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Modeling and application of ring stiffness condition for radial-axial ring rolling

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Abstract

Radial-axial ring rolling (RARR) is an advanced rotary forming technology for manufacturing various seamless rings, especially for large scale rings. A primary problem for RARR is to facilitate rolling process stability and form a ring with good dimension and performance. However, RARR is an extremely complex dynamic rolling process with high flexibility. To reasonably control guide roll is an important approach to keep rolling process stable during RARR. In this paper, a mathematical model of ring stiffness condition for RARR was established based on the force method. Then the influence factors to ring stiffness were discussed, especially the section bending moment factor. To verify the ring stiffness model, finite element (FE) simulation was adopted. In addition, a comparison of different ring stiffness models was made. It can be found the proposed stiffness model has a high accuracy. Furthermore, a control method of the pressure in the hydraulic cylinder to adjust the guiding force based on the stiffness model was proposed. By FE simulation of RARR, an appropriate adjustment coefficient to determine the guiding force was obtained. Finally, an experiment of RARR for a large ring was carried out. The rolling process was very smooth and steady, and a super-large ring with diameter more than 9 meters was manufactured successfully.

Keywords: Radial-axial ring rolling; Guide roll; Ring stiffness condition; Mathematical modeling; Guiding force.
1 Introduction

Ring rolling is an advanced rotary forming technology to manufacture seamless rings, such as bearing races, ring gears, flanges, aero-engine casings, etc. Compared with traditional manufacturing technologies, it has advantages of high producing efficiency and processing precision, low energy consumption and material cost, good microstructure and performance. Over 160 years, ring rolling has evolved into a relatively mature technology [1, 2], with amount of research work in theory [3-6], technique [7-10] and equipment [11, 12].

As a typical ring rolling method, RARR is suited to manufacture large scale rings at high temperature. During RARR, as shown in Fig. 1, the thickness of a ring is reduced by the radial extrusion of the main roll and mandrel, and the height is reduced by the axial extrusion of the upper and lower conical rolls. In radial deformation area, the main roll makes active rotary motion and the mandrel makes radial feed movement. In axial deformation area, the upper conical roll makes downward feed movement and the two conical rolls both make active rotary motion. When the thickness and height of the ring reduce, the diameter of the ring enlarges. The two conical roll also move backwards to keep them contact with the ring surface during rolling process. The two guide rolls are arranged respectively in the two sides of the main roll to steer the ring and keep the rolling process stable.

Fig. 1. Forming principle of RARR
From the forming principle of RARR above, it can be seen that RARR is an extremely complex dynamic rolling process with high flexibility. Recently, researches on RARR are mainly focused on FE modeling method and process simulation. For instance, Xie et.al [13] developed a rigid–viscoplastic dynamic explicit finite element code to simulate ring rolling. Davey and Ward [14] proposed a practical method for FE simulation of RARR using ALE flow formulation instead of conventional Lagrangian approach. Kim et al. [15] put forward a dua-mesh approach to RARR simulation with high computational efficiency. Simulations of microstructure evolution during RARR have been also tried [16-18]. By FE simulation, deformation behaviors during RARR and effects of technological parameters on RARR process have been revealed systematically [19-23]. Besides, macro–microscopic deformation laws during blank forging and ring rolling process have been investigated [24].

To facilitate rolling process stable and form a ring with good dimension and performance, some rolling conditions have been proposed. To make the ring rotate smoothly, biting condition in radial and axial rolls should be satisfied [25]. To make the ring produce plastic deformation, the penetrating condition in radial and axial deformation areas should be met [25]. Furthermore, to make rolling process stable, a steady forming condition for RARR should be guaranteed [26]. In the process of real manufacturing, waste ring product with distorted shape is an important problem, as shown in Fig. 2. One key reason is that the force applied on the guide roll is inappropriate.

Fig. 2. Waste ring products with distorted shape
At present, some scholars have made studies on the guide roll of ring rolling. Hua et al. [27] adopted a theoretical method to predict the motion rule of guide roll by calculating the ring size during cold radial ring rolling process of a groove ball ring. Li et al. [28] proposed a method to control the guide rolls by adjusting the pressure of hydraulic mechanism in 3D-FE radial ring rolling simulation. Xu et al. [29] put forward a plastic instability criterion for radial ring rolling and calculated the bending moment of ring under the action of guide rolls by the force method. Forouzan et al. [30] put forward employing thermal spokes method to simulate the guide roll effect in FE analysis of radial ring rolling process. However, compared with radial ring rolling, the rolling system of RARR is more complex. A pair of conical rolls is involved in the deformation and formed rings are in big size. Therefore, it is more important for RARR to control the guide rolls. Xu et al. [31] adopted the limit analysis method to establish the relationship between the allowed maximum force and the angle of the guide roll, and pointed out the most suitable angle of the guide roll. Hua et al. [32] proposed a ring stiffness condition for RARR, but it made a simplification to calculate the bending moment and emphasized on the permitting maximum outer radius of a ring. However, the research emphases above were not on how to control the force of guide roll based on the guide roll system of RARR.

This study aims to provide a scientific theoretical basis and reliable control method of guide roll during RARR. Firstly, a mathematical model of ring stiffness condition for RARR was established based on the force method. Then the influence factors to ring stiffness were discussed and the predicted results were compared with FE simulation and other ring stiffness models. Finally, a control method of the pressure in the hydraulic cylinder to adjust the guiding force based on the stiffness model was proposed and applied in a RARR experiment of a super-large ring successfully.
2 Mathematical modeling of ring stiffness condition for RARR

2.1 Establishment of the ring stiffness model

During RARR, the ring is deformed under the pressure of the main roll and mandrel in the radial direction and the pressure of the two conical rolls in the axial direction. Assuming the rotational motions are ignored, the mechanical model for RARR can be regarded as a circular beam with rectangular section fixed in the radial deformation area A and axial deformation area B, as shown in Fig. 3(a). The pressure of the guide roll applied on the ring is simplified as a concentrated force \( F_g \) pointing to the center of the circle. To simplify the model, the forces of the two guide rolls are supposed to be equal and the cross-sections of the rolled ring are supposed to be unified. Thus, it is a symmetric model, and a half model can be used to analyze the mechanical condition, as shown in Fig. 3(b). It is a three times statically indeterminate structure. Generally, the force method and displacement method are two basic and effective means to calculate statically indeterminate structures. In this paper, the force method is selected to analyze the mechanical condition. Based on the force method, the fixed joint at section B is removed and equivalent support reaction is added, as shown in Fig. 4. \( X_1, X_2 \) are the support forces in horizontal and vertical directions respectively, and \( M_3 \) is the support moment.

![Fig. 3. Simplified sketch of mechanical model for RARR: (a) a complete model; (b) a half model.](image-url)
Fig. 4. Equivalent mechanical model based on the force method

Firstly, the section bending moment of the circular beam caused by each force should be obtained. The section bending moment of the circular beam \( M_g(\varphi) \) caused by the guiding force \( F_g \) can be defined as

\[
M_g(\varphi) = \begin{cases} 
0 & (0 \leq \varphi < \alpha) \\
F_g R_a \sin(\alpha - \varphi) & (\alpha < \varphi \leq \pi)
\end{cases}
\]  

where \( R_a \) is the average diameter of the rolled ring, the section \( P \) is any section on the beam and \( \varphi \) is the central angle of the arc \( A P \), \( \alpha \) is the position angle of guide roll.

Then the section bending moment of the circular beam \( M_1(\varphi) \) caused by the unit load \( \bar{X}_1 \) in the same direction as \( X_1 \) can be expressed as

\[
\bar{M}_1(\varphi) = -R_2 (1 + \cos \varphi) \quad (0 \leq \alpha \leq \pi)
\]  

The section bending moment of the circular beam \( M_2(\varphi) \) caused by the unit load \( \bar{X}_2 \) in the same direction as \( X_2 \) can be expressed as

\[
\bar{M}_2(\varphi) = -R_2 \sin \varphi \quad (0 \leq \alpha \leq \pi)
\]  

The section bending moment of the circular beam \( M_3(\varphi) \) caused by the unit load \( \bar{X}_3 \) in the same direction as \( X_3 \) can be expressed as

\[
\bar{M}_3(\varphi) = -1 \quad (0 \leq \alpha \leq \pi)
\]  

Then the generalized displacement caused by each force at section B should be calculated. The generalized displacement \( \Delta_{1g} \) caused by \( F_g \) at the section B in the same direction as \( X_1 \) can be calculated by
\[
\Delta_{1g} = \int_0^\pi \frac{M_x(\varphi)M_z(\varphi)}{EI} R_x d\varphi = -\frac{FR^3}{EI} \left[ \frac{1}{4} \cos \alpha (\cos 2\alpha - 1) + \frac{1}{4} \sin \alpha \sin 2\alpha + \frac{1}{2} \alpha \sin \alpha + 1 - \cos \alpha \right]
\]

where \( E \) is the elastic modulus, \( I \) is the inertia moment.

The generalized displacement \( \Delta_{2g} \) caused by \( F_g \) at the section B in the same direction as \( X_2 \) can be calculated by

\[
\Delta_{2g} = \int_0^\pi \frac{M_x(\varphi)M_z(\varphi)}{EI} R_x d\varphi = -\frac{FR^3}{EI} \left[ \frac{1}{4} \sin \alpha (\cos 2\alpha - 1) + \frac{1}{4} \cos \alpha \sin 2\alpha - \frac{1}{2} \alpha \cos \alpha \right]
\]

The generalized displacement \( \Delta_{3g} \) caused by \( F_g \) at the section B in the same direction as \( X_3 \) can be calculated by

\[
\Delta_{3g} = \int_0^\pi \frac{M_x(\varphi)M_z(\varphi)}{EI} R_x d\varphi = \frac{FR^3}{EI}(\cos \alpha - 1)
\]

The generalized displacement \( \delta_{11} \) caused by \( \overline{X_1} \) at the section B in the same direction as \( X_1 \) can be calculated by

\[
\delta_{11} = \int_0^\pi \frac{M_z(\varphi)M_z(\varphi)}{EI} R_x d\varphi = \frac{2\pi R^3}{3EI}
\]

The generalized displacement \( \delta_{22} \) caused by \( \overline{X_2} \) at the section B in the same direction as \( X_2 \) can be calculated by

\[
\delta_{22} = \int_0^\pi \frac{M_z(\varphi)M_z(\varphi)}{EI} R_x d\varphi = \frac{\pi R^3}{2EI}
\]

The generalized displacement \( \delta_{33} \) caused by \( \overline{X_3} \) at the section B in the same direction as \( X_3 \) can be calculated by

\[
\delta_{33} = \int_0^\pi \frac{M_z(\varphi)M_z(\varphi)}{EI} R_x d\varphi = \frac{\pi R^3}{2EI}
\]

The generalized displacement \( \delta_{12} \) caused by \( \overline{X_2} \) at the section B in the same direction as \( X_1 \) is equal to the generalized displacement \( \delta_{21} \) caused by \( \overline{X_1} \) at the section B in the same direction as \( X_2 \), and can be calculated by

\[
\delta_{12} = \delta_{21} = \int_0^\pi \frac{M_z(\varphi)M_z(\varphi)}{EI} R_x d\varphi = \frac{2R^3}{EI}
\]
The generalized displacement $\delta_{13}$ caused by $X_3$ at the section B in the same direction as $X_1$ is equal to the generalized displacement $\delta_{31}$ caused by $X_1$ at the section B in the same direction as $X_3$, and can be calculated by

$$\delta_{13} = \delta_{31} = \int_0^s \frac{M_2(\phi)M_1(\phi)}{EI} R_s d\phi = \pi R_s^2$$ (12)

The generalized displacement $\delta_{23}$ caused by $X_3$ at the section B in the same direction as $X_2$ is equal to the generalized displacement $\delta_{32}$ caused by $X_2$ at the section B in the same direction as $X_3$, and can be calculated by

$$\delta_{23} = \delta_{32} = \int_0^s \frac{M_3(\phi)M_1(\phi)}{EI} R_s d\phi = \frac{2}{\pi} R_s^2$$ (13)

Based on the force method, the regular equation of this three times statically indeterminate structure can be expressed as

$$\begin{align*}
\delta_{11} X_1 + \delta_{12} X_2 + \delta_{13} X_3 + \Delta_{1g} &= 0 \\
\delta_{21} X_1 + \delta_{22} X_2 + \delta_{23} X_3 + \Delta_{2g} &= 0 \\
\delta_{31} X_1 + \delta_{32} X_2 + \delta_{33} X_3 + \Delta_{3g} &= 0
\end{align*}$$ (14)

Then the equivalent support reaction at section B can be obtained

$$X_1 = \frac{1}{\pi} F_x \alpha \sin \alpha$$ (15)

$$X_2 = -\frac{1}{2R_y} \left[ F_x R_x \alpha \sin \alpha + \pi F_x R_y \left( \frac{\pi}{8-\pi^2} \cos \alpha - \frac{\pi}{8-\pi^2} - \frac{2}{8-\pi^2} \alpha \cos \alpha \right) \\
+ \frac{2}{8-\pi^2} \sin \alpha - \frac{1}{\pi} \alpha \sin \alpha \right] + F_x R_y (\cos \alpha - 1)$$ (16)

$$X_3 = F_x R_y \left( \frac{\pi}{8-\pi^2} \cos \alpha - \frac{\pi}{8-\pi^2} - \frac{2}{8-\pi^2} \alpha \cos \alpha + \frac{2}{8-\pi^2} \sin \alpha - \frac{1}{\pi} \alpha \sin \alpha \right)$$ (17)

Finally, the section bending moment of the circular beam at section P can be calculated by

$$M_{1(\phi)} = F_x R_y Q_{1(\phi, \alpha)}$$ (18)

where $Q_{1(\phi, \alpha)}$ is the section bending moment factor and it can be expressed as
Then the section bending stress $\sigma$ can be calculated by the following formula

$$\sigma = \frac{M_{\phi}^s}{W_z} = \frac{6RF_{g}Q_{(\alpha, \phi)}}{BH^2}$$

To satisfy the stiffness condition, the maximum section bending stress should satisfy the following expression

$$\sigma_{\text{max}} = \left. \frac{M_{\phi}^s} {W_z} \right|_{\text{max}} = \frac{6RF_{g}Q_{(\alpha, \phi)}^s} {BH^2} \leq \left[ \sigma \right] = \sigma_s$$

Thus, to avoid distorting the ring shape, the guiding force should satisfy the stiffness condition

$$F_{g} \leq F_{g-\text{max}} = \frac{BH^2\sigma_s}{6RF_{g}Q_{(\alpha, \phi)}^s}$$

### 2.2 Discussion of the ring stiffness model

According to Eq. (22), the ring stiffness condition in RARR is related to its geometric dimension and yield strength, and also has relation with the maximal absolute value of the section bending moment factor $\left| Q_{(\alpha, \phi)} \right|_{\text{max}}$. Bigger geometric dimension of ring cross section, smaller geometric dimension of ring radius and bigger yield strength of ring material can make the ring to bear bigger force before being distorted, which are good for improving the ring stiffness. As we can see from Eq. (19), the expression of the section bending moment factor $Q_{(\alpha, \phi)}$ is related to the position angle of guide roll $\alpha$ and the angle of ring section position $\varphi$, but the detail is not clear. Therefore, a further study on the section bending moment factor $Q_{(\alpha, \phi)}$ is made in this section. The Fig. 5 shows a simple relation between $Q_{(\alpha, \phi)}$, $\alpha$ and $\varphi$. It can be seen that the distribution of $Q_{(\alpha, \phi)}$...
along the ring section changes from state I to state II when $\alpha$ increases. That is to say, when $\alpha$ is smaller than a certain valve, the distribution state of $Q_{(\varphi, \alpha)}$ likes an inverted letter N. The maximal absolute valve of the section bending moment factor $|Q_{(\alpha, \varphi)}|$ may only occur in section A, section B, section C or section D. Section A is between the main roll and mandrel in the ring. The absolute valve of the section bending moment factor at section A can be expressed as

$$|Q_{(\alpha, 0)}| = \sin \alpha - \frac{\alpha \sin \alpha}{\pi} - \frac{\pi \cos \alpha - \pi - 2 \alpha \cos \alpha + 2 \sin \alpha}{8 - \pi^2}$$  \hspace{1cm} (23)$$

Section B is between the two conical rolls in the ring. The absolute valve of the section bending moment factor at section B can be calculated by

$$|Q_{(\alpha, \varphi)}| = \left| \frac{\alpha \sin \varphi}{\pi} - \frac{\pi \cos \alpha - \pi - 2 \alpha \cos \alpha + 2 \sin \alpha}{8 - \pi^2} \right|$$  \hspace{1cm} (24)$$

Section C is contacted with the guide roll in the ring. The absolute valve of the section bending moment factor at section C can be determined by

$$|Q_{(\alpha, \varphi)}| = \left| \frac{\alpha \sin \cos \alpha}{\pi} - \frac{(1 - \cos \alpha) \sin \alpha}{2} + \frac{\pi (\pi \cos \alpha - \pi - 2 \alpha \cos \alpha + 2 \sin \alpha) \cos \alpha}{2(8 - \pi^2) \sin \alpha} \right|$$  \hspace{1cm} (25)$$

Section D is between section C and B. A peak valve of $Q_{(\varphi, \alpha)}$ occurs at this section. When $\alpha$ is bigger than a certain valve, the distribution state of $Q_{(\varphi, \alpha)}$ likes an inverted letter W. In this case, another peak valve of $Q_{(\varphi, \alpha)}$ occurs at section E which is between section A and C. The derivatives of $Q_{(\varphi, \alpha)}$ with respect to $\varphi$ at section D and E are both equal to 0.

Then the derivative of $Q_{(\varphi, \alpha)}$ with respect to $\varphi$ can be calculated by the following expression

$$\frac{\partial Q_{(\varphi, \alpha)}}{\partial \varphi} = \begin{cases} -\cos(\alpha - \varphi) + \frac{\alpha \sin \alpha \sin \varphi}{\pi} - \frac{(1 - \cos \alpha) \cos \varphi}{2} - \frac{\pi \cos \alpha - \pi - 2 \alpha \cos \alpha + 2 \sin \alpha \cos \varphi}{2(8 - \pi^2)} & (0 \leq \varphi < \alpha) \\ \frac{\alpha \sin \varphi}{\pi} - \frac{(1 - \cos \alpha) \cos \varphi}{2} + \frac{\pi (\pi \cos \alpha - \pi - 2 \alpha \cos \alpha + 2 \sin \alpha) \cos \varphi}{2(8 - \pi^2)} & (\alpha < \varphi \leq \pi) \end{cases}$$  \hspace{1cm} (26)$$

When $\frac{\partial Q_{(\varphi, \alpha)}}{\partial \varphi} = 0$, the position angle $\varphi$ of section D and E for different $\alpha$ can be calculated, as
shown in Fig. 6. The certain value of $\alpha$ is about 60.3°. Then the value of $|Q_{(\varphi, \alpha)}|$ at section A, B, C, D and E for different $\alpha$ can be calculated, as shown in Fig. 7. The maximal value of $|Q_{(\varphi, \alpha)}|$ for different $\alpha$ can be only at section A, B or C. When $0^\circ \leq \alpha \leq 46.3^\circ$, the maximal value of $|Q_{(\varphi, \alpha)}|$ is at section A. When $46.3^\circ < \alpha \leq 62.5^\circ$ or $79.5^\circ < \alpha \leq 90^\circ$, the maximal value of $|Q_{(\varphi, \alpha)}|$ is at section C. When $62.5^\circ < \alpha \leq 79.5^\circ$, the maximal value of $|Q_{(\varphi, \alpha)}|$ is at section B. The derivative of $|Q_{(\varphi, \alpha)}|$ with respect to $\varphi$ at section A, B, C, D and E for different $\alpha$ can be calculated.

![Fig. 5. Relationship between $\varphi$, $\alpha$ and $Q_{(\varphi, \alpha)}$.](image)

![Fig. 6. Position of section E and D under different $\alpha$.](image)

The derivative of $|Q_{(\varphi, \alpha)}|$ with respect to $\alpha$ at section A, B and C can be calculated, as shown in Fig. 8. As it can be seen, the maximal value of $|Q_{(\varphi, \alpha)}|$ at section A occurs when $\alpha=30^\circ$. For section B, the maximal value occurs when $\alpha=63.6^\circ$. And for section C, the maximal value occurs when $\alpha=51.5^\circ$. Finally, the maximal absolute value of the section bending moment factor
\[ |Q_{(\alpha,\phi)}|_{\text{max}} \] for different \( \alpha \) can be calculated, as shown in Fig. 9. It can be seen the value of 
\[ |Q_{(\alpha,\phi)}|_{\text{max}} \] depends on the position angle of guide roll \( \alpha \). The maximal value of 
\[ |Q_{(\alpha,\phi)}|_{\text{max}} \] occurs when \( \alpha=30^\circ \). When \( 0^\circ \leq \alpha \leq 30^\circ \), \( 46.3^\circ < \alpha \leq 51.5^\circ \) or \( 62.5^\circ < \alpha \leq 63.6^\circ \), the value of 
\[ |Q_{(\alpha,\phi)}|_{\text{max}} \] increases with position angle of guide roll. And when \( 30^\circ \leq \alpha \leq 46.3^\circ \), 
\( 51.5^\circ < \alpha \leq 62.5^\circ \) or \( 63.6^\circ < \alpha \leq 90^\circ \), the value of 
\[ |Q_{(\alpha,\phi)}|_{\text{max}} \] decreases with position angle of guide roll. Therefore, the influence of 
\[ |Q_{(\alpha,\phi)}|_{\text{max}} \] to the ring stiffness can be come down to the position angle of guide roll \( \alpha \). As we can see, the position angle of guide roll \( \alpha \) has a complex influence to the ring stiffness. When the position angle of guide roll \( \alpha \) is smaller than \( 30^\circ \), the ring stiffness is weakened as \( \alpha \) increasing. And when \( \alpha \) is bigger than \( 30^\circ \), the ring stiffness can be improved generally as \( \alpha \) increasing. However, the ring stiffness is weakened when 
\( 46.3^\circ < \alpha \leq 51.5^\circ \) or \( 62.5^\circ < \alpha \leq 63.6^\circ \).

Fig. 7. Relationship between \( \alpha \) and \( |Q_{(\phi,\alpha)}| \) for different sections
2.3 Verification by FE Simulation

To verify the result above, a static FE simulation for a half circular beam is made in ABAQUS. The all degrees of freedom at the two ends of the half circular beam A and B are limited. The load applied by guide roll to ring is on section C with two component forces $F_x$ and $F_y$ respectively in direction x and y, as shown in Fig. 10(a). The value of the resultant force of $F_x$ and $F_y$ is equal to the guide force $F_g$ and the direction is pointed to the center of the circle. A rectangular profile is selected for the beam, as shown in Fig. 10(b). The yield strength of the ring material is 40 MPa. The elastic modulus is 116.7449GPa. And the poisson ratio is 0.3. The element type is selected as beam element B31.
When the average radius of the circular beam $R_a = 4.596$ m, the length of the profile $a = 0.308$ m and the width $b = 0.308$ m, the angle of the guiding force $\alpha = 22.66^\circ$, the resultant force of $F_x$ and $F_y$ is equal to $2.5 \times 10^5$ N, the simulation results are shown in Fig. 11. It can be seen the beam produced plastic bending deformation which meant the stiffness of the beam is destroyed. The maximum stress and strain occur at section A. So a further study about the relationship between the guiding force and the deformation performance at section A is made, as shown in Fig. 12. When the guiding force increases to $1.85 \times 10^5$ N, the Mises stress at section A reaches the yield strength and the material begins to produce plastic bending deformation. According to Eq. (22), the maximum allowed guiding force $F_{g-max}$ can be calculated, the valve is $1.853 \times 10^5$ N. Thus it indicates the proposed stiffness model can predict the maximum allowed guiding force accurately.

Fig. 11. Distribution of stress and strain on the beam: (a) equivalent stress; (b) equivalent strain.
Based on the FE model above, when the resultant force of $F_x$ and $F_y$ is equal to $1.85 \times 10^5$ N, it is a critical state to produce plastic instability. The simulative results of section bending moment and stress are shown in Fig. 13. According to Eqs. (18) and (20), the theoretical results of section bending moment and stress can be calculated. Fig. 14 shows a comparison of simulative and theoretical results of section bending moment and stress. It can be found the results have a good agreement.

![Fig. 12. Evolution of Mises stress and equivalent plastic strain under different guiding force.](image)

**Fig. 12.** Evolution of Mises stress and equivalent plastic strain under different guiding force.

![Fig. 13. Simulative results of section bending moment and stress: (a) section bending moment; (b) section bending stress.](image)

**Fig. 13.** Simulative results of section bending moment and stress: (a) section bending moment; (b) section bending stress.
According to Eq. (21), the maximum section bending stress for a ring with certain size and material mainly depends on $|Q_{(\varphi, \alpha)}|_{\text{max}}$. According to Fig. 7, it can be seen the maximum value of $|Q_{(\varphi, \alpha)}|$ occurs at different position when the position angle of guide roll $\alpha$ changes. Thus, to make the beam produce plastic deformation under different angles of guiding force is tried. The simulative result is shown in Fig. 15. As it can be seen when $\alpha = \{22.66^\circ, 50^\circ, 70^\circ, 90^\circ\}$, the earliest position to produce plastic deformation is respectively at section A, section C, section B, section C. And from Fig. 7, the following law can be found. When $0^\circ \leq \alpha \leq 46.3^\circ$, the maximal value of $|Q_{(\varphi, \alpha)}|$ is at section A. When $46.3^\circ < \alpha \leq 62.5^\circ$ or $79.5^\circ < \alpha \leq 90^\circ$, the maximal value of $|Q_{(\varphi, \alpha)}|$ is at section C. When $62.5^\circ < \alpha \leq 79.5^\circ$, the maximal value of $|Q_{(\varphi, \alpha)}|$ is at section B. Thus, this law can explain the phenomenon appearing in Fig. 15.
Fig. 15. The earliest position to produce plastic deformation: (a) $\alpha=22.66^\circ$, $F_g=1.85\times10^5$ N; (b) $\alpha=50^\circ$, $F_g=2.2\times10^5$ N; (c) $\alpha=70^\circ$, $F_g=2.4\times10^5$ N; (d) $\alpha=90^\circ$, $F_g=2.9\times10^5$ N.

2.4 Comparison of different ring stiffness models

Recently, scholars have proposed some stiffness models for RARR. In this section, a comparison between some typical models and our model is made.

The stiffness model proposed by Xu et al. (X-model) [31] can be described as the following formula

$$F_g \leq F_{\text{X-max}} = \frac{B H^2 \sigma_s}{R_s} K_{X(\alpha)}$$

where $F_{\text{X-max}}$ is the maximum allowed guiding force based on X-model, the coefficient

$$K_{X(\alpha)} = \frac{1}{2\sin \alpha} \left\{ 1 + \sin \frac{\alpha}{2} \left[ 1 + \left( \frac{1 + \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} \right)^2 \right] \right\}.$$

The stiffness model proposed by Hua et al. (H-model) [32] can be described as the following formula

$$F_g \leq F_{\text{H-max}} = \frac{B H^2 \sigma_s}{R_s} K_{H(\alpha)}$$

where $F_{\text{H-max}}$ is the maximum allowed guiding force based on H-model, the coefficient

$$K_{H(\alpha)} = \frac{1}{6\sin \alpha}.$$

To make an easy comparison, the stiffness model proposed by us (D-model) can be also described as the following formula
\[ F_g \leq F_{Dg\text{-}max} = \frac{BH^2\sigma_s}{R_s} K_{D(a)} \]  \hspace{1cm} (29)

where \( F_{Dg\text{-}max} \) is the maximum allowed guiding force based on D-model, the coefficient \( K_{D(a)} = \frac{1}{6Q_{(a,\alpha)}^{\max}} \).

As it can be seen, the forms of the stiffness models above are similar. The difference is in the coefficient \( K_{(a)} \), which is a function of \( \alpha \). When \( \alpha \) increases from 0 \(^\circ\) to 180 \(^\circ\), the coefficient \( K_{(a)} \) for the three models can be calculated, as shown in Fig. 16. It shows that all the valves of coefficient \( K_{(a)} \) have a fast decrease when \( \alpha \) is smaller than 15 \(^\circ\), and then become a steady level. Besides, the value of \( K_{D(a)} \) is between the values of \( K_{X(a)} \) and \( K_{H(a)} \).

![Fig. 16. Comparison of the coefficient \( K_{(a)} \) for the three models.](image)

To make a further comparison of the predicted accuracy of the three models above, the maximum allowed guiding force based on the three models can be calculated, according to Eqs. (27), (28) and (29). The angle of guide roll and the geometry size of the ring are two important factors to the maximum allowed guiding force. So the maximum allowed guiding force under different angles of guide roll and geometry sizes of a ring during rolling process for the three models are analyzed. Furthermore, corresponding FE simulations based on Section 2.3 are made. To make an easy comparison of the predicted accuracy of the three models, the ratios of the theoretical values based on the three models to the simulative valves are calculated, as shown in Fig. 17. It
indicates the results of D-model have a good agreement with the simulative results. The results of X-model are bigger than the simulative results and the results of H-model are smaller than the simulative results. In addition, the angel of guide roll has a little influence to the accuracy for D-model, but an obvious influence for X-model and P-model. A larger ring size has a better accuracy for D-model and H-model, but a worse accuracy for X-model.

![Graphs showing the comparison of the predicted accuracy of the maximum allowed guiding force for the three models: (a) under different angles of guide roll; (b) under different radiuses of a ring.]

**Fig. 17.** Comparison of the predicted accuracy of the maximum allowed guiding force for the three models: (a) under different angles of guide roll; (b) under different radiuses of a ring.

### 3 Application of ring stiffness model for RARR

#### 3.1 Theoretical analysis

For RARR mill, the movement of guide roll is controlled by the hydraulic cylinder. When the length of the hydraulic cylinder is extended or shortened, the guide roll installed on the swing arm can swing forward or backward around the hinge D, as shown in **Fig. 18**. The force of guide roll $F_g$ applied on the ring is controlled by adjusting the pressure $p$ in the hydraulic cylinder. Therefore, it is necessary to establish a relationship between $p$ and $F_g$. 
To satisfy the mechanical equilibrium condition, the following equation should be ensured

\[ F_p L_1 \sin \delta = F_g L \sin \gamma \]  

(30)

where \( F_p \) is the force of hydraulic cylinder, \( L_1 \) is the length of beam JD, \( L \) is the length of beam DG, \( \gamma \) is the supplementary angle of \( \angle DGO_1 \), \( \delta \) is the angle of \( \angle DJE \).

According to the geometrical relationship, the values of \( \gamma \) and \( \delta \) can be calculated by the following formulas

\[ \gamma = \pi - \arccos \frac{L^2 + (R_g + R)^2 - (R + R_m - x_D)^2 - y_D^2}{2L(R_g + R)} \]  

(31)

\[ \delta = \arccos \frac{L^2 + (x_D + L_1 \sin \alpha_D - x_E)^2 + (y_D - L_1 \cos \alpha_D - y_E)^2 - (x_D - x_E)^2 + (y_D - y_E)^2}{2L_1 \sqrt{(x_D + L_1 \sin \alpha_D - x_E)^2 + (y_D - L_1 \cos \alpha_D - y_E)^2}} \]  

(32)

where \( R_g \) is the radius of the guide roll, \( R \) is the outer radius of the ring, \( R_m \) is the radius of the main roll, \( x_D \) and \( y_D \) are the coordinate of hinge D in direction x and y respectively, \( x_E \) and \( y_E \) are the coordinate of hinge E in direction x and y respectively, the angle \( \alpha_D \) is given by
\[
\alpha_b = \arctan \frac{R + R_m - x_D}{y_D} + \arccos \frac{y_D^2 + (R + R_m - x_D)^2 + L_2^2 - (R_y + R)^2}{2L_2 y_D + (R + R_m - x_D)^2} + \arccos \frac{L_2^2 + L_1^2 - L_2^2}{2L_1}
\] (33)

where \(L_2\) is the length of beam JG.

Finally, the pressure \(p\) in the hydraulic cylinder can be expressed as

\[
p = \frac{F_g L \sin \gamma}{\pi r_h^2 L_1 \sin \delta}
\]

(34)

where \(r_h\) is the inner radius of the hydraulic cylinder.

Therefore, if the suitable force of guide roll \(F_g\) applied on the ring can be determined, the pressure \(p\) in the hydraulic cylinder can be also determined. From the analysis above, it can be seen the maximum allowed guiding force \(F_{g\text{-max}}\) is determined by Eq. (22). So the pressure \(p\) should satisfy the following condition

\[
p \leq p_{\text{max}} = \frac{B H^2 \sigma_s L \sin \gamma}{6 \pi R |Q_{(\alpha, \phi)}|_{\text{max}} r_h^2 L_1 \sin \delta}
\]

(35)

Thus, during RARR, the pressure \(p\) can be set as

\[
p = k_g p_{\text{max}} = \frac{k_g B H^2 \sigma_s L \sin \gamma}{6 \pi R |Q_{(\alpha, \phi)}|_{\text{max}} r_h^2 L_1 \sin \delta}
\]

(36)

where \(k_g\) is an adjustment coefficient and \(0 < k_g \leq 1\).

Therefore, the force applied on the guide roll can be determined by

\[
F_g = k_g F_{g\text{-max}} = \frac{k_g B H^2 \sigma_s}{6 R |Q_{(\alpha, \phi)}|_{\text{max}}}
\]

(37)

3.2 FE analysis

Based on the theoretical analysis above, in this section, FE simulation for RARR of a super-large ring is carried out. The dimension of the ring and other primary rolling parameters are shown in Table 1. The material of the ring is 42CrMo, and its true stress-strain curves at different temperature \(T\) and strain rate \(\dot{\varepsilon}\) are shown in Fig. 20. The physical properties including thermal conductivity, specific heat capacity, etc. are provided by Pan [33]. The key technologies for FE
modeling of RARR are referenced to Zhou [25].

Table 1 Primary rolling parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of main roll D_{m} (mm)</td>
<td>1350</td>
</tr>
<tr>
<td>Diameter of mandrel D_{m1} (mm)</td>
<td>600</td>
</tr>
<tr>
<td>Diameter of guide rolls D_{g} (mm)</td>
<td>500</td>
</tr>
<tr>
<td>Angle of the axial rolls γ (°)</td>
<td>35</td>
</tr>
<tr>
<td>Outer diameter of blank D_{0} (mm)</td>
<td>3500</td>
</tr>
<tr>
<td>Inner diameter of blank d_{0} (mm)</td>
<td>2412</td>
</tr>
<tr>
<td>Axial height of blank B_{0} (mm)</td>
<td>544</td>
</tr>
<tr>
<td>Outer diameter of rolled ring D (mm)</td>
<td>9500</td>
</tr>
<tr>
<td>Inner diameter of rolled ring d (mm)</td>
<td>8884</td>
</tr>
<tr>
<td>Axial height of rolled ring B (mm)</td>
<td>308</td>
</tr>
<tr>
<td>Temperature of rolls (°C)</td>
<td>80</td>
</tr>
<tr>
<td>Temperature of blank (°C)</td>
<td>1100</td>
</tr>
<tr>
<td>Temperature of environment (°C)</td>
<td>20</td>
</tr>
<tr>
<td>Heat transmission coefficient (N·s^{-1}·mm^{-1}·°C^{-1})</td>
<td>10</td>
</tr>
<tr>
<td>Heat convection coefficient (N·s^{-1}·mm^{-1}·°C^{-1})</td>
<td>0.02</td>
</tr>
<tr>
<td>Heat radiation coefficient (N·s^{-1}·mm^{-1}·°C^{-4})</td>
<td>0.7</td>
</tr>
<tr>
<td>Friction coefficient</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Fig. 20. True stress-strain curves of 42CrMo at different temperature T and strain rate (a)
\( \varepsilon = 0.01; \) (b) \( \varepsilon = 0.1; \) (c) \( \varepsilon = 1; \) (d) \( \varepsilon = 10 \) [22].

From Fig. 20, it can be seen the stress is varied with strain, strain rate and temperature. During RARR, the temperature in the ring surface area decreases, but in the ring center area it keeps almost constant. Thus, the strength of the material increases gradually in a certain extent during rolling process. However, to simplify the calculation, the stress at initial temperature (1100 °C) is selected as a basis. In addition, the unstable deformation generally happens at a low strain rate. So the stress level of the material is selected at \( T = 1100 \, ^\circ\text{C} \) and \( \varepsilon = 0.01 \), the strength valve is about 40 MPa. Combined with the dimension of the ring and rolling mill, the theoretical pressure in hydraulic cylinder and force applied on the guide roll can be calculated according to Eqs. (36) and (37). The result when \( k_g = 1 \) is shown in Fig. 21. It can be found the pressure in hydraulic cylinder and force applied on the guide roll should be reduced gradually during rolling process.

![Fig. 21. Theoretical pressure in hydraulic cylinder and force applied on the guide roll during RARR.](image)

To simplify the FE model, the guide roll is connected with a swing arm. The swing arm can revolve around an endpoint. A equivalent moment is applied on the endpoint to simulate the force applied on the guide roll, as shown in Fig. 22. The equivalent moment can be calculated by the following expression
When \( k_g = 1 \), the value of the equivalent moment during rolling process is shown in Fig. 23. It shows the moment is relatively large at initial rolling stage, and then it reduces gradually as the ring diameter enlarging.

Fig. 23. Theoretical guiding moment applied on the guide roll during RARR.
Fig. 24. Forming results of the ring after RARR: (a) $k_g=1.5$; (b) $k_g=1$; (c) $k_g=0.2$.

Fig. 24. shows the forming results of the ring after RARR when selecting different values of $k_g$. It can be seen the ring is destroyed when $k_g=1.5$. As the force applied on the guide roll is too large, more than the maximum allowed force. The stiffness condition cannot be satisfied. So the ring produces bending deformation, and the failure shape is similar to Fig. 2. This phenomenon reflects the accuracy of the theoretical stiffness model in a certain extent. When $k_g=1$, the ring produces little bending deformation, but it is not obvious and the rolling process can finish. When $k_g=0.2$, the shape of the ring is good and the rolling process is stable. Therefore, the force applied on the guide roll should slightly smaller than the maximum allowed valve.

To make a further study on the suitable guiding force, a range of the adjustment coefficient $k_g$
from 0.02 to 1 is selected. When the guiding force is changed, the shape of the ring may be changed during rolling process. Thus, the roundness error $e_R$ is introduced to evaluate the ring roundness. It can be defined as

$$e_R = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (R_i - R_a)^2}$$ (39)

where $n$ is the number of selected points on the ring outer surface, $R_a$ is the average outer radius of the ring, $R_i$ is the outer radius at point $i$.

Fig. 24 describes the roundness of the ring during RARR under different adjustment coefficients. It can be seen when the adjustment coefficients $k_g$ is selected too large or small, the roundness error of the ring increases. Larger guiding force can make the ring produce bending deformation, leading to the ring unstable. Smaller guiding force cannot support the ring in a stable status, leading to the ring wagging. Therefore, the adjustment coefficients $k_g$ should be selected in an appropriate range to offer a suitable guiding force. It can be found when the adjustment coefficient $k_g$ is selected in a range of 0.1 to 0.5, the ring roundness error can be kept in a lower level. That is to say, it is beneficial to form a good shape ring when the pressure $p$ in the hydraulic cylinder is controlled in a suitable level with $k_g=0.1$~$0.5$.

![Fig. 25. Roundness of the ring during RARR under different adjustment coefficients.](image-url)
3.3 Experimental verification

Through the analysis above, a suitable range of the pressure $p$ in the hydraulic cylinder is determined. Then a rolling experiment for the super-large ring was conducted on a lager RARR mill. The details about the ring and the rolling mill are as the description in Section 3.2. Before ring rolling, the pressure $p$ in the hydraulic cylinder was set in a level when $k_g=0.2$. The experimental valve waved but kept in a similar level as the set during rolling process, as shown in Fig. 26. By this control method, the rolling process was stable and formed a super-large ring with good dimensional accuracy successfully, as shown in Fig. 27. Thus, it indicates the control method of the pressure $p$ in the hydraulic cylinder based on our stiffness model is feasible.

![Fig. 26. Theoretical and experimental values of the pressure in hydraulic cylinder.](image)

![Fig. 27. Experimental result of RARR for a super-large ring.](image)

4 Conclusions

In this study, a mathematical model of ring stiffness condition for RARR was established
based on the force method. Then the influence factors to ring stiffness were discussed. Furthermore, the predicted results were compared with FE simulation and other ring stiffness models. Based on the stiffness model, a control method of the pressure in the hydraulic cylinder to control the guiding force was proposed and applied in a RARR experiment of a super-large ring successfully. The main conclusions are drawn as follows:

(1) Bigger geometric dimension of ring cross section, smaller geometric dimension of ring radius and bigger yield strength of ring material are good for improving the ring stiffness. When the position angle of guide roll $\alpha$ is smaller than $30^\circ$, the ring stiffness is weakened as $\alpha$ increasing. And when $\alpha$ is bigger than $30^\circ$, the ring stiffness can be improved generally as $\alpha$ increasing. However, the ring stiffness is weakened when $46.3^\circ < \alpha \leq 51.5^\circ$ or $62.5^\circ < \alpha \leq 63.6^\circ$.

(2) Compared with FE simulative result, the theoretical result based on our stiffness model has a good agreement. And the predicted result is more accurate than other two stiffness models.

(3) The pressure in hydraulic cylinder and force applied on the guide roll should be reduced gradually during ring rolling process. Through selecting suitable adjustment coefficient $k_g$ to control the pressure in hydraulic cylinder and force applied on the guide roll can keep ring rolling process stable and form a ring with good dimensional accuracy.

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