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Analysis of the Least Squares Approach to Broadband Beamspace Beamforming

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Abstract—In this paper, we present a comprehensive comparison of different structures for broadband beamforming. We focus on both the tapped delay line (TDL) and the least squares (LS), beamspace approaches. Through simulations we confirm the superiority of the beamspace method (i.e., less complex and better frequency invariance). However, its anti-jamming ability is reduced due to non-orthogonal beams. We show how to mitigate this via a reduced rank approximation of the autocorrelation matrix.

Keywords—Beamspace beamforming; frequency invariance; eigendecomposition;

I. INTRODUCTION

Adaptive beamforming is one of the major implementations of array signal processing and has wide applications in various areas such as radar, sonar, communications and medical diagnosis, to name just a few [1]. While most research in the past has mainly focused on narrowband signals, in recent past broadband signals have received more attention. This can be mainly attributed to superior beamforming performance. For the reason that each broadband signal consists of many different frequency components, the beamforming weights should be different for different frequencies, which unavoidably increases the computational complexity [2]. Nevertheless, broadband beamspace adaptive beamforming is an effective method to alleviate this problem. In this approach, the array elements are initialized with fixed weights to form several beams pointing in different directions; thereafter the output of each beam is followed by a single variable weight to adjust the output adaptively [3]. Since only one adaptive weight is required in each beam, it is computationally more efficient.

The key problem in broadband beamspace beamforming is the design of the transfer matrix, which transforms the received signal from the element space into the beam space by forming several beams.

All these beams should meet two conditions: one is to be frequency invariant and the other is to be linearly independent [4]. The frequency invariant beamformer (FIB), which has been extensively studied, can form beams pointing to the signal of interest with a constant beamwidth. In recent years two main approaches to FIB design have emerged. (i) The first method is the Inverse Discrete Fourier Transform (IDFT) based method, and in [5] two-dimensional (2-D) FIR fan filters are used to construct a multibeam forming network in a tapped delay line (TDL) structure. This idea is then extended to 3-D structures based on the multi-dimensional IDFT [6]. Subsequently, a broadband beamspace adaptive beamformer based on this technique is proposed in [4]. (ii) The second approach is the Spatial Response Variation (SRV) based method. In [7] the SRV approach is defined to measure the fluctuation of the array spatial response within the desired frequency band. Recently, a least squares (LS) cost function is first combined with SRV constraints to control the frequency invariant property in the frequency band of interest [8]. This method can provide a closed-form solution and reduces the computational complexity. Moreover, it can easily be applied to different array structures. Therefore, we adopt the LS approach to design linearly independent frequency invariant beams. But it is worth noting that there is a trade-off between the frequency invariance and the output performance [4]. When the beams are not exactly orthogonal, the output will deteriorate. However, using eigendecomposition of the correlation matrix is an effective way to alleviate this problem [3].

So this paper is organized as follows. An introduction to the traditional TDL structure for broadband beamforming and a review about the LS approach to the FIB design are first provided in Section 2. Afterwards, beamspace adaptive beamforming and the eigendecomposition technique are introduced in Section 3. Then, a design example is presented in Section 4 to verify the proposed method and conclusions are drawn in Section 5.

II. THE LEAST SQUARES APPROACH TO BROADBAND FREQUENCY INVARIANT BEAMFORMING

A. Broadband beamforming with a TDL structure

A broadband beamformer with a TDL structure is shown in Fig. 1. Here M is the number of elements of the uniform linear array and N is the number of taps associated with each sensor. Suppose the direction of arrival (DOA) of the received signal is θ , the inter-element spacing is d , and the sampling period of the TDL is T_s , then the output can be expressed as:

$$y[n] = \sum_{m=0}^{M-1} \sum_{k=0}^{N-1} w_{m,k}^* x_{m,k}[n] \quad (1)$$

where $*$ denotes the conjugate operation and $x_{m,k}[n]$ is the output signal of the m -th antenna and the k -th tap. The angle and frequency dependent response can be written as:

$$P(\omega, \theta) = \sum_{m=0}^{M-1} \sum_{k=0}^{N-1} w_{m,k}^* e^{-j\omega(\tau_m + kT_s)} \quad (2)$$

where ω is the angular frequency and $\tau_m = m(d/c)\sin\theta$ ($m = 1, 2, \dots, M-1$) is the delay between the m -th sensor and the zero-phase reference point. It can be expressed in vector form as:

$$P(\omega, \theta) = \mathbf{w}^H \mathbf{s}(\omega, \theta) \quad (3)$$

where “ H ” denotes the conjugate transpose operation, \mathbf{w} is the coefficient vector, and $\mathbf{s}(\omega, \theta)$ is the steering vector, where:

$$\begin{aligned} \mathbf{w} &= [w_{0,0}, \dots, w_{M-1,0}, \dots, w_{0,N-1}, \dots, w_{M-1,N-1}]^T \\ \mathbf{s}(\omega, \theta) &= \mathbf{s}_{T_s}(\omega) \otimes \mathbf{s}_{\tau_m}(\omega, \theta) \end{aligned} \quad (4)$$

and \otimes denotes the Kronecker product with

$$\begin{aligned} \mathbf{s}_{T_s}(\omega) &= [1, e^{-j\omega T_s}, \dots, e^{-j\omega(N-1)T_s}]^T \\ \mathbf{s}_{\tau_m}(\omega, \theta) &= [e^{-j\omega\tau_0}, e^{-j\omega\tau_1}, \dots, e^{-j\omega\tau_{M-1}}]^T. \end{aligned} \quad (5)$$

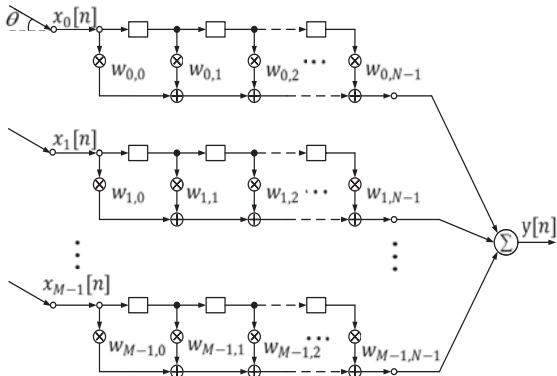


Fig. 1: Broadband beamforming with a TDL structure.

Then the Frost beamformer [9] to calculate the weights can be formulated as follows:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} \quad \text{subject to} \quad \mathbf{C}^H \mathbf{w} = \mathbf{f} \quad (6)$$

where $\mathbf{R}_{xx} = E[\mathbf{x}[n]\mathbf{x}^H[n]]$ is the covariance matrix of the received array signal $\mathbf{x}[n] = [x_0[n], x_1[n], \dots, x_{M-1}[n]]^T$, $E[\cdot]$ represents the expectation, \mathbf{C} is the constraint matrix, and \mathbf{f} is the response vector with one entry being unity and the rest being zero.

The well-known solution to (6) can be obtained by the standard Lagrange multiplier method, and is given by:

$$\mathbf{w}_{opt} = \mathbf{R}_{xx}^{-1} \mathbf{C} (\mathbf{C}^H \mathbf{R}_{xx}^{-1} \mathbf{C})^{-1} \mathbf{f}. \quad (7)$$

B. The LS approach to FIB design

In order to design an FIB, the response variation (RV) is first introduced. This is a parameter to control the frequency invariant property, and is given by [8]:

$$RV = \sum_{f_n \in \Omega_i} \sum_{\theta_k \in \Theta_{FI}} |\mathbf{w}^H \mathbf{s}(f_n, \theta_k) - \mathbf{w}^H \mathbf{s}(f_r, \theta_k)|^2 \quad (8)$$

where Ω_i and Θ_{FI} represent respectively the frequency range of interest and the direction range in which frequency invariance is considered, and f_r denotes the fixed reference frequency. Then we sample the frequency and angle ranges uniformly to get the sample points (f_n, θ_k) , and the frequency invariance can be realized by minimizing RV over \mathbf{w} . Note that if we minimize RV in the whole angle range, we only need to minimize the spectrum energy of the beamformer at the reference frequency f_r over the sidelobe region Θ_s , while maximizing the response at the reference frequency in the look direction θ_r ($\theta_r \in \Theta_m$), where Θ_m represents the mainlobe region. This can be written as:

$$\min_{\mathbf{w}} \sum_{\theta_k \in \Theta_s} |\mathbf{w}^H \mathbf{s}(f_r, \theta_k)|^2 \quad \text{subject to} \quad \mathbf{w}^H \mathbf{s}(f_r, \theta_r) = 1. \quad (9)$$

Then we can add the RV element to the LS cost function with a trade-off coefficient β and combine (8) and (9) together to get [8]:

$$\begin{aligned} \min_{\mathbf{w}} \sum_{i=0}^{I-1} \sum_{k=0}^{K-1} |\mathbf{w}^H \mathbf{s}(f_i, \theta_k) - \mathbf{w}^H \mathbf{s}(f_r, \theta_k)|^2 \\ + \beta \sum_{\theta_k \in \Theta_s} |\mathbf{w}^H \mathbf{s}(f_r, \theta_k)|^2 \\ \text{subject to} \quad \mathbf{w}^H \mathbf{s}(f_r, \theta_r) = 1 \end{aligned} \quad (10)$$

where I and K represent the number of samples over the frequency and the angle ranges in which frequency invariance is considered. Then we can rewrite (10) as:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{Q} \mathbf{w} \quad \text{subject to} \quad \mathbf{C}^H \mathbf{w} = \mathbf{f} \quad (11)$$

with:

$$\mathbf{Q} = \sum_{i=0}^{I-1} \sum_{k=0}^{K-1} (\mathbf{s}(f_i, \theta_k) - \mathbf{s}(f_r, \theta_k))(\mathbf{s}(f_i, \theta_k) - \mathbf{s}(f_r, \theta_r))^H + \beta \sum_{\theta_k \in \Theta_s} \mathbf{s}(f_r, \theta_k) \mathbf{s}(f_r, \theta_k)^H. \quad (12)$$

The solution (similar to (7)) is:

$$\mathbf{w}_{opt} = \mathbf{Q}^{-1} \mathbf{C} (\mathbf{C}^H \mathbf{Q}^{-1} \mathbf{C})^{-1} \mathbf{f} \quad (13)$$

where $\mathbf{C} = \mathbf{s}(f_r, \theta_r)$ and $\mathbf{f} = 1$.

III. BROADBAND BEAMSPACE ADAPTIVE BEAMFORMING

A. Broadband beamspace processor

In this section, we extend the technique presented in previous section to form P fixed independent frequency invariant beams to cover the range of azimuthal angles of interest. One of these beams is the main beam pointing in the direction of the signal of interest and the remaining $P - 1$ beams are the auxiliary beams pointing in the remaining directions. The appropriate structure is shown in Fig. 2 (with $\mathbf{x}[n]$ as defined for Fig. 1), where FIB_0 is the main beam followed by an arbitrarily fixed weight g_0 , and the other FIB_p ($p = 1, \dots, P - 1$) are the auxiliary beams which are connected to variable weights (g_p). The output is given by:

$$y[n] = g_0^* b_0[n] + \mathbf{g}^H \mathbf{b}[n] \quad (14)$$

where

$$\mathbf{g} = [g_1, g_2, \dots, g_{P-1}]^T \quad (15)$$

$$\mathbf{b}[n] = [b_1[n], b_2[n], \dots, b_{P-1}[n]]^T.$$

Then the output power can be written as:

$$P_{out} = E[|y[n]|^2] = E[|g_0^* b_0[n] + \mathbf{g}^H \mathbf{b}[n]|^2]. \quad (16)$$

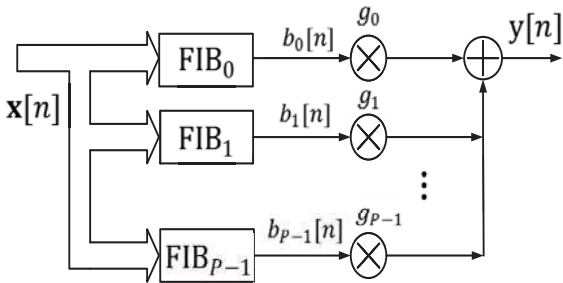


Fig. 2: Broadband beamspace processor structure [4].

The weights can be calculated by minimizing P_{out} while maintaining the constraint main beam weight, which can be expressed as:

$$\min_{\mathbf{g}} E[|g_0^* b_0[n] + \mathbf{g}^H \mathbf{b}[n]|^2] \quad \text{subject to } g_0 = \text{constant}. \quad (17)$$

This gives:

$$\mathbf{g}_{opt} = -\mathbf{R}_{bb}^{-1} \mathbf{r}_0 g_0 \quad (18)$$

where $\mathbf{R}_{bb} = E[\mathbf{b}[n] \mathbf{b}^H[n]]$ and $\mathbf{r}_0 = E[\mathbf{b}[n] b_0^*[n]]$. Finally, note that each FIB block in Fig. 2 is designed using the LS approach in section II.B.

B. Eigendecomposition

The broadband beamspace structure can show good results as regards both frequency invariance and the anti-jamming performance when the beams are completely orthogonal as shown in Fig. 3. However, the incomplete orthogonality of beams in Fig. 4 can lead to the deterioration of the output. In order to alleviate this problem, the eigendecomposition of the correlation matrix \mathbf{R}_{bb} is employed.

Suppose that the number of interferers is J , then the first J largest eigenvalues of \mathbf{R}_{bb} represent the effect of the interferers, and the remaining smaller $P - 1 - J$ eigenvalues approximately represent the effects of leakage and noise. So the correlation matrix can be rewritten as:

$$\mathbf{R}_{bb} = \sum_{j=1}^{P-1} \lambda_j \mathbf{z}_j \mathbf{z}_j^H \quad (19)$$

where \mathbf{z}_j is the eigenvector corresponding to λ_j . And the inverse matrix is given by:

$$\mathbf{R}_{bb}^{-1} = \sum_{j=1}^{P-1} \frac{1}{\lambda_j} \mathbf{z}_j \mathbf{z}_j^H. \quad (20)$$

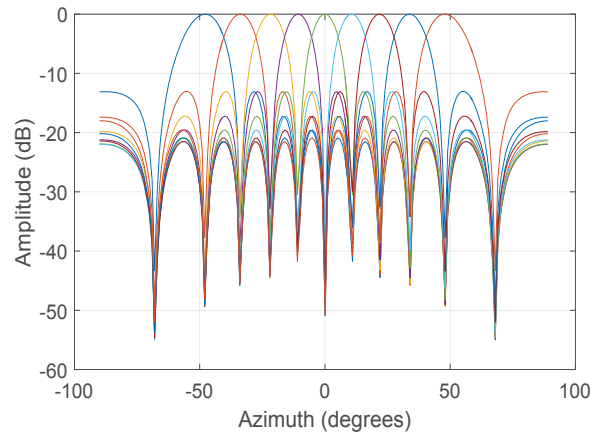


Fig. 3: Nine completely orthogonal beams in Fig. 2 for $P = 9$.

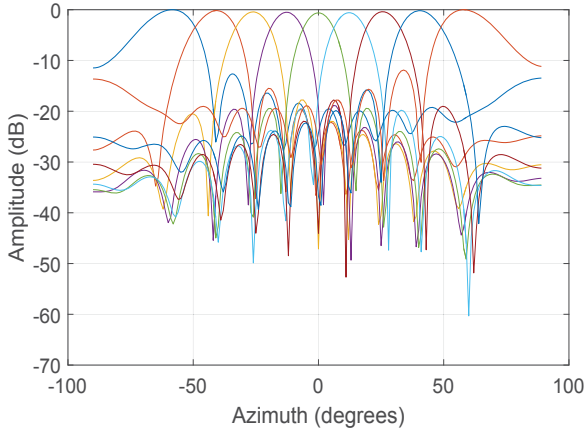


Fig. 4: Nine beams, not all orthogonal in Fig. 2 for $P = 9$.

By examining the magnitude of the eigenvalues $\{\lambda_j\}_{j=1}^P$ we can estimate J and thus divide the signals received by the auxiliary beams into interference space and noise plus signal leakage space. Then using just the largest J eigenvalues we approximate \mathbf{R}_{bb}^{-1} by the reduced rank version:

$$\hat{\mathbf{R}}_{bb}^{-1} = \sum_{j=1}^J \frac{1}{\lambda_j} \mathbf{z}_j \mathbf{z}_j^H \quad (J < P). \quad (21)$$

And so the new weights become:

$$\hat{\mathbf{g}}_{opt} = -\hat{\mathbf{R}}_{bb}^{-1} \mathbf{r}_0 g_0. \quad (22)$$

IV. DESIGN EXAMPLE

The simulation is based on a uniform linear array with $M = 12$ sensors, $N = 5$ taps, and the inter-element spacing $d = \lambda/2$ where λ is the wavelength corresponding to the highest frequency component. Suppose there is one desired linear frequency modulated (LFM) signal arriving from 30° with a bandwidth of 10MHz (40MHz to 50MHz), a signal-to-noise-ratio (SNR) of 0dB. The interference has a bandwidth of 10MHz (40MHz to 50MHz), a SNR of 30dB that comes from the direction of -40° . The sampling frequency is twice the highest frequency and the number of snapshots is 512. Fig. 5 shows how the 2-D beam patterns vary with frequency when we use a traditional TDL structure. We can see that although it has a good anti-jamming property, the beam patterns are different at different frequencies. Additionally, when the number of sensors (M) and the number of delays (N) increase, the computational complexity increases rapidly.

But if we adopt the frequency invariant beamspace method, then the dimension of the correlation matrix

(\mathbf{R}_{xx}) can be reduced from $MN \times MN$ to $P \times P$ (for \mathbf{R}_{bb}). Moreover, since the RV constraint is employed, this method can also achieve a good frequency invariant property. Fig. 6 shows the beam patterns at different frequencies when the LS based frequency invariant beamspace method is adopted. From this result, we can see that it has good frequency invariance, but the anti-jamming performance has deteriorated.

However if we employ eigendecomposition to the beamspace method, then the new result is shown in Fig. 7. Here we can see that now it not only achieves good frequency invariance but also maintains the anti-jamming property.

V. CONCLUSION

In this paper, we gave an overview of the general problem of broadband beamforming and focused on

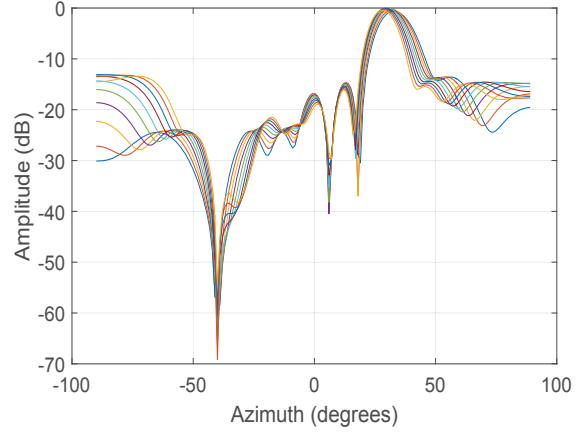


Fig. 5: 2-D beam patterns at different frequencies using the TDL structure.

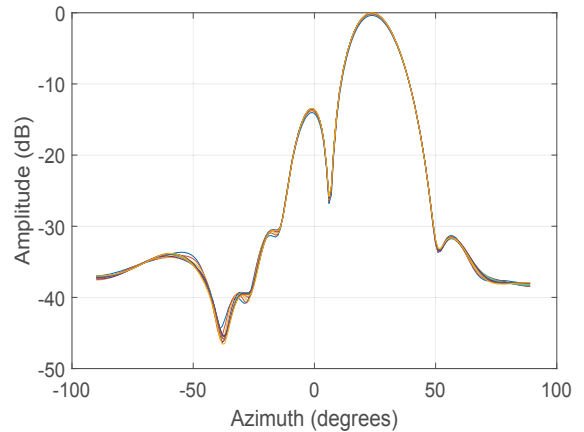


Fig. 6: 2-D beam patterns at different frequencies using the frequency invariant beamspace structure.

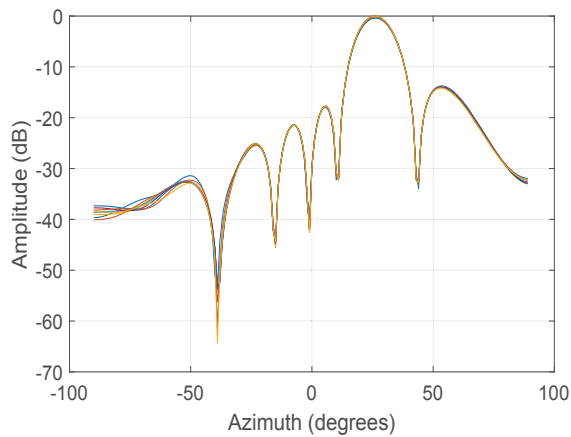


Fig. 7: 2-D beam patterns at different frequencies using the frequency invariant beamspace structure with eigen-decomposition.

published results for both the TDL structure and the beamspace method. We looked at three possibilities: (i) TDL design; (ii) LS beamspace design; (iii) LS beamspace design with reduced rank autocorrelation matrix.

Our simulations showed that while (ii) has reduced

complexity and better frequency invariance than (i), its anti-jamming performance is inferior. This was remedied in (iii) by using a reduced rank approximation of \mathbf{R}_{bb} to mitigate the effect of the non-orthogonal beams (see Fig. 4).

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