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Average-Cost Pricing and Dynamic Selection Incentives in the Hospital Sector

Abstract

This study investigates dynamic incentives to select patients for hospitals which are remunerated according to a prospective payment system of the DRG type. Using a model with patients differing in severity within a DRG, we show that price dynamics depend on the extent of hospital altruism and the relation between patients’ severity and benefit. Upwards and downwards price movements over time are both possible. In a steady state, DRG prices are unlikely to give optimal incentives to treat patients. Depending on the level of altruism, too few or too many patients are treated. DRG pricing may also give incentives to treat low-severity patients even though high-severity patients should be treated.

Keywords: Hospitals; DRGs; selection; severity.

JEL Classification: I11, I18, L13, L44.
1 Introduction

Prospective payment systems based on Diagnosis Related Groups (DRG) are now in use world-wide to finance hospitals. They were first introduced in the US to replace cost-based reimbursement rules which were associated with high health care expenditure. In most cases, a fixed tariff is paid for each patient, creating strong incentives to contain costs.\(^1\) Cost efficiency, however, is only one aspect of the design of payment systems. Prospective payment systems also influence how and which patients are treated. One problem is that hospitals may save on the quantity and quality of services (Ellis and McGuire 1986, 1990), in particular if demand is not responsive to quality.\(^2\) This study focuses on the incentives to select patients created by DRG-based payment. In particular, it has been argued that hospitals have a financial incentive to ‘dump’ patients, ie to avoid patients whose care requires high spending (Dranove, 1987; Newhouse, 1983).\(^3\)

Our focus is on DRG payment systems which use average-cost pricing. This is the standard practice in many countries (Quentin et al., 2011; Cots et al., 2011). An economic justification for average-cost pricing has been advanced by Shleifer (1985). He argues that this rule is essential in creating ‘yardstick competition’ between hospitals. It gives each hospital the incentive to invest efficiently in cost reduction. Prices will then fall to the efficient level, mimicking a competitive market. However, Shleifer does not consider that hospitals can also influence their cost by selecting patients.

DRG price adjustment is based on past average costs. The time lag to measure costs is typically one or two years. Previous literature has abstracted from this adjustment and analyzed the incentives to select patients in a static framework in which the DRG price is administratively set. Static models are more tractable but given the dynamic nature of average-cost pricing rule, we do not know whether the equilibrium described within the static models is actually correct and analogous to the one obtained in a more realistic dynamic framework. We fill this gap in

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\(^1\) Hospitals receive additional payments only for ‘outliers’ for exceptionally high expenses.

\(^2\) If demand is responsive to quality, hospitals with poor quality will be penalised by lower revenues (Chalkley and Malcomson, 1998a; Ma, 1994). For other dimensions of quality, however, further measures may be necessary, in particular monitoring of standards. Furthermore, cost sharing can be useful in enhancing quality (Chalkley and Malcomson, 1998b).

\(^3\) Varying the intensity of service with the severity of a patient is a related strategy (Allen and Gertler, 1991; Ellis 1998). Hospital may try to overprovide services to low-severity patients (also known as ‘creaming’) and to underprovide services to high-severity patients (‘skimping’).
knowledge. Our paper provides the first theoretical analysis of average-cost DRG pricing with lagged adjustment within a general framework which allows for benefit to increase or reduce with severity, and with different degrees of altruism. Indeed, we show that the dynamic aspects of the average-cost pricing rule can have a profound effect on the incentives to select patients over time. We highlight that dumping can get worse over time: dumping reduces the average cost, which with a lag further reduces the DRG price; in turn, this further reinforces dumping, and so on until the steady state is achieved. These dynamics are missed by static models which effectively assume instantaneous adjustment. This case arises if the DRG price leads to a selection of patients with lower average cost than the DRG price. The opposite case can also arise. When hospitals are altruistic, they may treat patients with average costs above the DRG price. The DRG price must rise, reducing dumping. It is even possible that no more patients are dumped. The dynamics of DRG pricing may start at an interior solution and end up in a corner solution where patients with all severities are treated. This cannot arise in a static model where in equilibrium either some or all patients are treated. The analysis therefore highlights that to properly understand DRG incentives, a dynamic analysis is required. As we do not know the extent to which hospitals are altruistic, both scenarios with low and high altruism are of practical relevance. Since the equilibrium within a dynamic framework might differ from a static one, we provide a welfare analysis and investigate whether DRG pricing gives the incentives to treat the right patients. Differently from previous literature (reviewed in more detail below) we highlight the criticality of assuming patients’ benefit (or net benefit) to increase or reduce with severity. Both scenarios are plausible depending on the treatment. The analysis highlights that not all scenarios are equally problematic, and refinement of DRG policies needs to take the specificities of each DRG into account in relation to benefits, costs, altruism and their relation with severity.

In order to analyze the dynamic selection incentives, we make three key assumptions. Firstly, we consider that providers are, at least to some extent, altruistic. This assumption is common in the theoretical health economics literature where it has long been recognised that providers (doctors and nurses) are concerned about the care provided to patients (Ellis and McGuire, 2012).

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1986; Chalkley and Malcomson, 1998b; Jack, 2005; Brekke, Siciliani and Straume, 2011, 2012).\textsuperscript{5}

Secondly, we assume that DRGs do not perfectly capture the expected cost of each patient as perceived by providers. In particular, asymmetric information between payer and provider puts constraints on the ability to refine DRGs. Increasing the number of DRGs to better capture patients’ expected costs can also aggravate the problem of upcoding.

Thirdly, we start from the premise that hospitals are able to select patients. For example, patients can be rejected by stating that a treatment has little chance of success or by pretending to have no more capacities. Providers can exploit the asymmetry of information which characterizes the doctor-patient relation (Arrow, 1963; McGuire, 2000). The assumption is consistent with the extensive empirical clinical variation literature which shows great unexplained variations in surgery rates across small areas for a number of surgical and medical treatments (Phelps, 2000). Publicly-funded systems are characterised by large excess demand and pervasive rationing across and within treatments (Siciliani, 2014). The assumption of patient selection is in line with previous literature such as De Fraja (2000) and Malcomson (2005) (and the assumption of ‘dumping’ in the seminal paper by Ellis, 1998). Both studies assume that hospitals are profit maximisers. Given our assumption of altruistic providers, the amount of selection is less dramatic since altruism broadly weakens, but does not necessarily eliminate, incentives for selection. None of these papers considers dumping when DRG prices are set at the average cost.

In our model, providers balance the benefit from the DRG price and from the altruistic motive against the cost of treating a patient. Only patients will be treated for whom these benefits exceed cost. This defines a ‘marginal patient’ such that the benefit from the DRG price and from the altruistic motive is equal to the expected cost of treating this patient. Price dynamics arise if the average cost resulting from the providers’ treatment decisions differs from the DRG price. Then the price will be adjusted in the next period, giving new incentives for providers to treat patients.

For non-altruistic hospitals only a downward movement of prices is possible. In this case, the marginal patient is characterized by cost equal to the DRG price. As long as hospitals treat

\textsuperscript{5}This assumption is also made in the literature on motivated agents in the broader public sector, where the agent is assumed to share, to some extent, the objective function of the principal (Francois, 2000; Murdock, 2002; Besley and Ghatak, 2005; Prendergast, 2007).
patients with different severities, average cost must be lower than the DRG price implying that it must fall in the next period. Altruistic hospitals, by contrast, are willing to treat patients which cause losses. Then average cost can be above the DRG price. In this case altruism has a double effect on patients’ selection. Not only does higher altruism lead to more patients being treated but with a delay also increases the tariff which further increases the number of patients treated. In equilibrium, it is possible that all patients are treated.

After investigating hospital incentives under current payment systems, we conduct a welfare analysis. Here it is decisive whether patients’ benefit decreases or increases with severity. For instance, if a treatment requires a good physical constitution to recover, patients’ benefit is likely to decrease with severity. This suggests that patients below a critical severity threshold should receive treatment. Higher severity can also be associated with higher patients’ benefit, for example, when higher severity causes higher pain without treatment. Then, it can be optimal that only high-severity patients are treated.

We find that DRG pricing is unlikely to implement the first-best allocation. The two crucial factors are the level of altruism and the relationship between patients’ benefit, costs and severity. Low altruism implies that patients with high severity are dumped even though benefits exceed costs. With high altruism too many patients are treated. Depending on how patients’ benefit decreases or increases with severity, either too many high-severity or too many low-severity patients are treated. Finally, hospitals can have incentives to treat low-severity patients while it would be optimal from a first-best perspective to treat those with high severity. In this case, DRG pricing may give completely the wrong incentives. Hospitals treat low-severity patients while the first-best calls for the treatment of high-severity patients only.

We proceed as follows. In Section 2, we present the model and derive the dynamics of DRG pricing and equilibrium prices. Section 3 characterises the first-best allocation and evaluates the outcome of the DRG system. In Section 4, we discuss the implications for empirical analyses and policy. Section 5 concludes.
2 The Model

We focus on one DRG. There are \( N \) identical hospitals that provide treatment to patients. In each hospital, patients with a given diagnosis differ in severity of illness \( s \) which is distributed over the support \([s, \bar{s}]\) with density function \( f(s) \) and cumulative distribution function \( F(s) \). We assume that \( f(s) \) does not vary with time \( t \). The expected cost of treating a patient with severity \( s \) is \( c(s) \), and is increasing in severity, \( c'(s) > 0 \).\(^6\)

A patient’s benefit of hospital treatment is given by \( b(s) \). We assume that the benefit is positive and can increase or decrease with severity, \( b'(s) \geq 0 \). \( b'(s) > 0 \) captures the case when a higher severity is associated also with higher capacity to benefit from treatment because higher severity causes higher pain without hospital treatment. For example, those with a higher severity may suffer from higher arthritis pain and benefit more from a hip replacement. \( b'(s) < 0 \), by contrast, can describe a treatment such as a complicated surgery which requires a good physical constitution to recover. Patients’ severity is observed by providers but not by the payer. Both scenarios are plausible and likely to vary by DRG.

Patients have a passive role and accept to undertake the care recommended by the provider (as in Ellis and McGuire, 1986; Ellis, 1998; De Fraja, 2000; Malcomson, 2005). We adopt a model in discrete time. Hospitals receive a DRG price in each period \( t \) which is denoted by \( p_t \) for patients with the same diagnosis. The DRG price is set according to the average cost of all patients who are treated with the same diagnosis in the previous period. We therefore assume that the pricing rule is exogenously given and that the regulator has committed to this commonly observed pricing rule (as in Hafsteinsdottir and Siciliani, 2010). We also assume that \( N \) is sufficiently large and can therefore ignore any strategic effect of each individual hospital choice on future prices.\(^7\)

Hospitals are risk neutral and partially altruistic. Altruism (or motivation) is captured by the parameter \( \alpha \) with \( 0 \leq \alpha \leq 1 \).\(^8\) The provider’s altruistic gain from each patient treatment

\(^6\)We do not model explicitly incentives to contain costs, eg by introducing a cost-containment effort variable as in some of the studies cited above. This is because we focus on a purely prospective DRG system. The provider is residual claimant and will choose optimal cost-containment effort. Therefore, adding such a variable would make the presentation more complex without adding significant additional insights.

\(^7\)For small \( N \) each hospital may for example strategically admit more severe patients to secure higher prices in the future.

\(^8\)We interpret the parameter \( \alpha \) as altruism. More generally, this parameter could encompass other sources of
is $ab(s)$. A selfish provider ($\alpha = 0$) cares only about the difference between revenues and costs. In each period the provider treats the patient if $p_t \geq c(s) - ab(s)$, i.e., if the price at least covers the ‘net cost’, defined as the expected cost minus the altruistic gain. In an interior solution, the marginal treated patient has severity $s_t$ which is implicitly defined by

$$p_t = c(s_t) - ab(s_t),$$  \hspace{1cm} (1)

i.e., the DRG tariff is equal to the marginal net cost. Whether the provider treats low-severity or high-severity patients under this interior solution depends on specific assumptions about altruism, the benefit and the cost function.\(^9\) Furthermore, we assume that altruistic hospitals are able to sustain losses for some time. This possibility arises out of equilibrium because altruistic hospitals are willing to treat high-severity patients who cause losses. In equilibrium, however, the DRG price equals current average cost and hospitals have a balanced budget.\(^10\)

In the following we distinguish two different scenarios, increasing and decreasing net cost with severity over the support $[\underline{s}, \bar{s}]$:\(^11\)

(I) **Net cost increasing with severity**: $c'(s) - \alpha b'(s) > 0$ in $[\underline{s}, \bar{s}]$. In such case, low-severity patients are treated in an interior solution. This scenario always arises if patient’s benefit decreases with severity, $b'(s) < 0$: low-severity patients have lower costs and higher benefits, making these patients attractive for providers both for profit and altruistic reasons. Hence, providers treat patients with severity below or equal to $s_t$ in period $t$ (defined in eq. (1)). This scenario can still arise if patient’s benefit does not vary with severity, or if it increases with severity and either altruism or the marginal benefit from higher severity is sufficiently small.

(D) **Net cost decreasing with severity**: $c'(s) - \alpha b'(s) < 0$ in $[\underline{s}, \bar{s}]$. In such case, high-severity patients are treated in an interior solution. This scenario obtains when patients’ benefit increases with severity and $ab'(s)$ is sufficiently high: although high-severity patients are

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\(^9\)Patients who are not treated by the hospital can still receive some health care outside of the hospital, such as a drug treatment through the family doctor.

\(^10\) In Section 2.3, we consider the possibility of losses in more detail.

\(^11\) These scenarios are not exhaustive as a non-monotonous relation between net cost and severity is also possible. To keep the exposition focused, we concentrate on the two clear-cut cases.
more costly, their benefit from treatment (weighted by the degree of altruism) is sufficiently high that providers have an incentive to treat all patients with severity above $s_t$ in period $t$ despite the fact that more severity implies higher treatment costs. This scenario requires positive altruism.

In the following we discuss the two different scenarios separately, first starting by the case where providers have an incentive to focus on low-severity patients. We assume that in each period $t$, the DRG price is set according to the average cost of the patient treated in the previous period. We do not consider any hospital-specific DRG price adjustments because we assumed that all hospitals are identical.

2.1 Scenario I: net cost increasing with severity

In Scenario I the net costs is increasing with severity (with $c'(s) - \alpha b'(s) > 0$ in $[\hat{s}, \overline{s}]$), and therefore providers select low-severity patients if they do not treat all patients. Price dynamics can be examined in a phase diagram in which the variables $p_t$ and $p_{t-1}$ are plotted against each other (see Figure 1). In Appendix A.1, we formally derive the function $p_t(p_{t-1})$ based on the average-cost pricing rule. If $p_t(p_{t-1})$ is above the 45° line, the price in $t - 1$ is below average cost and the price increases. Conversely, where $p_t(p_{t-1})$ is below the 45° line the price in $t - 1$ exceeds average cost and must fall in the next period. Equilibria are given by the intersections of $p_t(p_{t-1})$ with the 45° line.

The minimum price under scenario I which induces providers to treat patients is equal to the net cost of the patient with lowest severity and thus given by $p^I_{\text{min}} \equiv c(\hat{s}) - \alpha b(\hat{s})$. Correspondingly, we define $p^I_{\text{all}} \equiv c(\overline{s}) - \alpha b(\overline{s})$ as the price above which providers treat all patients. Such price is equal to the net cost of the patient with highest severity. Furthermore, we show in Appendix A.1 that the function $p_t(p_{t-1})$ is increasing for intermediate prices $p^I_{\text{min}} \leq p_{t-1} < p^I_{\text{all}}$. This result arises because in scenario I a higher price implies a higher marginal severity. Providers are therefore willing to treat more costly patients. This increases average cost and therefore the price in the next period.

Figure 1 displays three possible results depending on the level of altruism.\(^{12}\) The functions

\(^{12}\)Figure 1 is based on numerical examples which are detailed in the Appendix.
Figure 1: Altruism and Price Dynamics
start at the minimum price \( p_t(p^I_{\text{min}}) = c(s) \). Once the price \( p^I_{\text{all}} \) is reached, all patients are treated leading to average cost \( c^\mu \). We assume an initial price in period 0 above the net cost of patients with lowest severity, ie \( p_0 \geq p^I_{\text{min}} \), so that some patients are treated in the initial period. There is always a a unique equilibrium with the equilibrium price \( p^* \).

Let us first consider in Figure 1.a the special case with zero altruism where providers only maximise profits. In this case, the price function evaluated at the lowest price is on the 45° line. For higher prices, \( p_t(p_{t-1}) \) must be below the 45° line. This is because profit-maximising providers treat patients only as long as the price covers cost. The cost of the marginal patient in period \( t-1 \), \( c(s_{t-1}) \) thus corresponds to \( p_{t-1} \). Average cost of these patients must be below \( c(s_{t-1}) \) because cost increases with severity, leading to a lower price in the next period. Thus, in equilibrium only the lowest severity patients are treated such that \( s^* = s \) and \( p^* = p^I_{\text{min}} = c(s) \).

The arrows in Figure 1.a. display the price movements for an initial price \( p_0 \). The DRG system induces dumping which in the following periods reduces the average cost of treatment and therefore the DRG price, leading to increased incentives for dumping. Prices converge to the equilibrium price, implying an almost complete unravelling of the market. This is a rather negative result. It implies that DRG pricing has the potential to generate a dynamic development towards the 'bottom'. Given an initial (high) price, providers have an incentive to select patients, and treat only those with low severity. This in turn implies that the following period the price will be lower, which will induce even further selection and reduce marginal severity, and so on.

Next, we turn to altruistic providers. Figure 1.b illustrates the case with intermediate altruism. The price function evaluated at the lowest price \( p_t(p^I_{\text{min}}) \) is above the 45° line because hospitals are willing to treat the lowest severity patients at a price below cost \( c(s) \). The price \( p_t(p^I_{\text{all}}) \) lies below the 45° line. The equilibrium is unique with price \( p^* \) between \( p^I_{\text{min}} \) and \( p^I_{\text{all}} \). For an initial price \( p^0_1 \) below the unique equilibrium \( p^* \), prices increase over time, for an initial price \( p^2_0 \) above \( p^* \), prices decrease over time. Although the dynamics is not as extreme as in Figure 1.a, some selection takes place.

The downward movement of prices in Figure 1.b could reflect the price and selection dynamics when cost reimbursement rules are replaced with a DRG system. Under the old regime,
hospitals have strong incentives to admit patients since expensive patients were simply reim-
bursed. Furthermore, incentives for cost efficiency are low. This implies high average cost and 
therefore a high initial DRG price. Thus, falling prices and costs cannot only be explained by 
higher cost-containment incentives of the DRG system but also by selection incentives that get 
reinforced over time.

Figure 1.b also shows that with altruism, an upward price movement of prices can arise for 
sufficiently low initial price. Altruistic providers are willing to treat the lowest-severity patient 
even though the price is below cost which corresponds to average cost in this case. In the next 
period, the price must increase. Thus, altruism has as a double effect on patients’ selection. Not 
only does it lead to more patients being treated but with a delay also increases the tariff which 
thus further increases the number of patients treated.

In Figure 1.c, altruism is high. The price $p^I_{all} = c(s) - \alpha b(s)$ for which providers are willing to 
treat all patients is therefore low causing the price $p_t(p^I_{all})$ to be above the 45° line. This implies 
that all patients are treated in equilibrium at price $p^* = c^a$. Only increasing prices are possible: 
since providers’ altruism is high, they are willing to treat patients whose costs are larger than 
the DRG price. This in turn implies a higher tariff in the next period which further encourages 
providers to treat high-cost patients.

So far, we have only considered cases with an unique equilibrium. Multiple equilibria are 
possible. Figure 2 shows an example leading to two stable (A and C) and one instable equilibrium 
(B). More than three equilibria are also possible, with the total number being odd and one 
more stable equilibrium than unstable equilibrium. With multiple equilibria, both decreasing 
and increasing prices are possible depending on the initial price.

The degree of altruism is therefore critical in characterising whether prices are decreasing or 
increasing over time. In Appendix A.1.2, we prove the following proposition:

**Proposition 1** If the net cost is increasing in severity, there is a unique and stable equilibrium 
if

- hospitals are non-altruistic. In this equilibrium, $p^* = c(s)$ and $s^* = s$, ie only patients with 
  lowest severity are treated. Prices can only be decreasing.
- hospital altruism exceeds the critical level $\hat{\alpha} = (c(\bar{s}) - c^\mu)/b(\bar{s})$. In this equilibrium, $p^* = c(\bar{s})$ and $s^* = \bar{s}$, i.e. all patients are treated. Prices can only be increasing.

For intermediate levels of altruism, there is at least one stable equilibrium in which providers treat patients up to a severity level $s^*$ in $[\underline{s}, \bar{s}]$. Depending on the initial price, decreasing or increasing prices are possible.

Further results on the role of altruism can be obtained by a comparative static analysis. Interior equilibria are described by the following two equations for equilibrium price and severity, $p^*$ and $s^*$:

$$p^* = \frac{\int_{\underline{s}}^{s^*(p^*)} c(s)f(s)ds}{F(s^*(p^*))},$$  \hspace{1cm} (2)

$$p^* = c(s^*(p^*)) - \alpha b(s^*(p^*)).$$  \hspace{1cm} (3)

In equilibrium, patients are treated in the interval $[\underline{s}, s^*(p^*)]$. Equation (2) simply states that the DRG equilibrium price must correspond to average cost of these patients. Equation (3) characterises the equilibrium selection decision by providers. It states that in an interior equilibrium, price must be equal to the net cost of the marginal patient. Using the right-hand sides,
we can further characterize the optimal marginal severity $s^*$ as

$$\alpha b(s^*) = c(s^*) - \int_{s^*}^{\infty} c(s)f(s)ds - \frac{\int_{s^*}^{\infty} c(s)f(s)ds}{F(s^*)}. \quad (4)$$

In equilibrium, the marginal severity below which patients are treated is defined such that the benefit from treatment (weighted by altruism) is equal to difference between the marginal cost and the average cost. Since the cost is increasing in severity, the marginal cost is always above the average cost (and the price) and the right-hand side of equation (4) is positive.

If providers are profit maximisers, then condition (4) is satisfied only if patients with lowest severity are treated so that marginal and average cost coincide. Any small positive level of altruism implies that patients above minimum severity are treated so that the cost of treating the marginal patient is higher than the average cost. Therefore, a corner solution (with $s^* = s$) cannot arise for positive altruism.

Condition (4) emphasises the critical role played by altruism. Differentiating (4) with respect to $\alpha$, we obtain that higher altruism leads to a higher marginal severity starting from a stable interior equilibrium and to a higher price: $\partial s^*/\partial \alpha > 0$ and $\partial p^*/\partial \alpha > 0$ (Proof in Appendix A.1.3). Higher altruism implies that providers are willing to treat more severe patients at the margin. This in turn implies higher cost which translates into a higher price.

**Proposition 2** In a stable interior equilibrium, the marginal benefit from treatment weighted by altruism equates the difference between the marginal and average cost of treatment. The price and the marginal equilibrium severity, below which patients are treated, increase with altruism.

### 2.2 Scenario D: net cost decreasing with severity

We now turn to scenario D where the net cost of treating a patient decreases with severity: $c'(s) - \alpha b'(s) < 0$. For a given price, assuming they do not treat all patients, providers will treat high-severity patients. This case can arise only if patient’s benefit increases with severity ($b'(s) > 0$) and altruism is sufficiently high.

The minimum price under scenario D which induces providers to treat patients is equal to the net cost of the patient with highest severity and thus given by $p^{D}_{\text{min}} \equiv c(\bar{s}) - \alpha b(\bar{s})$. Providers
will treat all patients for prices exceeding $p^D_{\text{all}} \equiv c(s) - \alpha b(s)$, the price is equal to the net cost of the patient with lowest severity. In Appendix A.2, we show that the function $p_t(p_{t-1})$ is decreasing for intermediate prices $p^D_{\text{min}} \leq p_{t-1} < p^D_{\text{all}}$. This result is in stark contrast to scenario I when net costs increases with severity. It arises because with net cost increasing with severity, a higher price reduces the marginal severity. Providers are therefore willing to treat further patients who have higher net cost, but lower cost. This decreases the average cost and therefore the price in the next period.

Figure 3 illustrates. The function $p_t(p_{t-1})$ is falling in the range $[p^D_{\text{min}}, p^D_{\text{all}}]$ and then flat. The unique equilibrium is given by a price equal to the average cost, $p^* = c^\mu$. Price adjustment is fast. Starting in period 0, consider first the initial price $p^1_0$. In the next period the price rises above $c^\mu$ because providers only treat high-severity patients. This price is above $p^D_{\text{all}}$ which implies that all patients are treated in period 1. In period 2, the price is thus $p_2 = c^\mu$ and the equilibrium is reached. Price convergence is even faster for an initial price above $p^D_{\text{all}}$, eg $p^2_0$ and $p^3_0$. Every patient is therefore treated in period 0. Thus, the price in the following period will be equal to the average cost $p_t(p^D_{\text{all}}) = c^\mu$. The price will remain at this level and the equilibrium is reached already after one period.

Proposition 3 summarises.
Proposition 3 If net cost is decreasing with severity, all patients are treated in the unique and stable equilibrium, and price is equal to the average cost of all patients. Depending on the initial price, the equilibrium price is reached in one period, or the price first rises and then falls to the equilibrium price in the second period.

2.3 The role of altruism

The level of altruism plays an important role in our analysis. First, price dynamics depend crucially on altruism. Low altruism makes a downward movement of prices likely, in particular if the system starts with a high initial price. With high altruism, the picture is more complex. An upward movement of prices is now possible in Scenario I. In Scenario D, which requires both high altruism and patient benefit increasing in severity, the price adjustment can be non-monotonous. Prices can fall after an initial price rise. In contrast to the case of low altruism, this price fall is accompanied by an increase in volume as providers extend treatment to low-severity patients.

Second, altruistic providers are willing to treat patients even though they make losses. In equilibrium, this case does not arise. However, losses can arise for several periods in the transition to the equilibrium. This is the case whenever the price is above the 45° line in the phase diagram. In Scenario D, this transition is short. In Scenario I, the speed of adjustment depends on the slope of the function $p_t(p_{t-1})$. If the slope is small, the equilibrium is quickly reached, if it is high, transition takes longer. In the Appendix, we show that the slope depends on the properties of the distribution function of $s$ and on how strongly net costs change with severity (see equation (A.3)). If $c'(s) - ab'(s)$ is large, $p_t(p_{t-1})$ is flat and the adjustment is fast. If $c'(s) - ab'(s)$ is small, the speed of adjustment is slow. In this case, hospitals can come under severe economic strain. As a consequence, altruistic behavior may be reduced, for example, if hospital management puts more pressure on physicians. This would shift the function $p_t(p_{t-1})$ downwards and lead to a faster adjustment.

Finally, an important result is that the equilibrium price can only increase with altruism. We have shown this for scenario I and confirmed this for scenario D which requires high altruism. This result is in sharp contrast with what obtained under optimal pricing rules where the
regulator can design the optimal price as a function of the parameters at stake, including altruism (eg Ellis and McGuire, 1986). Under an optimal pricing rule, higher altruism generally implies a lower price. Intuitively, higher altruism implies that the provider needs less to be incentivised through prices. When prices are determined by an average-cost rule, by contrast, high altruism tends to push up prices as providers are willing to treat high-severity patients. This drives up average costs and, with a lag, prices. To what extent this is in the regulator’s interest will be discussed in the following section.

3 A Normative Analysis

From a policy perspective, the crucial question is whether price adjustments over time give providers the incentive to treat the ‘right’ patients. This requires a normative analysis. In the following, we start from the premise that patients should be treated when benefits are (weakly) above costs: \( b(s) \geq c(s) \) for \( s \) over the support \([s, \bar{s}]\).\(^{13}\) Again, we distinguish between two relevant scenarios.

(L) Low-severity patients should be treated in an interior solution. Suppose that patient benefit is either decreasing with severity or increasing with severity but less steeply than cost: \( c'(s) - b'(s) > 0 \). Assuming an interior solution, the marginal patient is such that \( b(s^I) = c(s^I) \) and patients with severity \( s \) below first-best severity \( s^I \) should receive treatment and those with severity above \( s^I \) should not.

(H) High-severity patients should be treated in an interior solution. Suppose that patients benefit increases with severity more quickly than costs: \( c'(s) - b'(s) < 0 \). Then the marginal patient is still characterised by \( b(s^I) = c(s^I) \) but patients with severity above \( s^I \) should now be treated. If benefit is higher than cost for all patients over the support \([s, \bar{s}]\), then it is optimal to treat all patients.

\(^{13}\)We therefore do not consider provider benefit \( \alpha b(s) \) for the optimal treatment decision. In this, we follow Chalkley and Malcomson (1998b) who oppose a double counting of treatment benefits in social welfare calculations.
Scenarios L and H relate in the following way to scenarios I and D in the last section:

- Scenario L with \( c'(s) - b'(s) > 0 \) implies that provider’s scenario I with \( c'(s) - \alpha b'(s) > 0 \) holds for any \( \alpha \in [0, 1] \). Therefore, under scenario L we can ignore provider’s scenario D because the latter never arises (since \( c'(s) - \alpha b'(s) < 0 \) never holds).

- Scenario H with \( c'(s) - b'(s) < 0 \) arises only if benefit increases with severity, \( b'(s) > 0 \), and sufficiently steep. Here, on the provider’s side we need to distinguish between scenario I and D. Provider’s scenario I arises when low altruism implies \( c'(s) - \alpha b'(s) > 0 \) and scenario D when high altruism causes \( c'(s) - \alpha b'(s) < 0 \).

### 3.1 Scenario L: \( c'(s) - b'(s) > 0 \). It is optimal to treat low-severity patients

We start by considering scenario L where it is optimal to treat low-severity patients. If \( c(\overline{s}) - b(\overline{s}) > 0 \) and \( c(s) - b(s) < 0 \), an interior solution arises and it is optimal to treat patients with severity \( s \in [\underline{s}, \overline{s}] \). Recall that an interior solution obtains in provider’s scenario I if altruism is below a critical level \( \hat{\alpha} \) (see Proposition 1). In this case, the optimal provider’s equilibrium severity (see equation (4)) is characterised by

\[
\alpha b(s^*) + \frac{\int_{\underline{s}}^{s^*} c(s)f(s)ds}{F(s^*)} = c(s^*). \tag{4}
\]

When altruism is zero, only patients with lowest severity are treated and \( s^* = \underline{s} < \overline{s} \). Equilibrium severity \( s^* \) increases with altruism, but as long as \( s^* < \overline{s} \) too few patients are treated. It would be optimal from a welfare perspective to treat patients with severities in \( (s^*, \overline{s}] \) as well (see Figure 4, case (a)). At the opposite side of the spectrum, if altruism is high, too many patients are treated (see Figure 4, case (b)). Comparing the first-best severity characterized by \( b(\overline{s}) = c(\overline{s}) \) with condition (4), this must be the case for \( \alpha = 1 \). There are some patients with high severity that should not be treated but instead are.

We can compute the level of altruism \( \tilde{\alpha} \) such that the severity chosen by the provider \( s^* \) is the same as the first-best severity \( s^f \) (by using \( b(s^f) = c(s^f) \) and condition (4):

\[
\tilde{\alpha} = 1 - \frac{\int_{\underline{s}}^{s^f} c(s)f(s)ds}{F(s^f)} \frac{1}{c(s^f)} < 1.
\]
The threshold is equal to one (the highest level of altruism) minus the ratio of the average and marginal cost of treatment when evaluated at the first-best severity. The ratio between the average and marginal cost is always below one since the marginal cost is increasing and low-severity patients are treated. Too few patients are treated for $\alpha < \tilde{\alpha}$, and too many patients are treated for $\alpha > \tilde{\alpha}$. Only by chance will providers implement the first-best solution.\(^{14}\)

\(^{14}\)Our results also extend to corner solutions. First, consider that it is optimal to treat all patients. A sufficient assumption is $c(s) - b(s) < 0$. Then too few patients will be treated if altruism is below $\tilde{\alpha}$. It is also possible that a DRG is active even though nobody should be treated. This case applies if $c(s) - b(s) > 0$. Then individuals will nevertheless be treated if the initial price is sufficiently high, i.e. above $p_{\text{min}}^t = c(s) - \alpha b(s)$. 

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Proposition 4 Suppose that in the first-best solution it is optimal to treat low-severity patients 
\[(c'(s) - b'(s)) > 0, c(s) - b(s) > 0 \text{ and } c(s) - b(s) < 0.\]
Then,

- too few patients are treated for sufficiently low altruism \[(\alpha < \bar{\alpha} = 1 - \frac{AC(s_f)}{c(s_f)} < 1);\]
- too many patients are treated for sufficiently high altruism \[(\bar{\alpha} < \alpha < 1).\]

3.2 Scenario H: \[c'(s) - b'(s) < 0.\] It is optimal to treat high-severity patients

In this case, high-severity patients should be treated in the first-best solution, i.e., patients with 
severity above \(s^f\). If \(c(s) - b(s) > 0\) and \(c(\overline{s}) - b(\overline{s}) < 0\), an interior solution arises and it is 
optimal to treat patients with severity \(s \in [s^f, \overline{s}]\).

Recall that this case arises only if benefit increases with severity, i.e., \(b'(s)\) is positive and 
sufficiently steep. Here, on the provider’s side we need to distinguish between scenario I and 
D. Provider’s scenario I arises when \(c'(s) - \alpha b'(s) > 0\) and scenario D when \(c'(s) - \alpha b'(s) < 0\).
Assuming for simplicity that \(c'(s)/b'(s)\) is constant, we can determine a critical level of altruism 
\(\bar{\alpha} \equiv c'(s)/b'(s)\) such that for \(\alpha < \bar{\alpha}\) scenario I arises and above which scenario D arises.

In scenario I, hospitals do not treat high-severity patients when altruism is below the threshold \(\hat{\alpha}\) (see Proposition 1). This is a worrying case since the DRG system gives the wrong 
incentives. In Figure 4 case (c), this result is particularly marked as the incentives are completely 
wrong. In case (d), for higher altruism, at least some patients are treated who should be treated.

If altruism is above the threshold \(\hat{\alpha}\) in scenario I or if scenario D obtains, all patients are 
treated and the DRG system induces too many patients to be treated as in Figure 4 case (e).
The intuition is that altruism is so strong that all are treated (\(\alpha > \hat{\alpha}\) in Scenario I) or that 
DRG pricing based on the highest-cost types makes it attractive to treat low-cost types as well 
(\(\alpha > \bar{\alpha}\) and thus Scenario D).\(^{15}\)

\(^{15}\)Also for this scenario, it is straightforward to consider corner solutions. It is optimal to treat all patients if 
\(c(\overline{s}) - b(\overline{s}) < 0\). For sufficiently low altruism, the DRG system will not induce this result. Individuals with high 
severity will be dumped. Again, it can happen that a DRG is active even though nobody should be treated. This 
\(\text{case applies if } c(\overline{s}) - b(\overline{s}) > 0\). Providers may nevertheless treat patients. Consider the case of perfect altruism 
\(\alpha = 1\). Then scenario D obtains. If the initial price is at least \(p_{D\min}^D = c(\overline{s}) - b(\overline{s})\), then an equilibrium will 
be reached in which all are treated even though \(b(s) < c(s)\) for all patients.
Proposition 5 Suppose that in the first-best solution it is optimal to treat high-severity patients \((b'(s) > 0; c'(s) - b'(s) < 0 \text{ and } c(s) - b(s) > 0)\). Then,

- for sufficiently low altruism \((\alpha < \min\{\hat{\alpha}; \check{\alpha}\})\) there are some patients with low severity who are treated although benefits are below costs;
- too many patients are treated for sufficiently high altruism \((\alpha > \min\{\hat{\alpha}; \check{\alpha}\})\).

The analysis assumes so far that the net cost of treatment is either increasing or decreasing with severity. There may be cases where the net cost is increasing with severity both for low-severity and high-severity patients, but is decreasing with severity for middle-severity patients. In such cases, it would be optimal to treat middle-severity patients but not low- and high-severity patients.

The scenario could arise for example if benefit increases with severity at a decreasing rate and cost is linear in severity. It is not optimal to treat low-severity patients since benefits are too low despite the low cost of treatment. Neither it is optimal to treat high-severity patients: the additional benefit is not offset by the relatively higher costs.

Our normative analysis suggests that DRG payments are particularly problematic when net cost decreases with severity since patients with high severity should be treated but are not. The issue remains even under the scenario just described if under the DRG system the provider has incentive to treat only part of the patients with middle severity, i.e., those with lower severity within this group. Our key lessons carry over even if net cost is non-monotonic in severity.

4 Implications for empirical analyses and policy

4.1 Identifying problematic DRGs

Our normative analysis highlights that DRG systems have undesirable properties. In order to identify problematic DRGs, it is important to determine whether net cost of treatment is increasing or decreasing with severity. This is where empirical analyses could help regulators. In this section, we outline potential data and methods.

Cost and benefits of health treatments are regularly measured and employed in economic evaluations of healthcare treatments to decide whether a new treatment should be provided by a
public insurer (Drummond et al., 2015). These are generally based on clinical trials on a sample of patients’ population who need a specific treatment. Benefits can be measured on a continuous scale, eg through Quality Adjusted Life Years (QALYs), and converted in monetary terms through monetary social values. There is therefore a wealth of information that could be used to compute benefits and costs, and then to relate to appropriate measures of severity. Simply, for each patient in the sample the net cost of treatment (cost minus benefit) can be computed and then correlated with severity. Non-linear effects may be accounted for by regressing net costs with polynomials of severity or by segmenting severity in different bands. This seems in line with emphasis to provide cost-effective analysis by severity sub-groups (Sculpher and Gafni, 2001; Willan et al., 2004).

Computing net costs by severity for a large number of relevant hospital DRGs is however not without challenges. Economic evaluations cover only a subset of treatments provided by hospitals. Moreover, some may be based on relatively small samples raising issues of external validity. An alternative is to recur to administrative hospital data which cover all patients treated. A good recent example is Patient Reported Outcomes (PROMs) within the English National Health Service for four common elective procedures: hip replacement, knee replacement, hernia and varicose veins. These are collected on a routine basis, and measure the health of the patients before and after the surgery. A measure of benefit from treatment is given by the health gain, the difference in health (as measured by QALYs or other questionnaire) before and after the surgery. Severity can be measured by health before surgery or other condition-specific indicators (eg pain before the surgery). Costs within DRGs could be proxied by length of stay and transformed into costs by making assumptions about cost of stay per day.

PROMs data although increasingly available are still limited. Hospital administrative data do provide other health measures, such as in-hospital mortality though these tend to be infrequent events and the risk of mortality is negligible for common elective conditions. For the latter, emergency readmissions rates for different severity sub-groups could be used as proxies of the benefits. Simple correlations between length of stay and health outcomes might also suggest whether patients with higher severity, with longer length of stay, are more likely to benefit from treatment.
In summary, there are a number of approaches that could be developed to assess whether the net cost of treatment is increasing or decreasing with severity, or at least for developing an informed guess about this relation. Current DRG systems currently ignore such information (with few exceptions such as pay-for-performance schemes for quality). Therefore, moving in this direction may be beneficial to develop appropriate regulatory interventions.

4.2 Regulatory interventions

Suppose that the regulator can identify whether net cost increases or decreases with severity. In this section, we discuss possible regulatory interventions. For both scenarios, we can conclude that the DRG price system with prices based on lagged average cost cannot be expected to give providers the incentive to treat the ‘right’ patients. In particular, this applies for situations in which only a fraction of patients should be treated. Only by chance will providers treat only those patients whose benefits exceed costs. When only high-severity patients should be treated, the optimal solution can never be implemented.

From a regulatory perspective, corner solutions are not as problematic as interior solutions. If nobody should be treated, the regulator could simply eliminate a DRG. If all patients should be treated, altruism may be sufficiently high to induce providers to treat all. Furthermore, dumping can be severely punished. For interior solutions, by contrast, some patient selection is optimal and it is difficult for the payer to infer whether the right patients are treated. In this case one option is to abandon the average-cost rule and to set prices administratively to induce providers to treat the right patients.

Consider first an interior solution in scenario L in which all patients in $[s_a, s_f]$ should be treated. Scenario L implies that providers also prefer low-severity patients. By setting the price to $p_f = c(s_f) - \alpha b(s_f)$, the first-best treatment decision can be induced. However, hospitals costs and revenue will differ. Consider first that too few patients are treated in equilibrium ($s^* < s_f$ as in Figure 4 case (a)). Then the regulator must increase the price, eg to $p_0^f$ in Figure 1(b), creating a positive gap between price and average cost. Excessive profits could be avoided by making hospitals pay a lump-sum in advance but this can be difficult to implement. If too many patients are treated in equilibrium ($s^* > s_f$ as in Figure 4 case (b)), the regulator must
set the price at a lower level, eg price $p^1_0$ in Figure 1(b). This leads to losses for which hospitals must be compensated for by lump-sum transfers.

An interior solution in scenario H calls for treatment of all patients in $[s^f, s]$. This solution can only be implemented by an administrative price $p^f = c(s^f) - \alpha b(s^f)$ if providers also have a preference for high-severity patients. This price must be lower than the equilibrium price, eg price $p^1_0$ in Figure 3. Hospitals run losses and lump-sum transfers become necessary. If, however, providers have a preference for low-severity patients, no administrative price can implement the first-best solution. The regulator’s and providers’ objectives are fundamentally opposed.

These limitations of the price-mechanism in inducing first-best treatment decisions put forward the question whether alternative institutional arrangements can lead to better outcomes. For DRGs in which the average-cost rule does not lead the right treatment decisions, contracting combined with audits is an alternative. In particular, consider the last case in which the regulator wants high-severity patients to be treated but providers prefer low-severity patients. The regulator can specify in a contract that only high-severity should obtain treatment and set payments equal to average cost of this group. Regular audits check whether hospitals deviate from this rule. For patients with severity below the specified threshold, no payment is made. A drawback of this solution is that it causes additional expenditure and administrative burden. However, this may be justified to ensure that the right patients obtain treatment.

5 Conclusions

This study is the first to investigate hospitals’ dynamic incentives to select patients when prices are calculated according to the average-cost pricing rule. We considered a range of assumptions regarding the degree of providers’ altruism and patients’ benefit and cost functions. Previous studies focused on static incentives to select patients and assumed administratively set DRG prices. Many countries, however, base their DRG prices on lagged average cost. In our dynamic framework, we have analyzed how this pricing rule influences providers’ incentives over time.

We showed that hospitals’ dynamic incentives to select patients depend on the degree of providers’ altruism and the relation between patients’ severity and its benefit. For sufficiently low altruism, the incentive to avoid expensive patients with high severity may be reinforced over
time until a steady-state is reached, leading to a dynamic development towards the ‘bottom’. The introduction of a DRG price system generates incentives to dump patients which implies an even lower price in the future period which further encourages selection and dumping and so on. For a stable steady-state equilibrium, we showed that higher altruism leads to higher equilibrium prices. This is in sharp contrast with what obtained under optimal pricing rules where the optimal price reduces with altruism (Ellis and McGuire, 1986).

An upward movement of prices is also possible. Altruistic providers may treat patients whose costs are much larger than the DRG price. This in turn implies with a delay a higher DRG tariff which encourages providers to treat further high-cost patients. This mechanism can reinforce an equilibrium with no dumping. If altruism is high and patient benefit increases with severity, another pattern can arise. Providers prefer to treat high-severity patients and prices can then first rise. This creates incentives to extend treatment to low-severity patients and prices fall again.

Our dynamic analysis provides an explanation for downwards or upwards DRG price movements over time. Whether volume of treatment follows the price depends on the scenario. If providers have a preference for treating low-severity patients, lower prices are associated with a lower volume of treatment. If provider prefer to treat high-severity patients, however, a price fall is accompanied by an increase in volume as providers extend treatment to low-severity patients.

Welfare implications of current DRG systems have also been derived. We assumed that patients should be treated if benefits exceed costs and compared this to the equilibrium of the DRG system. Based on this benchmark, we determined to what extent the DRG system gives incentives to treat the ‘right’ patients. We considered interior solutions in which only a fraction should be treated. We found that the optimal treatment decision is only implemented by chance. Both providers and the regulator must have a preference for the treatment of low-severity patients and altruism must be neither too low or too high. Otherwise severe deviations can arise:

- If patients’ benefit decreases with severity, low altruism causes too few patients to be treated.

- For high altruism, the problem is that too many (either of high-severity or of low-severity)
patients are treated.

- When patients’ benefit increases with severity (for example higher severity causes higher pain without treatment), then DRG pricing can lead to low-severity patients being treated even though high-severity patients should be treated. In the extreme, no patient is treated who should be treated.

While the incentives created by DRGs to dump patient are generally recognized, these results show that the problem is more complex. Dumping of high-severity patients which should be treated is only one possibility. DRG systems can also create incentives to treat high-severity patients which should not be treated. An example are complicated surgeries which can be very costly for patients in otherwise poor health state and should therefore be limited to individuals with low severity. Overall, our analysis shows that a DRG system in which prices are calculated according to the average-cost pricing rule cannot be expected to give the right incentives to treat patients.

We have discussed several approaches that could be developed to assess whether the net cost of treatment is increasing or decreasing with severity. This would allow to identify problematic DRGs. For these cases, an option is to use administrative prices. Then, however, costs and revenue will differ. The regulator must either require hospitals to pay a lump-sum to avoid excessive profits or pay lump-sum transfers to cover losses. This solution, however, is infeasible, if the regulator wants high-severity patients to be treated but providers have a preference for low-severity cases. Then an alternative is to contract hospitals for the treatment of high-severity patients and to audit regularly whether hospitals comply.

Our key assumption is that hospitals decide about treatment. In practice, there are limitations to this rule when it comes to the ‘dumping’ of patients. For example, patients may protest against not receiving treatment. Nevertheless, patients can be rejected by stating that a treatment has little chance of success or by pretending to be operating above capacity. Hospitals can be expected to meet less resistance when too many patients are treated. In our model, we assumed that all patients receive positive benefits from treatment and are therefore unlikely to protest if they get treated even though costs exceed benefits.

Our model makes a number of simplifying assumptions. First, we have not allowed explicitly
for the presence of capacity constraints. These may be relevant for health systems with lower spending and limited resources, which translate into excess demands. In such countries, one natural response would be for the regulator of the DRG system to set a lower price. Indeed, given a capacity constraint, the regulator can always set within our model a regulated price which is low enough to discourage providers to admit too many treatments. An alternative way to think about the role of capacity constraints is in terms of its impact on the cost structure. Hospitals with a more binding capacity constraints (given for example by a fixed number of beds) effectively face a higher marginal (monetary or non-monetary) cost of treating patients at the margin. Within our model, this would imply a higher marginal cost of treating a patient for a given severity, which in turn will reduce the number of patients who are treated in equilibrium (see Brekke et al., 2011, for an analogous approach). The cost function we adopt is general. We therefore conjecture that the key insights of our model will not be altered by larger marginal costs induced by tighter capacity constraints.

Second, providers are assumed to be homogeneous. This is for tractability reasons and in line with many theoretical models in the health economics literature. Introducing heterogeneity in costs across hospitals would make the model more realistic and generate additional results at the cost of greater analytical complexity but would not alter the key insights in terms of price dynamics and welfare properties. Hospitals with higher costs will treat fewer patients in equilibrium but will decide to choose high- or low-severity patients along the lines illustrated by the model. The problem for the regulator becomes more involved. The regulator would need to decide whether to allow for differential prices for hospitals with different cost structure. Its design is likely to depend on the degree of asymmetric information on costs and other institutional constraints (Miraldo et al., 2011).

Third, we have assumed that providers can select or dump patients. Our model does not allow for quality differences across patients, where providers could potentially skimp on high-severity patients by providing lower quality or cream-skim low-severity patients by providing higher quality to these patients (in line with Ellis, 1998). The extent to which hospitals have an incentive to differentiate quality across patients within a DRG is likely to depend on how different severity groups can observe quality differences and are able to respond to it. In our
model, such incentives would be mitigated by the presence of altruism. Moreover, the empirical literature suggests that demand responsiveness to quality remains low (Brekke et al., 2014). This implies that incentives to skimp on quality may be pervasive in DRG payments system, and may need to be countervailed by other mechanisms such as pay-for-performance schemes or quality audits and inspections. The incentives to skimp or cream-skim, if present, are driven by the profit margin on each patient. Our model highlights the dynamics generated by price setting based on past costs. When price increases over time, the incentives to skimp are likely to be diluted, and instead strengthened by period reductions in price.
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Appendix

A.1 Scenario I

A.1.1 Derivation of the first-order difference equation

In this scenario, providers select low-severity patients. The average cost in period \( t - 1 \) and thus the DRG price in period \( t \) is equal to

\[
p_t = \frac{\int s_{t-1}(p_{t-1}) c(s) f(s) \, ds}{F(s_{t-1}(p_{t-1}))} \equiv g(p_{t-1}), \tag{A.1}
\]

where \( F(s_{t-1}(p_{t-1})) = \int s_{t-1}(p_{t-1}) f(s) \, ds \) is the number of patients treated in the previous period and \( s_{t-1}(p_{t-1}) \) is the marginal severity in the previous period \( t - 1 \). It is defined by \( p_{t-1} = c(s_{t-1}) - \alpha b(s_{t-1}) \) and determined by the price in the previous period \( p_{t-1} \). The pricing rule is therefore recursive since the price in period \( t \) depends on the price in period \( t - 1 \), which is captured more succinctly by the function \( p_t = g(p_{t-1}) \). We define \( p^I_{\text{min}} \equiv c(\underline{s}) - \alpha b(\underline{s}) \) as the minimum price under scenario I which induces providers to treat patients, which is equal to the net cost of the patient with lowest severity. Similarly, we define \( p^I_{\text{all}} \equiv c(\overline{s}) - \alpha b(\overline{s}) \) as the price under scenario I above which providers treat all patients. Such price is equal to the net cost of the patient with highest severity. If the price is above such level, i.e. \( p_{t-1} \geq p^I_{\text{all}} \), then equation (A.1) implies that the price in the following period is equal to the average cost when all patients are treated \( c^\mu \equiv \int s c(s) f(s) \, ds \). Increasing net costs, \( c'(s) - \alpha b'(s) > 0 \), imply that the net cost of the patient with highest severity is higher than the net cost of the patient with lowest severity so that \( p^I_{\text{min}} < p^I_{\text{all}} \).

For prices \( p_{t-1} \) in the intermediate range \( [p^I_{\text{min}}, p^I_{\text{all}}) \), the price dynamics follows equation (A.1) which is a non-linear first-order difference equation. Price dynamics can therefore be summed up in

\[
p_t(p_{t-1}) = \begin{cases} 
    c^\mu & \text{if } p_{t-1} \geq p^I_{\text{all}} \\
    g(p_{t-1}) & \text{if } p^I_{\text{min}} \leq p_{t-1} < p^I_{\text{all}}.
\end{cases} \tag{A.2}
\]

For intermediate prices \( p^I_{\text{min}} \leq p_{t-1} < p^I_{\text{all}} \), we can implicitly differentiate the function
$g(p_{t-1})$ defined by (A.1). Using (1), we obtain

$$\frac{dp_t}{dp_{t-1}} = \frac{f(s_{t-1}(p_{t-1}))}{F(s_{t-1}(p_{t-1}))} \left[ c(s_{t-1}(p_{t-1})) - \frac{f(s_{t-1}(p_{t-1}) c(s) f(s) ds}{F(s_{t-1}(p_{t-1}))} \right] \frac{1}{c'(s) - \alpha b'(s)}. \quad \text{(A.3)}$$

The term in the square brackets in equation is positive since the cost of the marginal patient is always higher than the average cost (recall that net cost is increasing in severity). Furthermore, $c'(s) - \alpha b'(s) > 0$ by assumption. Thus, $dp_t/dp_{t-1}$ is strictly positive for $p_{t-1}^f \leq p_{t-1} < p_{t-1}^f$.

The equilibria that correspond to Figures 1.a to 1.c are based on linear benefits and costs functions: $b(s) = b_0 + b_1 s$, $c(s) = c_0 + c_1 s$ with $c_0 > 0$, $c_1 > 0$, $b_0 > 0$. Patients’ severity is distributed over the support $[\underline{s}, \bar{s}]$ with the uniform density function with a mass of one, $f(s) = 1$, (implying $\bar{s} - \underline{s} = 1$). Under scenario I we have $c'(s) - \alpha b'(s) > 0$ and therefore $c_1 - \alpha b_1 > 0$. This is always satisfied when $b_1 < 0$ but also if $b_1 > 0$ and altruism is sufficiently low, i.e. $\alpha < \tilde{\alpha} \equiv c_1/b_1$. Condition (1) which defines the marginal treated patients corresponds to $p_t = c_0 + c_1 s_t - \alpha (b_0 + b_1 s_t)$, implying $s_t = \frac{b_0 c_0 + c_1 s_t - c_0}{c_1 - \alpha b_1}$. Using (A.1), DRG pricing implies $p_t = c_0 + c_1 s_{t-1}$. Substituting $s_{t-1}$, we obtain the linear function:

$$g(p_t) = \frac{A}{2 (c_1 - \alpha b_1)} + \frac{c_1}{2 (c_1 - \alpha b_1)} p_{t-1}$$

with $A \equiv c_1 c_0 (c_1 - \alpha b_1) + c_0 c_1 + \alpha (c_1 b_0 - 2b_1 c_0)$. Moreover, $p_{t-1}^f \equiv (c_0 + c_1 \underline{s}) - \alpha (b_0 + b_1 \underline{s})$, $p_{t-1}^f \equiv (c_0 + c_1 \bar{s}) - \alpha (b_0 + b_1 \bar{s})$ and $c^\mu \equiv c_0 + c_1 (\bar{s} + \underline{s})$. Due to the linearity, there is always a unique equilibrium for $p_0 \geq p_{t-1}^f$. For the interior equilibrium which holds for $0 < \alpha < \tilde{\alpha} = c_1/[2(b_0 + b_1 \bar{s})]$, we obtain $p^* = \frac{A}{c_1 - 2a b_1}$, $s^* = c_0 + 2a b_0 / (c_1 - 2a b_1)$. Overall, we find:

(a) For $\alpha = 0$, we have $p^* = c(\underline{s})$, $s^* = \underline{s}$.

(b) For $0 < \alpha < \tilde{\alpha} = c_1/[2(b_0 + b_1 \bar{s})]$, we have an interior equilibrium with $p^* = \frac{A}{c_1 - 2a b_1}$, $s^* = \frac{c_1 + 2a b_0}{c_1 - 2a b_1}$.

(c) For $\alpha \geq \tilde{\alpha}$, we have $p^* = c^\mu$, $s^* = \bar{s}$.

Figures 1.a to 1.c are based on $b_1 = 0$, $b_0 = c_1 = 1$, $c_0 = 0.5$, $\underline{s} = 0$ and $\alpha \in \{0, 0.25, 0.75\}$.
A.1.2 Proof of Proposition 1

To characterise the solution, it is crucial whether the kink in the function \( p_t(p_{t-1}) \) is above or below the 45\(^\circ\) line. It is below if \( p_t(p^I_{all}) = c^\mu < p^I_{all} = c(\overline{\alpha}) - \alpha b(\overline{\alpha}) \), i.e., when the average cost of treating all patients is lower than the net cost of treating the patient with highest severity. We therefore find that decreasing prices are possible (for a sufficiently high initial price) if and only if

\[
p_t(p^I_{all}) = p^I_{all} = c(\overline{\alpha}) - \alpha b(\overline{\alpha}) \iff \alpha < \hat{\alpha} \equiv \frac{c(\overline{\alpha}) - c^\mu}{b(\overline{\alpha})} > 0,
\]

where \( \hat{\alpha} \) is critical level of altruism below which not all patients are treated in equilibrium and prices can be decreasing if the initial price is sufficiently high. For altruism above \( \hat{\alpha} \), prices can only increase until they reach the highest possible level, i.e., the average cost of all patients. In Figure 1.a and 1.b, altruism is below \( \hat{\alpha} \). Figure 1.c shows the case of high altruism \( \alpha \geq \hat{\alpha} \). The unique stable equilibrium is given by \( p^* = c^\mu \) and everybody is treated.

A.1.3 Comparative statics with respect to \( \alpha \)

Differentiating (4) with respect to \( \alpha \), we obtain

\[
\frac{\partial s^*}{\partial \alpha} = \frac{b(s^*)}{-\left[\alpha b(s^*) - c'(s^*) + \frac{f(s^*)}{F(s^*)} \left( c(s^*) - \frac{\int_0^{s^*} c(s)f(s)ds}{F(s^*)} \right)\right]}. \tag{A.4}
\]

The stability condition requires

\[
\left. \frac{dp_t}{dp_{t-1}} \right|_{s^*} = \frac{f(s^*)}{F(s^*)} \left[ c(s^*) - \frac{\int_0^{s^*} c(s)f(s)ds}{F(s^*)} \right] \frac{1}{-[\alpha b(s) - c'(s)]} < 1,
\]

which implies that the denominator in \( \frac{\partial s^*}{\partial \alpha} \) is positive. Moreover,

\[
\frac{\partial p^*}{\partial \alpha} = \frac{f(s^*)}{F(s^*)} \left( c(s^*) - \frac{\int_0^{s^*} c(s)f(s)ds}{F(s^*)} \right) \frac{\partial s^*}{\partial \alpha} > 0.
\]
A.2 Scenario D

Since now patients in the interval \([s_{t-1}, \bar{s}]\) are treated, the DRG price in period \(t\) is given by

\[
p_t = \frac{\int_{s_{t-1}(p_{t-1})}^{\bar{s}} c(s) f(s) ds}{1 - F(s_{t-1}(p_{t-1}))} \equiv h(p_{t-1}),
\]  

(A.5)

where \(1 - F(s_{t-1}(p_{t-1})) = \int_{s_{t-1}(p_{t-1})}^{\bar{s}} f(s) ds\) is the number of patients treated and \(s_{t-1}(p_{t-1})\) is the marginal severity in the previous period \(t - 1\), defined by \(\alpha b(s_{t-1}) + p_{t-1} = c(s_{t-1})\).

As above, we define the minimum price which induces providers to treat some patients. Because in this case providers have a preference for high-severity patients, the price in scenario D is given by

\[
p_{D_{\text{min}}} \equiv c(\bar{s}) - \alpha b(\bar{s}),
\]

which is the net cost of treating the patient with highest severity. Furthermore, we define 

\[
p_{D_{\text{all}}} \equiv c(s) - \alpha b(s)
\]

as the price above which providers treat all patients, which is equal to the net cost of treating the patient with lowest severity. The assumption \(c'(s) - \alpha b'(s) < 0\) implies that 

\[
p_{D_{\text{min}}} < p_{D_{\text{all}}}.
\]

If \(p_{t-1} \geq p_{D_{\text{all}}}\), the price in the following period is equal to the average cost when all patients are treated, 

\[
\mu = \int_{s_{t-1}(p_{t-1})}^{\bar{s}} c(s) f(s) ds.
\]

For intermediate prices \(p_{t-1} \in [p_{D_{\text{min}}}, p_{D_{\text{all}}}])\), the price dynamics follow the non-linear first-order difference equation (A.5) which implicitly defines the function \(p_t = h(p_{t-1})\). Price dynamics can therefore be summed up in

\[
p_t(p_{t-1}) = \begin{cases} 
  c^\mu & \text{if } p_{t-1} \geq p_{D_{\text{all}}} \\
  h(p_{t-1}) & \text{if } p_{D_{\text{min}}} \leq p_{t-1} < p_{D_{\text{all}}}. 
\end{cases}
\]

(A.6)

For intermediate prices \(p_{D_{\text{min}}} \leq p_{t-1} < p_{D_{\text{all}}}\), we can implicitly differentiate (A.5). Using (1), we obtain

\[
\frac{dp_t}{dp_{t-1}} = \frac{f(s_{t-1}(p_{t-1}))}{1 - F(s_{t-1}(p_{t-1}))} \left[ c(s_{t-1}(p_{t-1})) - \frac{\int_{s_{t-1}(p_{t-1})}^{\bar{s}} c(s) f(s) ds}{1 - F(s_{t-1})} \right] \frac{1}{c'(s) - \alpha b'(s)}.
\]

(A.7)

The term in the square brackets in equation (A.7) is negative since the cost of the marginal patient is lower than the average cost. Furthermore, \(c'(s) - \alpha b'(s) < 0\) by assumption. Thus, \(dp_t/dp_{t-1}\) is strictly negative for \(p_{D_{\text{min}}} \leq p_{t-1} < p_{D_{\text{all}}}\) and a higher price in a period decreases the price in the next period.
The equilibrium corresponding to Figure 3 is based on linear benefit and cost functions and a uniform density function as above. Scenario D arises if \( ab_1 > c_1 \) or, equivalently, \( \alpha > \hat{\alpha} \equiv c_1/b_1 \). Patients are treated if \( \alpha (b_0 + b_1 s_t) + p_t \geq c_0 + c s_t \), leading to \( s_t = \frac{c_0 - \alpha b_0 - p_t}{\alpha b_1 - c_1} \). Using (A.5), DRG pricing implies \( p_t = c_0 + \frac{c_1}{2} + \frac{c_1 (c_0 - \alpha b_0)}{2(\alpha b_1 - c_1)} - \frac{c_1}{2(\alpha b_1 - c_1)} p_{t-1} \), with \( p_{min}^D = c_0 + c_1 \mathfrak{f} - \alpha b_0 - \alpha b_1 \mathfrak{f}, \ p_{all}^D = c_0 + c_1 \mathfrak{f} - \alpha b_0 - \alpha b_1 \mathfrak{f}, \) and \( c'' = c_0 + c_1 (\mathfrak{f} + \mathfrak{g})/2 \). In equilibrium, we have a corner solution with \( s^* = \mathfrak{f} \) and \( p^* = c_0 + \frac{c_1 (\mathfrak{f} + \mathfrak{g})}{2} \).

Noting that \( \hat{\alpha} = \frac{\alpha}{b_1} > \hat{\alpha} = \frac{c_1}{2(b_0 + b_1 \mathfrak{f})} \), all patients are treated in this equilibrium. To draw Figure 3, we have used \( b_0 = b_1 = 2, \ c_1 = 1, \ c_0 = 1.5, \ \mathfrak{f} = 0 \) and \( \alpha = 0.6 \).

### A.3 Examples for Figure 3.1

As in our other examples, we assume that benefit and cost is linear in severity, the density function is uniform and that the provider has an interior solution. Scenario L arises if \( c_1 > b_1 \) and \( b_0 > c_0 \), so that \( s^f = \frac{b_0 - c_0}{c_1 - c_0} \). Scenario I applies with \( s^* = \frac{2 c_1 + 2 b_0}{c_1 - 2 b_0} \). Using the parameter values \( b_1 = 0, \ b_0 = c_1 = 1, \ c_0 = 0.5, \ \mathfrak{f} = 0 \), we obtain \( s^f = 0.5 \) and \( s^* = 2 \alpha \). Thus, \( \hat{\alpha} = 0.25 \).

For \( \alpha = 0.2 \), we have \( s^* = 0.4 \) and case (a) obtains. \( \alpha = 0.4 \) implies \( s^* = 0.8 \) and therefore case (b). Scenario H arises if \( b_1 > c_1 \) and \( c_0 > b_0 \), so that \( s^f = \frac{c_0 - b_0}{b_1 - c_1} \), and \( \frac{c_2}{b_1} \equiv c_1/b_1 \). If \( \alpha > \frac{c_1}{b_1} \) then scenario D arises and \( s^* = \mathfrak{f} \). If \( \alpha < c_1/b_1 \) then scenario I arises and \( s^* = \frac{2 c_1 + 2 b_0}{c_1 - 2 b_0} \) if \( 0 < \alpha < \hat{\alpha} = \frac{c_1}{2(b_0 + b_1 \mathfrak{f})} \) and \( s^* = \mathfrak{f} \) if \( \alpha < \alpha < \hat{\alpha} \). Using the parameter values \( b_0 = b_1 = 2, \ c_1 = 1, \ c_0 = 2.5, \ \mathfrak{f} = 0 \), we obtain \( s^f = 0.5, \ \hat{\alpha} = 0.125 \) and \( \hat{\alpha} = 0.5 \). Scenario I with \( s^* = \frac{4 \alpha}{1 - 4 \alpha} \) holds for \( 0 < \alpha < 0.125 \), scenario D with \( s^* = \mathfrak{f} \) for \( \alpha > 0.125 \). The figures (c) to (e) correspond to \( \alpha \in \{0.05, 0.1, 0.2\} \) with \( s^* \in \{0.25, 0.67, 1\} \)