gradient. The half space is assumed to be continuous with different materials for each layer. The properties of the materials in both layers are described using linear elastic parameters (Young's modulus E and Poisson's ratio ν) and Mohr-Coulomb criterion parameters (Cohesion c, friction angle φ and dilation angle ψ). The materials are assumed to be homogenous, isotropic, and elastic-perfectly plastic with associated flow rule satisfied.

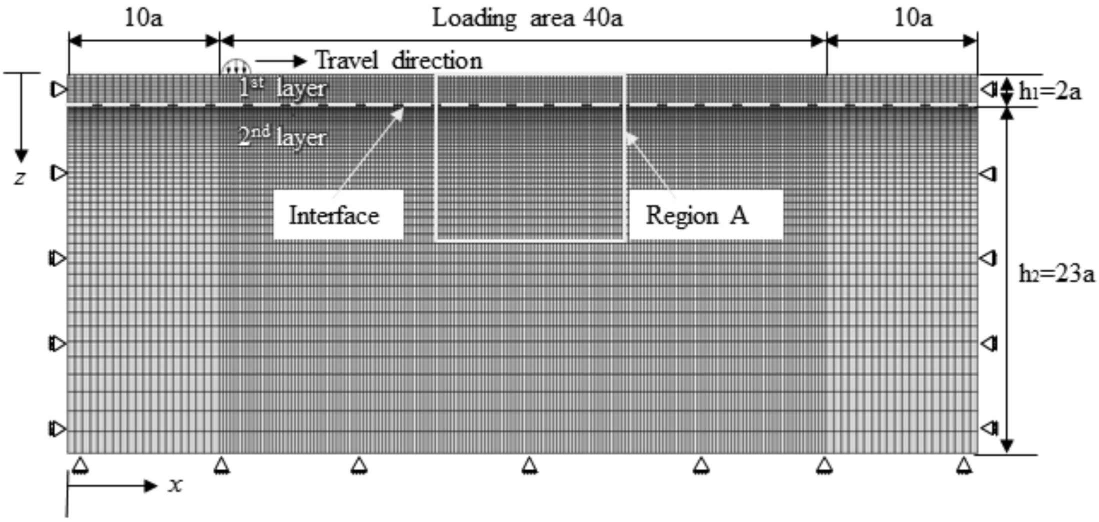


Fig. 2. FE model and boundary conditions.

According to the lower-bound shakedown theory proposed by Melan in 1938, a structure will shakedown if a self-equilibrated residual stress field exists such that its superposition with load-induced elastic stress field does not exceed yield criterion anywhere in the structure. Therefore, residual stresses developed in the elastic-plastic structure play a pivotal role in applying shakedown theory to pavement problems. In previous lower-bound shakedown solutions, statically-admissible residual stress fields were used; therefore actual residual stress fields developed in the pavements were not considered. In the present study, FE elastic-plastic analysis is carried out to obtain the actual residual stresses developed in the layered pavements under repeated moving traffic loads. As exhibited in Fig. 2, the contact load is applied along the travel direction from one side to another. After each loading pass, the applied load is removed thoroughly to investigate the stresses remaining in the pavement structure (known as residual stresses). After a few loading passes, it is found that the residual stresses are nearly independent of the travel direction and do not change with increase of loading pass. Finally, a static surface load with the same magnitude of moving load is applied in the middle of its surface to examine the status of the structure. If the total stress state of each point in the pavement does not violate the yield criterion, no yielding area can be visualised in the contour plot of ABAQUS. This means the applied load is below the pavement shakedown limit and the whole structure has fully recovered to elasticity. Further load cycles cannot increase the pavement plastic deformation, so that the pavement structure is in a shakedown state. In contrast, yielding area in the contour plot means the applied load is above the pavement shakedown limit and the whole structure is still in a non-shakedown status. Several simulations with different load magnitudes around the theoretical shakedown limit of Wang and Yu (2013a) were performed to determine the numerical shakedown limit. Accordingly, the numerical shakedown limit and the residual stresses obtained by present approach can be compared with the theoretical results.

RESULTS AND COMPARISON

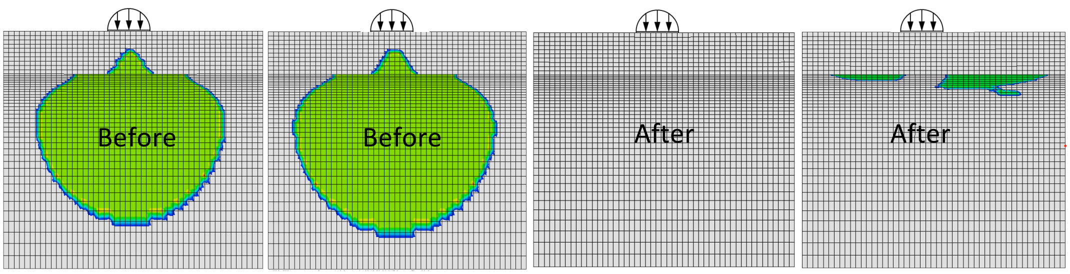
Shakedown and Non-shakedown

A two-layered pavement is designed as shown in Table 1, for which the theoretical shakedown limit is equal to $18.7c_2$ (Wang and Yu, 2013a). Fig. 3 gives the yielding areas in Region A before and after the loading passes when $p_0=18.7c_2$ and $p_0=19.3c_2$. Clearly, when $p_0=18.7c_2$, large and non-continuous yielding areas were generated under the static load before loading passes (Fig. 3a) but no yielding area can be found after limited number of loading passes (Fig. 3c). However, when $p_0=19.3c_2$, relatively larger yielding areas were generated before the loading passes (Fig. 3b) and two small yielding areas are still observed after loading passes (Fig. 3d). Therefore, for such two-layered pavement system, $p_0=18.7c_2$ leads to the shakedown state whereas $p_0=19.3c_2$ results in the non-shakedown state. This means the shakedown limit should be in between $18.7c_2$ and $19.3c_2$. Finally, the numerical shakedown limit ($18.9c_2$) is determined by undertaking more simulations using different magnitudes of load between $18.7c_2$ and $19.3c_2$ and it shows a good agreement with the theoretical shakedown limit.

Table1. Parameters for the two layered soil material

Layer	Friction Angle φ (°)	Dilation Angle ψ (°)	Stiffness Ratio E_1/E_2	Strength Ratio c_1/c_2	Poisson's Ratio ν	1st Layer Thickness (h_1/a)
1st Layer	30	30	10	10	0.2	2
2nd Layer	0	0			0.49	

(**Note**: The location of the 1st layer and the 2nd layer can refer to Fig. 2. In this paper, the subscript 1 means the 1st layer and the subscript 2 means the second layer. E is elastic modulus and c is cohesion.)

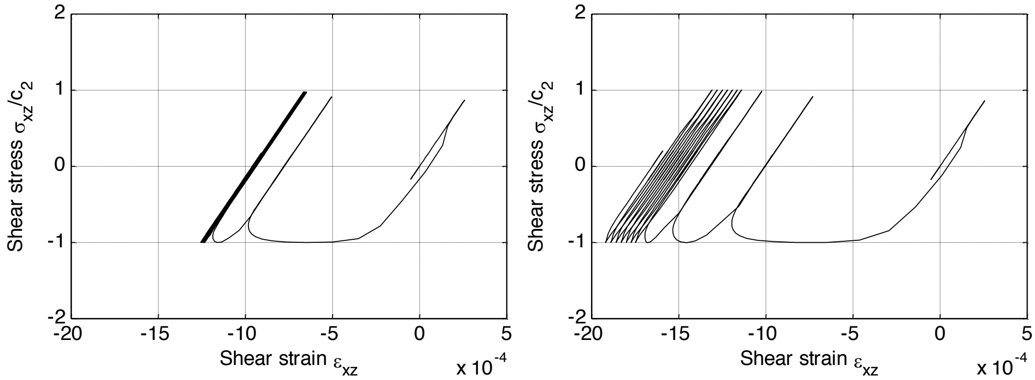


(a) p_0=18.7c$_2$ **(b) p_0=19.3c$_2$** **(c) p_0=18.7c$_2$** **(d) p_0=19.3c$_2$**

Fig. 3. Indication of yielding areas in Region A before and after loading passes

Stress and Strain Response

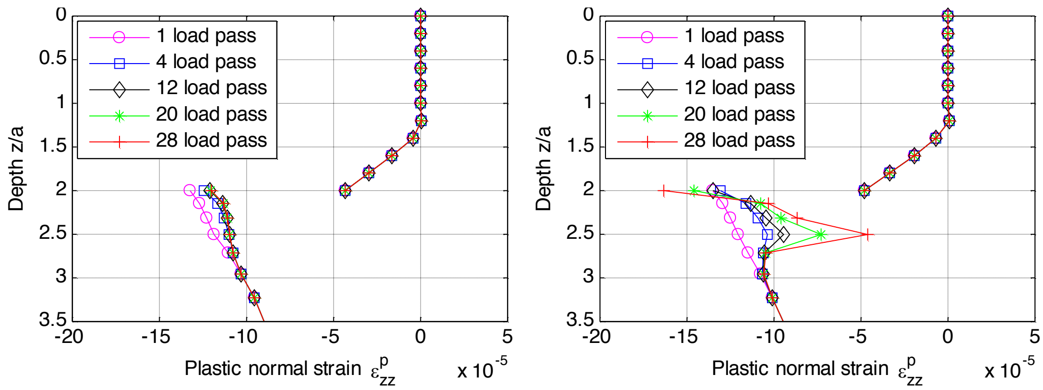
Stress and strain response curves provide a perspicuous way to help understand the notion of shakedown. A point in the middle section beneath the interface is selected as a representative to illustrate the shear stress and shear strain response during the loading pass. As shown in Fig. 4, when the load applied ($18.7c_2$) is less than the numerical shakedown limit ($18.9c_2$), the stress- strain response becomes fully elastic after a few passes of load; otherwise, the plastic deformation increases continuously after every load cycle (see Fig. 4(b)).



(a) p_0=18.7c$_2$ **(b) p_0=19.3c$_2$**

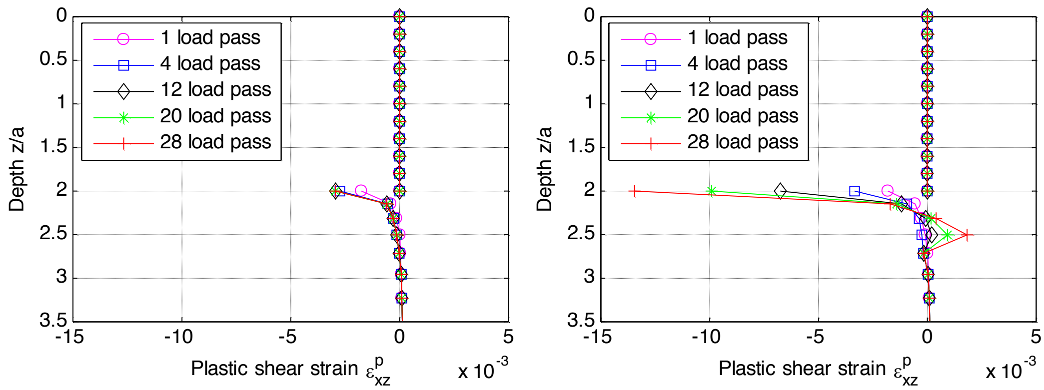
Fig. 4. Shear stress-strain response cycles beneath the interface in the middle section during the 10 passage of load.

Fig. 5 and Fig. 6 demonstrate the development of plastic normal strain and plastic shear strain under different load levels in the two-layered pavement. It can be seen that, when the applied load is above the shakedown limit, the amount of plastic normal strain and plastic shear strain increase at each load cycle and this will lead to structure failure.



(a) p_0=18.7c$_2$ **(b) p_0=19.3c$_2$**

Fig. 5. Development of plastic normal strain.



(a) p_0**=18.7c**$_2$ **(b)** p_0**=19.3c**$_2$

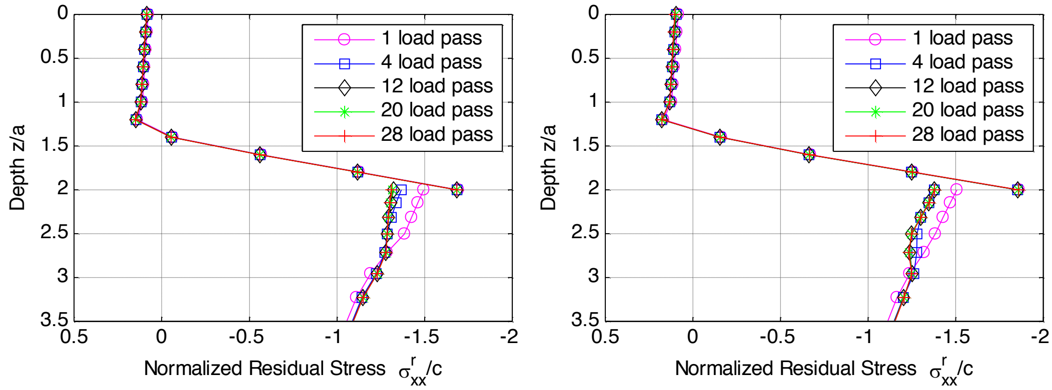
Fig. 6. Development of plastic shear strain.

Residual Stress Fields

It has been noted by Wang (2011) that in a pavement structure, the horizontal residual stress field in the travel direction may increase at the most critical points to help the structure shake down, thus in this paper, the development of horizontal residual stress field is analysed. Fig. 7 shows the horizontal residual stress field in the middle section of the pavement model when the load is at or above the theoretical shakedown limit (p_0=18.7c$_2$). Here, the residual stresses of each layer are normalised with respect to their own cohesion respectively. From these figures, it is clear that the residual stress fields in the first layer change barely with increasing loading passes, while some changes are observed in the second layer nearby the interface. This means the second layer is more critical than the first layer and the critical point is close to the interface. This is in agreement with the theoretical finding of Wang and Yu (2013a) where the critical point of this particular pavement structure is on the top of the second layer. Wang and Yu (2011) also indicated that the actual horizontal residual stress field should lie between two critical residual stress fields: minimum larger roots and maximum smaller roots when the applied load is no larger than the shakedown limit. Therefore, the fully-developed residual stress field obtained by the numerical shakedown analysis can be checked by comparing with the critical residual stress fields. As shown in Fig. 8, the maximum smaller roots and minimum larger roots are calculated according to the theoretical method when p_0=18.7c$_2$. The solid curve indicates fully-developed horizontal residual stresses obtained by numerical method with same magnitude of load level. It lies between the two critical residual stress fields with some negligible errors.

Effect of Stiffness Ratio

Fig. 9 demonstrates the influence of material stiffness ratio E_1/E_2 on 2D numerical shakedown limits and theoretical shakedown limits (Wang and Yu 2013a). The numerical shakedown limits agree well with the theoretical shakedown limits. An optimum stiffness ratio can be found which provides the maximum resistance to pavement failure, i.e. the shakedown limit is maximized. Moreover, according to the contour plots of ABAQUS, it is not difficult to find that the peak point manifests the change of the critical points locations from the second layer to the first layer.



(a) p_0=18.7c$_2$ **(b) p_0=19.3c$_2$**

Fig. 7. Development of horizontal residual stresses.

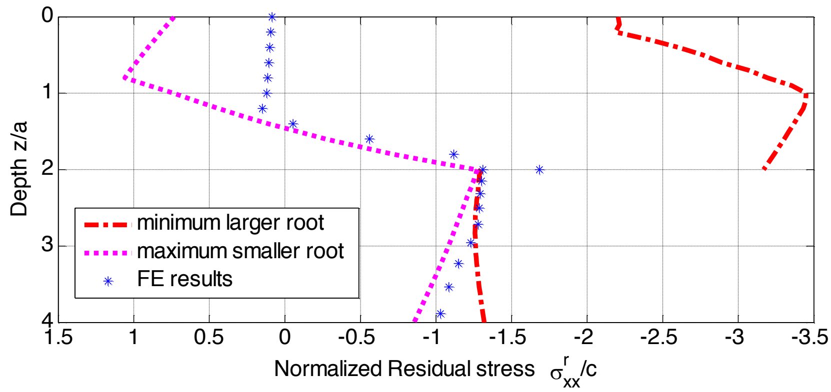


Fig. 8. Comparison between critical residual stress fields and FE calculated residual stress fields in two layered pavement when p_0=18.7c$_2$.

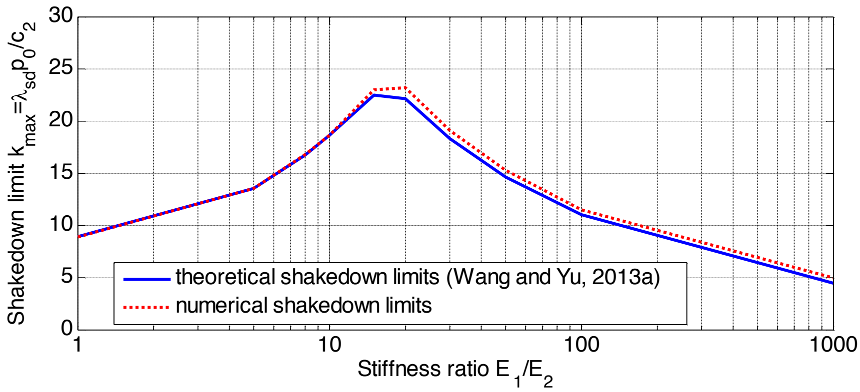


Fig. 9. Shakedown limits versus stiffness ratio when c_1/c_2=10, h_1/a=2 and $\varphi_1=\psi_1$=30°

CONCLUDING REMARKS

The numerical approach developed in this paper has been proved to be a valid and visible way to estimate the shakedown limit of the pavement structure. The results show that the numerical shakedown limits are consistent with the theoretical shakedown solutions (Wang and Yu 2013a) for 2D two-layered pavement model.

In addition, the numerical approach provides information about the development of residual stresses and plastic strains which cannot be obtained by theoretical shakedown solutions. The fully-developed residual stress field obtained from numerical approach lies between the two critical residual stress fields calculated by theoretical method.

According to the numerical results, the residual stresses are nearly independent of travel direction and are fully-developed after a limited number of loading passes. The development of residual stresses plays an important role in helping the structure reach shakedown status, thereby preventing further yielding. However, if the applied load is too large, the fully developed residual stresses will not be able to prevent continuing plastic strain which can lead to ratchetting and distress of pavement structure.

For two-layered pavement structure, the stiffness ratio E_1/E_2 is one of the major factors which affect the shakedown limits. It is found that there is an optimum stiffness ratio which would provide the maximum resistance to pavement failure. The influence of other factors, such as strength ratio (c_1/c_2), friction angle and layer thickness on shakedown limits will be investigated in future.

Compared with the classical shakedown analysis based on the static/kinematic shakedown theorems, this approach can be easily extended to more complicated or non-standard materials, e.g. materials follow non-associated plastic flow rule or strain-hardening materials.

Further work will focus on extension of the current numerical approach to consider other pavement materials, multi-layered pavements and three-dimensional numerical modeling.

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