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https://doi.org/10.1016/j.solener.2016.09.021

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Photothermal Conversion Efficiency of Nanofluids: An Experimental and Numerical study

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Abstract: This work investigated experimentally the photothermal conversion efficiency (PTE) of gold nanofluids in a cylindrical tube under natural solar irradiation conditions, and compared with a developed 3-dimensional numerical model. The PTE of gold nanofluids was found to be much higher than that of pure water, and increased non-linearly with particle concentration, reaching 76% at a concentration of 5.8 ppm. Significant non-uniform temperature distribution was identified both experimentally and numerically, and a large uncertainty can be caused in the PTE calculation by using only one temperature measurement. A mathematical model was also developed to calculate the absorption efficiency without knowing the temperature field, which can be used to predict the theoretical PTE for nanofluids based on their optical properties only.

Keywords: nanoparticle; nanofluid; solar energy; photothermal conversion efficiency, direct absorption

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<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Definition</th>
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<tbody>
<tr>
<td>A</td>
<td>surface area exposed to solar radiation (m²) / absorbance (-)</td>
</tr>
<tr>
<td>aₙ</td>
<td>Mie coefficient to compute the amplitudes of the scattered field (-)</td>
</tr>
<tr>
<td>bₙ</td>
<td>Mie coefficient to compute the amplitudes of the scattered field (-)</td>
</tr>
<tr>
<td>c</td>
<td>specific heat capacity (J/(kg · K))</td>
</tr>
<tr>
<td>cₚ</td>
<td>specific heat capacity (J/(kg · K))</td>
</tr>
<tr>
<td>D</td>
<td>particle diameter (m)</td>
</tr>
<tr>
<td>E</td>
<td>spectral emissive power (W/m³)</td>
</tr>
<tr>
<td>fᵣ</td>
<td>volume concentration (-)</td>
</tr>
<tr>
<td>h</td>
<td>convection coefficient (W/(m² · K))</td>
</tr>
<tr>
<td>I</td>
<td>radiative intensity (W/m²)</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity (W/(m · K))</td>
</tr>
<tr>
<td>kᵣ</td>
<td>imaginary part of the complex refractive index of the based fluid (-)</td>
</tr>
<tr>
<td>L</td>
<td>optical depth (m)</td>
</tr>
<tr>
<td>m</td>
<td>mass (kg) / relative refractive index (-)</td>
</tr>
<tr>
<td>n</td>
<td>complex refractive index (-) / order of accuracy</td>
</tr>
<tr>
<td>q</td>
<td>heat flux (W/m²)</td>
</tr>
<tr>
<td>Q</td>
<td>efficiency factor for Mie scattering (-)</td>
</tr>
<tr>
<td>Qᵣ</td>
<td>radiative heat source in heat transfer equation</td>
</tr>
<tr>
<td>R</td>
<td>radius of cylinder experimental tube (m)</td>
</tr>
<tr>
<td>r</td>
<td>radius in integrating process (m)</td>
</tr>
<tr>
<td>ˢ</td>
<td>one specific direction which contains infinitesimal pencil of rays</td>
</tr>
<tr>
<td>T</td>
<td>temperature (°C)</td>
</tr>
<tr>
<td>t</td>
<td>time (s)</td>
</tr>
<tr>
<td>u</td>
<td>velocity (m/s)</td>
</tr>
</tbody>
</table>
Greek symbols

\( \beta \) extinction coefficient (m\(^{-1}\))

\( \Phi \) scattering phase function

\( \varepsilon \) spectral emissivity

\( \eta \) efficiency (-)

\( \kappa \) absorption coefficient (m\(^{-1}\))

\( \lambda \) wavelength of light in vacuum (m)

\( \sigma \) scattering coefficient (m\(^{-1}\))/Stefan-Boltzmann constant = \(5.670 \times 10^8\) (W/(m\(^2\)-K\(^4\)))

\( \Omega \) solid angle

\( \rho \) density

\( \psi_n \) spherical Bessel function of order \( n \)

\( \xi_n \) spherical Bessel function of order \( n \)

Superscripts

- average value

\( \rightarrow \) vector quantity

Subscripts

\( \text{abs} \) absorption

\( \text{amb} \) ambient

\( \text{b} \) black body

\( \text{ext} \) extinction

\( \text{f} \) fluid

\( \eta \) wavelength range

\( \text{i} \) direction number of light

\( \text{n} \) nanoparticle

\( \text{out} \) outlet

\( \text{p} \) particle
sca scattering
s scattering
w water
1 Introduction

The concerns over excessive use of fossil energies and increasing environmental problems have accelerated rapid development of solar energy technologies [1]. However, the difficulties in efficiently collecting solar energy and converting it into useful energies (i.e., either electricity or heat) limit the extensive utilization of solar energy [2]. Most of solar thermal collectors have ‘tube-in-plate’ arrangements, which absorb solar energy on their surfaces and transfer heat to a working fluid running inside the tubes. Such an arrangement is surface-limited, i.e., relying on the transfer of heat from a tube surface to the fluid inside. This would produce a large temperature difference between the fluid and the absorber especially for high temperature applications (i.e., solar thermal power plants) [3], and result in a limited solar energy utilization efficiency.

The concept of volumetric solar energy absorption, i.e., certain materials are seeded in a working fluid to absorb solar energy directly within the fluid itself, was originated in 1970’s and coined as direct absorption solar collector (DASC) [4]. In the concept, selective tube materials are used to allow most of the solar energy pass through the wall and into the fluid, but prevent the radiation leakage from the fluid, forming a ‘thermal trapping’ phenomenon[5]. In this way, the highest temperature exists in the fluid and the overall conversion efficiency from solar energy to heat can be largely improved due to reduced re-radiation heat loss.

The use of nanoparticles as effective absorption media is a recent development. It has been reported that adding very diluted particles into base fluid could enhance the radiative absorbing efficiency and improve the overall heat transfer rate due to their large specific surface areas [6]. A range of nanoparticles including metal (such as Cu, Au, and Ag), metal oxide (such as TiO₂, Al₂O₃) and carbon materials [7–11] have been investigated under laboratory [3,12–14] and natural sunlight conditions [15–17]. Some results were very encouraging. For instance, the solar conversion efficiency of a 0.01% graphite nanofluid was found to be as high as 122.7% of that of a conventional surface absorbing collector [12]. Some metallic nanoparticles such as gold and silver have also drawn wide attentions because of their Surface Plasmon Resonance effects (SPR)
For these kinds of materials, the resonance frequencies of conduction electrons are usually in the visible-light spectrum, which is weakly absorbed by most of the heat transfer fluids but occupies nearly half of the total solar radiation energy [20]. Zhang et al. [14] showed that a very low concentration of gold nanoparticles (i.e., mass concentration of 0.0028%) could increase the photothermal conversion efficiency (PTE) of the base fluid by 20%, reaching an impressive specific absorption rate (SAR) of ~10 kW/g under laboratory conditions. In another study conducted outdoor, up to 144% enhancement in the stored thermal energy was obtained for 6.5 ppm silver nanoparticle-based direct absorption under natural sunlight conditions [15]. However, it shall be noted that only one temperature was measured in most of the published work [21–25] and a uniform temperature assumption was used to calculate the energy efficiency, neglecting the temperature distribution within the fluid. Considering a reduced radiative intensity along the path of the absorbing liquid, there shall exist large temperature non-uniformity in the fluid, whose neglect may lead to inaccurate calculations for the PTE and disguise some key parameters (such as the optical depth) in optimizing the collector design.

Quite a few studies [3,24,26–28] have built numerical models to simulate the radiative and heat transfer process in nanofluids. Unfortunately, most of these simulative studies were based on the ideal solar spectrum, and ignored that the spectral emissive power distribution is affected by the atmosphere’s absorption, especially in the infrared spectrum. As the radiation properties of nanoparticles are highly spectral dependent, any numerical work should consider the solar spectrum associated with realistic experimental conditions. A theoretical method to predict photothermal efficiency directly based on nanoparticle’s basic physical properties with respect to particle loadings and optical depth has yet to be established.

To overcome the issues reviewed above, a detailed investigation of the photothermal conversion characteristics of gold nanoparticle dispersions was conducted both experimentally and numerically. Gold nanoparticles were synthesized via a one-pot reaction, and the experiments were performed under natural
sunlight conditions with multiple temperature measurement. A 3-dimensional model was developed to simulate the experimental results, which was followed by a parametric investigation of the influence of particle concentration, solar radiation intensity and receiver geometrical parameters on the solar conversion efficiency. A new method to theoretically predict the photothermal conversion efficiency of nanoparticle dispersions was proposed according to the radiative transfer equation.

2 Experimental investigation

2.1 Gold nanoparticle dispersions formulation

In this study, a one-step method [29] was used to produce gold nanoparticle dispersions and different concentrations were prepared. A typical procedure is introduced below as an illustration, i.e., CASE 6 in Table 1. Here $2.5 \times 10^{-7}$ mol $\text{HAuCl}_4$ was dispersed into 50 ml DI water in a three-necked flask under heating. A magnetic blender was used to stir the liquid until boiling. Boiling was continued for 10 min and then 50 ml of $1 \times 10^{-5}$ mol/L sodium citrate was added. The solution turned dark blue within 30 seconds and the final color became wine red after being heated for an additional 20 minutes. The size and shape of CASE 6 identified by a transmission electron microscopy are shown in Fig. 1a. Table 1 represents 6 cases of gold nanoparticle dispersions. The dispersions were maintained good stability for over two months, and were used for the below experiments without further purification and separation.

An UV/Vis spectrometer (UV-1800 SHIMADZU UV Spectrophotometer) was applied to measure the absorption spectrum of the nanoparticle dispersions at different concentrations. As shown in Fig. 1b, due to the strong surface Plasmon resonance of gold nanoparticles in the visible light spectrum[14], a peak absorption wavelength was found at 526 nm for all the dispersions, which is consistent with the TEM analysis that the size of gold nanofluids was about 20 nm. According to the Beer-Lambert Law (known as Beer's Law) [30], also seen in Eq. 8 below, there is a linear relationship between the absorbance and the concentration of the sample, as shown in the inset.
2.2 Experimental settings

The experimental setup in this study is showed in Fig. 2a and b. The tubes used in this experiment were custom-made from high temperature resistant quartz glass. The tube contained a vacuum interlayer to reduce the impact of convection from outside air. The sample fluids were placed in the inside-tube with a diameter of 25 mm and length of 300 mm. The outside-tube had a diameter of 60 mm with two small-bore pipes, which were used to fix temperature sensors.

As shown in Fig. 2b, three T-type thermocouples (Omega TT-T-40-SLE) with a precision of ±0.5 K were placed evenly in the bottom, middle, top of the sample fluids along the optical depth, and two more thermocouples were applied to measure the air inside and outside the tube (i.e., the ambient), respectively. A data acquisition (Agilent 34970A) system was used to measure the thermocouple voltage signal and then transferred it into digital form, recorded in a PC under LabVIEW environment. A solar radiation intensity sensor was employed to measure the solar intensity, and the data was also recorded in the PC.

3 Experimental results analysis

3.1 Temperature variation

Example temperature curves of gold nanofluids and DI water under varied natural solar intensity are shown in Fig. 3a, together with the air temperature inside and outside the tube. A slow increase in DI water temperature is observed and the solar intensity (I) was varied from about 400 W/m² to 700 W/m². Clearly the temperature variation of both water and nanofluids lags behind, but gold nanofluids show much more rapid temperature increase under the same intensity than pure water. For example, the bulk temperature is increased by ~21 K after 60 minutes’ heating for a GNP concentration of 5.8 ppm, more than three times of pure water temperature rise. Here the average temperature from three thermocouples, i.e., \(\bar{T} = \frac{T_{TC1} + T_{TC2} + T_{TC3}}{3}\), was used to represent the fluid temperature.

To reveal the possible temperature difference inside the fluid, Fig. 3b shows the temperature profiles of
three thermocouples for 5.8 ppm gold nanofluid and DI water respectively under the same condition as Fig. 3a. Consistent large temperature difference was found for the nanofluid. A maximum 2.6 K temperature difference was found for nanofluid after 28 minutes’ illumination, but for DI water, the maximum temperature difference was in a relative small region (i.e., less than 0.5 K). Considering the potential large temperature difference in the nanofluid, as will be revealed by the numerical model, the location of the thermocouple would affect the calculated PTE significantly if only one measurement was used.

Fig. 3c shows an example of the influence of solar intensity variation on the PTE for 0.72 ppm gold nanofluid. For the constant solar intensity case, the temperature increased smoothly and reached the equilibrium point after two hours’ illumination; but for varied solar intensity (cloudy day), the same nanofluid exhibits a changing tendency, which follows the pattern of solar intensity variation. Generally, salient temperature difference among three thermocouples can be observed under both solar intensities, being larger for a higher solar intensity. Further non-uniform temperature distribution and its effects on the solar efficiency is discussed in section 5.

3.2 Efficiency and SAR

The average photothermal conversion efficiency (PTE) is defined as the ratio of the internal energy increase of the fluid to the total incoming radiation input:

\[
\eta = \frac{(c_p m_w + c_p m_n) \Delta T}{I A \Delta t} \approx \frac{c_p m_w}{I A} \frac{\Delta \bar{T}}{\Delta t}
\]

(1)

where \(\Delta \bar{T}\) is the average temperature difference \(\Delta \bar{T} = (\Delta T_{IC1} + \Delta T_{IC2} + \Delta T_{IC3}) / 3\). Comparing with the base water, thermal energy stored in gold nanoparticles is negligible owing to extremely low concentration: i.e., a maximum of 5.8 ppm in volume (0.01% in mass).

To quantify the capability of nanoparticles in absorbing solar energy, the specific absorption rate (SAR) is employed[15]:
The calculated PTEs within the measurement uncertainty of ±3% under two different solar intensities are shown in Fig. 4, which is in general similar to previous studies[14,15]. The PTE reaches 45.5% even at a relatively low concentration (0.36 ppm), increased by 163% comparing with the base water (17.3% at \( I = 950 \text{W/m}^2 \)). The highest PTE of gold nanofluids is 73.6%, which is more than 4 times of DI water and more promising than what found in previous studies [12,14,31]. The PTE increases rapidly under low particle concentrations and approaches asymptotically to a constant value when the concentration reaches a certain status (i.e., 1 ppm for \( I = 600 \text{W/m}^2 \)). A lower PTE value (i.e., 10% smaller) is found for the high solar intensity case, which is assumed to be related to an increased heat loss. Fig. 4 also shows that SAR rapidly decreases with the increasing concentration below 1 ppm, and the highest SAR reaches 2.715 kW/g at nanoparticle concentration of 0.36 ppm for \( I = 950 \text{W/m}^2 \) solar intensity, which is in the broad range of previous studies [14,15]. Further investigation with numerical results will be conducted regarding to PTE and SAR in section 5.

4 Numerical model of direct absorbing solar energy for nanofluids

4.1 Solar radiation and Mie scattering theory

In order to get better understanding of radiative heat transfer in gold nanofluids, a numerical model was built in this work. Realistic solar irradiation profile was calculated based on ASTM G173-03 Reference Spectra [32]. As shown in Fig. 5a, the solar irradiation on the ground is distinctly different from that at top of atmosphere, especially for spectral emissive power in infrared, which is due to the intense absorption of \( \text{H}_2\text{O} \) and \( \text{CO}_2 \) in atmosphere. The result from integrating the spectral emissive power with wavelength shows that nearly 99% of solar radiation energy at sea level exists in 0.2~3 \( \mu \text{m} \). As solar emissive power takes part of nearly 43% in infrared, which can be seen in the inset figure, considering the actual solar spectrum on the ground becomes essential for solar thermal applications. For the purpose of proper simplification [3], spectral emissive power
for sun (T=5762 K) and nanofluid (T=303 K) has been calculated and separated into two bands on wavelength
of 3 \mu m (see in Fig. 3b); Solar irradiation wavelength locates mainly below 3 \mu m with a peak at 480 nm,
while wavelength for nanofluid of 303 K is beyond 3 \mu m.

In the present modeling, the characteristic size employed in radiative transfer equation is as \( x_d = \pi D / \lambda \),
where D represents the diameter of nanoparticles. For our experimental study, the diameter of gold
nanoparticles is 20 nm characterized by TEM as shown in Fig. 1a. Although it is appropriate to use simplified
equations, i.e., the Rayleigh scattering approximation [30], to calculate the absorption coefficient, since the
diameter of suspended particles in the experiments are much smaller than the wavelength of irradiation
(\( x_d << 1 \)). The original Mie scattering equations [30] is preferred to identify the optical properties for spherical
nanoparticle suspensions in order to obtain detailed scattering parameters, such as the efficiencies for
scattering, absorption, backscattering, averaged absolute-square E-field. The Mie scattering equations can be
described by:

\[
\begin{align*}
a_n &= \frac{m\psi_n(mx)\psi_n'(x) - \psi_n(x)\psi_n'(mx)}{m\psi_n(mx)\xi_n'(x) - \xi_n(x)\psi_n'(mx)} \\
b_n &= \frac{\psi_n(mx)\psi_n'(x) - m\psi_n'(mx)\psi_n'(mx)}{\psi_n(mx)\xi_n'(x) - m\xi_n(x)\psi_n'(mx)}
\end{align*}
\] (2-a, b)

\[
Q_{\text{sc}}(\lambda) = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) \left[ |a_n|^2 + |b_n|^2 \right]
\] (2-c)

\[
Q_{\text{ext}}(\lambda) = \frac{2}{x^2} \sum_{n=1}^{\infty} (2n+1) \Re(a_n + b_n)
\] (2-d)

where the functions \( \psi_n(x) \) and \( \xi_n(x) \) are spherical Bessel functions[30] of order n (n= 1, 2,..) and the
primes refer to the derivatives with respect to the argument, and m represents the ratio of refractive indexes,
calculated by:

\[
m = \frac{n_{\text{particles}}}{n_{\text{fluid}}}
\] (3)
where \( n_{\text{particles}} \) and \( n_{\text{fluid}} \) are the complex refractive index [33–35] of gold and based fluid relative to the ambient medium, respectively. In consideration of relative low concentrations of nanofluids developed for solar thermal applications, particles should absorb and scatter light independently according to the scattering map [30]. With such a consideration, the absorption coefficient can be calculated from the below equation:

\[
\kappa(\lambda) = \kappa_p(\lambda) + \kappa_f(\lambda) = \frac{3\pi fQ_{\text{abs}}(\lambda)}{2D} + \frac{4\pi k_f(\lambda)}{\lambda}
\]  

(4)

4.2 Radiative transfer equation and heat transfer equation

The property of spectral intensity can be described simultaneously by the radiative transfer equation, known as RTE [30]:

\[
\hat{s} \cdot \nabla I_\eta = \kappa_\eta I_{b\eta} - \beta_\eta I_\eta + \frac{\sigma_{\eta q}}{4\pi} \int_{4\pi} I_\eta(\hat{s}_i)\Phi_\eta(\hat{s}_i,\hat{s})d\Omega_i
\]  

(5-a)

\[
\nabla \cdot q_\eta = \kappa_\eta \left(4\pi I_{b\eta} - \int_{4\pi} I_\eta d\Omega\right)
\]  

(5-b)

\[
\kappa_\eta = \beta_\eta - \sigma_{\eta q}
\]  

(5-c)

where \( I_\eta \) represents the radiative intensity of wavelength range \( \lambda_\eta_1 \rightarrow \lambda_\eta_2 \) in the direction \( \hat{s}_i \), \( I_{b\eta} \) is the re-emission of nanofluid, \( \Phi_\eta(\hat{s}_i,\hat{s}) \) is called the scattering phase function and describes the probability that a ray from one direction \( \hat{s}_i \), will be scattered into a certain other direction \( \hat{s} \), \( \kappa_\eta \), \( \beta_\eta \), and \( \sigma_{\eta q} \) are the absorption, extinction and scattering coefficient, respectively. Spectral radiative heat flux \( q_\eta \) can be obtained by integrating the radiative intensity with the solid angle \( \Omega \).

Transient heat transfer equation is shown as:

\[
\rho c_p \frac{\partial T}{\partial t} + \rho c_p \tilde{u} \cdot \nabla T + \nabla \cdot \tilde{q} = Q_t
\]  

(6-a)

\[
\tilde{q} = -k \nabla T
\]  

(6-b)

\[
Q_t = -\int_0^\infty q_\eta(\lambda)d\lambda
\]  

(6-c)

The boundary condition and initial condition are:

\[
-\vec{n} \cdot \vec{q} = \varepsilon \sigma \left(T_{\text{amb}}^4 - T^4\right) + h\left(T_{\text{amb}} - T\right)_{\text{contact-air}}
\]  

(7-a)
\[ \text{where } h \text{ is the convection coefficient due to convection from one end of the tube contacted with air outside,} \]

\[ \text{whose contribution to the result is small and a typical value of } 15 \text{ W/(m}^2\cdot\text{K)} \text{ is used in this work.} \]

### 4.3 Predicted absorption efficiency for nanofluids

For most of our applied particles, scattering contribution can be neglected according to the Mie theory. Furthermore, the radiative equations can be simplified as a 1-dimentional transfer process. Under these assumptions, an analytical solution for Eq. (6-a) can be obtained:

\[ E_\lambda(L) = E_{\lambda,0} e^{\beta L} + E_{\text{abs,}\lambda}(1 - e^{\beta L}) \]  

where \( E \) represents the spectral emissive power with unit of \( \text{W/m}^3 \). In order to investigate the spectrum behavior at wavelength below 1.1 \( \mu \text{m} \), which is the upper limit of our spectrophotometer, black body re-emissive radiation can be neglected due to the small intensity of re-emission, seen in Fig. 5b. Coupled with Eq. 5, Eq. 9 can be further simplified as:

\[ \log \left( \frac{E_{\text{out}}(\lambda)}{E_0(\lambda)} \right) = \beta_L \cdot L \cdot \log_{10} e = \left( \frac{3\pi Q_{\text{abs}}(\lambda)}{2D} \right) \cdot \log_{10} e \cdot f_\lambda \cdot L = A(\lambda) \]  

Eq.10 represents the analytic derivation process of the famous Beer-Lambert Law [30]. With these assumptions, a new method is proposed to evaluate the total absorption efficiency (ABE) for a given nanoparticle concentration and optical depth, which is the theoretical maximum possible photothermal conversion efficiency:

\[ \eta(L, f_\lambda) = \frac{\int_{0.2\mu m}^{3\mu m} E_0(\lambda) \left( 1 - 10^{-\alpha(\lambda, L) L} \right) d\lambda}{\int_{0.2\mu m}^{3\mu m} E_0(\lambda) d\lambda} = \frac{\int_{0.2\mu m}^{3\mu m} E_0(\lambda) (1 - e^{-\alpha(\lambda, L) L}) d\lambda}{\int_{0.2\mu m}^{3\mu m} E_0(\lambda) d\lambda} \]  

Further investigation will be discussed in next section with experimental results.

### 4.4 Solution methodology

A high-order algorithm has been used to solve Eqs. (3) ~ (5) and (11) to calculate coefficients related to
optical properties for nanofluids and photothermal conversion efficiency, with functions powered by Matlab

associated with COMSOL Multiphysics, similar to the one described by Kluczyk[36]. It should be noticed that
obtaining an analytical solution to Eqs. (6) ~ (8) is extremely difficult as the 3-dimentional transient heat
transfer equations coupled with transient RTE equations are complicated partial differential equations. A finite
element method (FEM) was employed to solve the equations numerically in COMSOL. A predefined Heat
Transfer with Radiation in Participating Media equations [30] together with user defined functions (radiative
transfer equation) were used to describe thermal and radiative energy transfer process. The discretization of the
simulative space was conducted with the appliance of a built-in non-structured meshing COMSOL algorithm.
The maximum element size inside nanofluid was chosen as 0.2 cm, and the maximum was chosen as 0.05 cm
for the surface with respect to radiative and convective heat loss, which mainly happens at the surface. A direct
solver called MUltifrontal Massively Parallel sparse direct Solver (MUMPS) with tolerance of $10^{-5}$ was
adopted to numerically solve the matrices assembled according to the governing equations and boundary
conditions described above. The initial and boundary conditions were originated from experimental
measurement (such as temperature of inside air and the ambient). With Mie scattering, RTE, transient heat
transfer equations coupled with varying boundary conditions, it is very demanding on the computational power.
The Advanced Research Computing (ARC) at University of Leeds is used to solve the equations in in parallel.

5. Numerical results and comparison

5.1 Validation against experimental data

The absorption coefficients for gold nanoparticles and working fluid (water) calculated by Mie scattering
theory (Eqs.3-5) can be seen in Fig. 6a, where the volume concentration of gold particles is 5.8 ppm. The
absorption coefficient which cannot be obtained through experiments is a key parameter for 3D heat transfer
and radiative transfer equations. As shown in Fig. 6a, gold nanoparticles contribute the absorption in the
visible light spectrum and the base fluid (water) is more effective in the infrared range. To examine the
reliability of our calculation, the absorbance from numerical results based on Eq.10 is compared with experimental value in Fig. 6b. The simulation result is generally in agreement with the experimental value. The experimental deviation in 600–800 nm is due to the existence of some bigger particles in the fluids, which could make the absorbance red shift.

Based on experimental boundary and initial conditions, as an example, the comparison of the simulation with experimental results under constant solar intensity (i.e., 950 W/m²) is shown in Fig. 7. The temperature rise of 1.45 ppm gold nanofluid is much higher than that of DI water, for example ~25 K for nanofluid and ~12 K for water. However, both nanofluid and water exhibit non-uniform temperature distribution during the illumination, as much as 4.4 K and 1.9 K temperature difference can be reached, respectively. The high temperature of the top layer (TC1) shows that solar energy is mostly absorbed in the surface layer. The low temperature at the bottom layer is related to effects of solar intensity decay along the optical path, and the limited heat conduction capacity. After about three hours’ heating, nanofluid can maintain approximately an equilibrium temperature (~ 57 °C), but for water the maximum is only ~ 45 °C, indicating that gold nanofluid even with very low concentration can significantly enhance the energy conversion from solar radiation to thermal form.

5.2 Temperature distribution profile inside the tube

Non-uniform temperature distribution can be further demonstrated through T-profile in 3-dimensional fluid (gold nanoparticles with volume concentration of 1.45 ppm, water as based fluid, under constant solar intensity, i.e., 950 W/m²), which can be seen in Fig. 8a. The result in this case shows that the temperature at the up middle of the tube along Y direction is higher than the bulk value. Generally, the highest temperature is located inside the nanofluid volume (about 0.8 cm to the illuminated surface), for example 44 °C in this case. Clearly larger temperature difference (i.e., more than 6 K) for the whole considered volume can be seen in Fig. 8b, comparing with the 3 thermocouple measurement in Fig. 7, which illustrates the danger of using individual
measured values to calculate PTE.

5.3 Efficiency prediction and comparison

Most of the prior studies such as Andrej et al. [3] were based on experimental or simulative temperature field to optimize the efficiency of nanofluid-based DASC. Here we report a mathematical method to predict the absorption efficiency (ABE), which represents the maximum possible efficiency for any nanofluid-based solar system, according to optical properties, seen in Eq. (11).

Fig. 9a shows the absorption efficient in different wavelength. Clearly comparing to the water case, much higher ABE is observed for gold nanofluids across the whole solar spectrum. The spectrum ABE increases with the particle concentration, and nearly 100% ABE is achieved for 5.8 ppm nanofluids in the visible light spectrum, which is related to the surface plasmon resonance phenomenon of gold nanoparticles. The small depression at ~900 nm is associated with the poor absorbency performance in the near-infrared, which can be explained by the Mie scattering theory.

Fig. 9b and c show the ABE as a function of optical length $L$ and particle concentration $v_f$. The initial efficiency of the nanofluids receiver increases rapidly with $L$, then asymptotically reaches 100%. This result is similar to Zhang et al.[14]’s experimental observation, where it showed clearly that the PTE increased nonlinearly with volume concentration but no proper explanation was provided. As shown in radiative transfer equations (Eq. 9), the radiative transport energy is consumed inside nanofluid exponentially, and the PTE should exhibit a nonlinear dependence on the particle concentration.

Fig. 9 also reveals the important parameters that determine the maximum possible solar receiver efficiency. The impact of the optical depth $L$ and particle concentration $f_v$ is embedded in the exponential term as $L \cdot f_v$ in Eq. 9. Fig. 9b reveals that both optical concentration and optical depth should be in a relative small values to achieve an optimized effect, as higher volume concentration of nanoparticle (i.e., more than 10 ppm for $L=0.05$ m) increases the efficiency slightly. Comparing to nanofluids, the efficiency depends more on the
optical depth to reach a relative high value (i.e., 25 m for 80%). Overall, employing GNP\'s enhances the ABE significantly compared with pure water.

In order to compare the prediction of ABE against experimental and simulative data (i.e., where the optical depth is not a constant), a modified equation is proposed by integrating the efficiency with the radius, as:

$$\eta\left(f_v\right) = \int_0^R \int_{0.2\mu m}^{3\mu m} \frac{E_0(\lambda)\left(1-e^{-2\xi(\lambda,f_v)\rho}\right)d\lambda}{R\int_{0.2\mu m}^{3\mu m} E_0(\lambda)d\lambda}$$

(11)

where $R$ is the radius of cylindrical tube. The ABE becomes an only function of volume concentration. The ABE is compared with three ways of photothermal conversion efficiencies based on temperature field, i) from one measured thermocouple, ii) from the average temperature measured by three thermocouples, and iii) from the simulated temperature field. For the simulated data, the efficiency is obtained by considering temperature difference in each computational nodes, as:

$$\eta = \sum_{i=1}^{n} c_a \rho_v V_i \Delta T_i$$

(12)

Fig. 10a shows reasonably good agreement from four different methods. It can be expected that ABE from Eq. 12 is the theoretical maximum efficiency, in which the scattering and heat leakage are not considered. It can be used as an efficient method for nanoparticle selection and solar collector optimization without the need of measuring the temperature field. Photothermal efficiency based only on one temperature point shows a large underestimation of the PTE for all samples. For example, the PTE is 12% lower than that from the experiment-determined average temperature at $f_v=5.8$ ppm.

What\'s more, the temperature used to determine PTE is usually at the beginning when the heat leak is not significant, the temperature difference (i.e., 4.4 °C for 1.45 ppm under 950 W/m² solar intensity) inside nanofluid is comparable to this temperature range (10-15 °C). Neglecting of non-uniform temperature distribution could cause significant inaccuracy (with an uncertainty of 29%-44%) in calculating the energy
efficiency. To illustrate such an effect, Fig. 10b shows the maximum differences in temperature and PTE based on the simulation result under a solar intensity of 1000 W/m$^2$ and particle size of 20 nm. Here the temperature elevation employed to determine PTE is 10 K from the beginning of the experiment. Depending on the measurement location, as high as 67% uncertainty could be obtained for the case of 5.8 ppm gold nanofluid.

The temperature non-uniformity issue will become more and more serious when the volume concentration of nanofluids increases or the radiation intensity increases (i.e., under focused solar intensities). Quite a few recent studies [13,16,17,37] have shown that under a focused solar light, i.e., via a typical Fresnel lens, rapid steam can be produced from plasmonic nanofluids albeit the bulk solution was still under subcooled conditions. In one study [16], by using very dilute gold nanoparticles (16.7 ppm) under a solar concentration of 1000 times, , steam generation efficiency was calculated as high as 80%, and only 20% of the absorbed solar energy was used to increase the bulk fluid temperature. However, there is still a strong debate if the steam can be produced around heated nanoparticles. Considering the potential large temperature difference in the fluids under a focused solar intensity, there is a possibility that steam could be produced on the strongly heated surface layer, where strong evaporation or even boiling can occur. Further exploration of this issue is ongoing, and will be presented in the future.

6 Conclusions

Both outdoor experiments and simulation were conducted in this work to analyses the photothermal conversion characteristics of gold nanofluids, and a new method was proposed to predict the theoretical efficiency based only the optical properties. The main conclusions can be summarized as:

1) The photothermal conversion efficiency of gold nanofluids is much higher than that of pure water, and increased non-linearly with particle concentration, reaching 76% at a concentration of 5.8 ppm.

2) Significant non-uniform temperature distribution was identified inside the fluid, indicating that the consideration of average temperature is needed to obtain a reliable PTE.
(3) A new method was developed to predict radiative absorption efficiency based on the optical properties of nanofluid, without the need of knowing the temperature field inside the fluid. The method can be used to identify the performance of nanoparticles and optimize solar absorbers efficiently.

(4) A radiative heat transfer model coupled with the Mie scattering theory was developed. This model can predict temperature profile successfully, which confirms the existence of large temperature difference inside nanofluids.

(5) The comparison of various efficiencies shows that Eq. (11) can be used to predict the theoretical maximum photothermal conversion efficiency, and using only one-point temperature measurement could cause significant inaccuracy, i.e., uncertainty of 67% for 12.75 ppm gold nanofluid at 1000 W/m² solar intensity.

ACKNOWLEDGEMENT

This work was supported by the EU Marie Curie Actions-International Incoming Fellowships (FP7-PEOPLE-2013-IIF-626576), and the author Haichuan Jin also acknowledges the financial support for his visiting study at the University of Leeds from the China Scholarship Council (CSC) under the Grant No.201506020031.
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Table captions:

Table 1 Different cases of gold nanoparticles dispersions.
Figure captions:

Fig. 1. Nanoparticle characterization: (a) TEM image of gold nanoparticles (CASE 6); (b) A: absorbance of gold nanoparticle dispersions under different volume fraction; B: peak absorbance variation with concentrations in volume.

Fig. 2. Experimental setup: (a) Soar thermal illumination experiment under nature sunlight conditions (located on the roof of Human Machine and Environment Engineering Building in Beihang University in Beijing, 39° 59' 5.49" North, 116° 21' 18.70" East); (c) A schematic illustration of experimental setup.

Fig. 3. (a) Temperature profile of differently diluted nanofluids under varied solar intensity (cloudy, solar intensity from about 400 W/m$^2$ to 700 W/m$^2$); (b) Experimentally obtained increasing temperature profile of TC1-TC3 located inside the tube, concentration of nanofluids is 5.8 ppm and solar intensity is varied. (c) Comparison of three thermocouple signals for 0.72 ppm gold nanofluid under different solar intensities: constant solar intensity (i.e., about 950 W/m$^2$) and varied solar intensity (i.e., from 400 W/m$^2$ to 700 W/m$^2$).

Fig. 4. Variation of photothermal conversion efficiency ($\eta$) and specific absorption rate (SAR) with volume concentration for gold nanofluids under different solar intensities.

Fig. 5. (a) ASTM G173-03 Reference Spectra from literature, inset shows the solar energy distribution along with wavelength in percentage (integrating spectral emissive power with wavelength divided by irradiation intensity). (b) Calculation of spectral emissive power for sun (T=5762 K) and nanofluid (T=303 K), where spectral distribution is separated into two bands, A ($\lambda<3000$ nm) and B ($\lambda>3000$ nm).

Fig. 6. (a) Real and imaginary parts of $M$ (Eq. (2)) and calculated absorption coefficients for gold nanoparticles ($K_p$), working fluid ($K_f$) and total ($K$) according to Mie scattering theory; (b) Absorbance from Eq. (10) in comparison with results from spectrophotometer.

Fig. 7. Experimentally obtained increasing temperature profile of TC1-TC3 located inside the tube, compared with numerical results from 3D model concentration of nanofluids is 1.45 ppm and solar intensity is constantly about 950 W/m$^2$.

Fig. 8. (a) Schematic for numerical simulation of a 3-D volumetric solar receiver based on gold nanofluid of concentration 1.45
ppm with normal nature solar radiation I=900 W/m², where the geometry parameters can be seen in (b), solar radiation is in -y
direction; (b) Temperature profiles in cross-section for nanofluid after 30 minutes’ illumination (Global Tilt) at x=0 cm.

Fig. 9. (a) Spectral efficiency which is defined as below in this paper:

\[ \eta_\lambda(\lambda) = \frac{\int \lambda \cdot E\lambda(\lambda)(1-e^{-\alpha(\lambda)L})d\lambda}{\int \lambda \cdot E\lambda(\lambda)d\lambda}; \]

(b) Absorption efficiency (ABE) as a function of optical length L for 5
different volume concentrations (inset: DI water); (c) ABE as a function of volume concentration f, for 5 different optical lengths.

Fig. 10. (a) Comparison of the experimentally obtained photothermal conversion efficiency, efficiency calculated from 3D model
and predicted absorption efficiency, result based on only one thermocouple; (b) Maximum efficiency uncertainty caused by
non-uniform temperature distribution in nanofluids.
Table 1 Different cases of gold nanoparticles dispersions

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Reagents</th>
<th>Reaction time</th>
<th>Color</th>
<th>Peak size</th>
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<tbody>
<tr>
<td>1</td>
<td>HAuCl₄: 100ml 2.4×10⁻⁶ mol/L, Sodium Citrate: 10ml 0.034mol/L</td>
<td>30 min</td>
<td>Wine red</td>
<td>9nm</td>
</tr>
<tr>
<td>2</td>
<td>HAuCl₄: 100ml 2.4×10⁻⁶ mol/L, Sodium Citrate: 5ml 0.034mol/L</td>
<td>30 min</td>
<td>Orange</td>
<td>15nm</td>
</tr>
<tr>
<td>3</td>
<td>HAuCl₄: 100ml 2.4×10⁻⁶ mol/L, Sodium Citrate: 2ml 0.034mol/L</td>
<td>40 min</td>
<td>Pink</td>
<td>25nm</td>
</tr>
<tr>
<td>4</td>
<td>HAuCl₄: 100ml 2.4×10⁻⁶ mol/L, Sodium Citrate: 0.7ml 0.034mol/L</td>
<td>60 min</td>
<td>Crystal violet</td>
<td>65nm</td>
</tr>
<tr>
<td>5</td>
<td>HAuCl₄: 100ml 2.4×10⁻⁶ mol/L, Sodium Citrate: 0.3ml 0.034mol/L</td>
<td>90 min</td>
<td>Ash black</td>
<td>120nm</td>
</tr>
<tr>
<td>6</td>
<td>HAuCl₄: 50ml 5×10⁻⁶ mol/L, Sodium Citrate: 50ml 1×10⁻⁵ mol/L</td>
<td>20 min</td>
<td>Amaranth</td>
<td>19nm</td>
</tr>
</tbody>
</table>
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(a) Diagram showing the relationship between temperature and time for different conditions.

(b) Diagram showing the temperature and time for TC1, TC2, TC3, and GNPs 5.8ppm conditions.
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(a) ASTM G173-03 Reference Spectra Derived from SMARTS (nm)

(b)

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(a)

Solar radiation
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$$\eta_{\lambda} = \frac{\int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} E_{\lambda}(\lambda) (1 - e^{-\alpha_{\lambda} L}) d\lambda}{\int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} E_{\lambda}(\lambda) d\lambda};$$

(b) Absorption efficiency (ABE) as a function of optical length $L$ for 5 different volume concentrations (inset: DI water); (c) ABE as a function of volume concentration $f_v$ for 5 different optical lengths.
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