Graph Theory and The Identity of Indiscernibles

Callum DUGUID[†]

ABSTRACT

The mathematical field of graph theory has recently been used to provide counterexamples to the Principle of the Identity of Indiscernibles. In response to this, it has been argued that appeal to relations between graphs allows the Principle to survive the counterexamples. In this paper, I aim to show why that proposal does not succeed.

1. The PII

The perennial issue of the Principle of the Identity of Indiscernibles has recently been under attack from the mathematical domain. More specifically, it has been claimed that counterexamples to it can be found in graph theory. A response to this, focussing on relations found between graphs, rather than within them, has also been offered. In this essay I aim to show why the latter position faces a problem in its details.

The Principle of the Identity of Indiscernibles (PII) states that any two objects which share all of their (suitably restricted) properties are identical. That is, indiscernibles must be identical. In the commonly given contrapositive form, the PII states that any two distinct objects must be discernible in a relevant respect. As is well known, allowing any property to discern two objects renders the principle trivial: for any x, take the property of being identical to x.

For the PII to be considered interesting, it is therefore necessary to restrict the range of properties that are permitted to discern objects. This is normally done by appeal to the notion of qualitative properties. However, this term is not obviously applicable to the mathematical properties that we will be considering. Further, different authors often take it to enforce additional constraints on which properties are permissible beyond merely ruling out those which trivialise the PII. For these reasons, I prefer to avoid referring to permissible properties as qualitative ones.

One way of approaching the constraint on properties is given by Ladyman, Linnebo and Pettigrew, who find the usual talk of 'qualitative' ambiguous and offer two ways in which it can be understood (2012). Call a property 'identity-involving' if its proper analysis involves the identity relation. Call a property 'object-involving' if its proper analysis makes appeal to particular objects. An immediately appealing way of deciding which properties are not permitted is to rule out identity-involving ones due to standard worries that these will prevent the PII from being a principle that reduces identity facts to non-identity facts. As haecceities, those properties that hold of only a single specific object, require both object names and the identity relation in their analysis, we might then permit object-involving properties to feature in the PII.

Unfortunately, permissibility of properties cuts across the object-involving and identity-involving distinctions. A ban on identity-involving properties would rule out superlative properties (and more generally, any property that expresses uniqueness). Yet properties like being the brightest star look like they express genuine differences between objects without trivialising the PII. Similarly, object-involving properties like being the daughter of x also express difference while not entailing that every object must therefore be discernible.

A more fruitful route of making the distinction – as far as this paper goes – is given by Rodriguez-Pereyra (2006). Let us define a property as being impermissible iff differing with respect to it is or may be differing numerically. That is, a trivialising property expresses only that the object which bears it is distinct.

As a consequence of this, we cannot say that the resulting PII is a reductive principle. Since there are non-trivialising properties which are also identity-involving, demonstrating that the PII

[†] School of Philosophy, Religion and History of Science, University of Leeds, Leeds, LS2 9JT, UK; Email: phcjd@leeds.ac.uk

holds for some domain does not suffice to show that the relevant identity facts are reducible to facts that do not involve identity. This may conflict with the motivation that some people have for accepting the principle. This is unfortunate, but it is the aim of this paper to show that even this weakened form does not hold in all domains. If successful, this entails that more motivated, stronger forms will fail as well.

Since the terminology commonly used in these debates is often taken to have subtly different meanings depending on the particular author, it will be helpful to be clear about which set of definitions I intend to work with. Those that I favour are given in a paper by Caulton and Butterfield (2012). In the Frege-Quine tradition, an 'object' is simply something which could be the referent of a singular term. When we have a formula that can be applied to one object and not another, we say the two objects are 'discernible'.

Saunders's revival of Quine's grades of discriminability reminded metaphysicians that discernibility can be found in varying degrees.[1] In order of least-discerning to most-discerning, Caulton and Butterfield distinguish intrinsic, absolute, relative and weak discernibility. This paper will only need to focus on two of these: absolute and weak discernibility. We say that two objects are absolutely discernible if there is some 1-place formula which applies to one of the objects and not the other. We say that two objects are weakly discernible if there is some 2-place formula that is satisfied by the objects as a pair but is not satisfied by either object taken with itself.

Under the reasonable assumption that the language we are working in is expressively adequate (that is, each formula in the language corresponds to some property and each property corresponds to some formula), the following two definitions are equivalent to the previous ones. Two objects are absolutely discernible if there is some property that one has but the other does not. Two objects are weakly discernible is there is some relation between the objects that neither object bears to itself.

If the discernibility that the PII is concerned with is taken to be absolute discernibility alone, then there is a plethora of counterexamples available to it. These come from several areas, such as: philosophical imagination, as in the case of Black's spheres (Black, 1952); physics, which gives us fermions and bosons; and mathematics, which presents the complex conjugates i and -i. In all three cases, there are two objects that appear to have all of the same properties and relations. Hence any defender of the PII must allow take the relevant discernibility to be more discerning. As the most discerning form is weak discernibility, it is this form of the PII that I am interested in.

Earlier it was claimed that we do not need to ban object-involving properties from featuring in the PII since they are not all trivialising. However, we may wish to heed the warning of Muller (2015), who bans such properties on account of them often being used in an illegitimate way.[2] With this tripartite distinction of discernibility in hand we are now in a position to do just that. By Ladyman, Linnebo and Pettigrew's Theorem 6.4, any form of discernibility in a language with object names available is equivalent to weak discernibility in a language without such names (Ladyman, Linnebo and Pettigrew, 2012, 172-173). In other words, adding properties that use object names will not discern any more objects than weak discernibility already does.

There is a practical benefit to making this move. While any trivialising object-involving properties ought to be ruled out by our impermissibility definition, exactly what counts as trivialising in the mathematical domain is not an easy matter. It is possible for impermissible properties to mistakenly get categorised as permissible via human error; following Muller in banning object-involving properties reduces the chances of this. While this may result in some acceptable properties being ignored, we know that there will be a weakly discerning relation available to discern the relevant objects.

As a consequence of the above discussion, haecceitistic properties, like being identical to b, will be counted as impermissible on two counts. Not only are they trivialising in the sense

[1] The original work is found in Quine (1976). See also Saunders (2003) for discussion.

[2] It is worth noting that appeal to object names is not normally taken to be a permissible move in the classic case of Black's spheres.

mentioned previously, they are also object-involving. In the context of graph theory, which will be the focus of this article, properties like being adjacent to b will also be ruled out: while not trivialising, they are object-involving. However, those that make no reference to vertex names, like being adjacent to 3 distinct vertices will be counted as permissible.[3]

Introducing weak discernibility allows the PII to survive the traditional counterexamples noted above as the objects in each can be weakly discerned. For example, in the case of Black's spheres, one is a mile apart from the other, but not a mile apart from itself. However, even the weaker form of the PII does not hold in all contexts. In particular, it has recently come under attack in the mathematical domain, where it is claimed that there exist graph-theoretic counterexamples.[4] In the remainder of this paper, I will first explain exactly what these counterexamples are, and why the revised form of the PII cannot accommodate them. I shall then introduce De Clercq's (2012) defence of the principle. The discussion will then turn to a problem with this defence, concluding that the PII does not hold in the mathematical domain in general.

2. Graph Theory

The basic idea behind the graph theoretic counterexamples is that graph theorists assume the distinctness of certain mathematical objects, but we can find no relation between the objects that discerns them. Hence either graph theorists are mistaken or the PII does not hold in graph theory. I will be assuming what Shapiro calls the faithfulness constraint: we aim to 'provide an interpretation that takes as much as possible of what mathematicians say about their subject as literally true' (2008, 289). On that basis the above conflict gives us reason to conclude that even the weak form of PII does not hold in graph theory.

To begin the discussion proper, we will need to first introduce a small amount of graph theory. In what follows, definitions of graph-theoretic terms are taken from Bondy and Murty (2007) and Diestel (2010).

A graph G is an ordered pair (V, E), where V is a set of vertices and E is a set of edges. Members of E are unordered pairs of vertices. A vertex is said to be incident to those edges that have it as an end-point. Two vertices are adjacent if they are incident to the same edge. The degree of a vertex is the number of edges it is incident to. To aid the reader in interpreting them, graphs are often represented by diagrams. This is normally done by drawing the vertices as dots or circles and the edges as lines that connect them.

As an example of a graph, let G_1 = (V, E), where V = {a, b, c} and E = {{a, b}, {b, c}, {c, a}}. This is the graph that would normally be represented by a triangle (see figure 1). It consists of three vertices and three edges, such that each pair of vertices are connected by an edge.

The main graph-theoretic property that we will be interested is that of isomorphisms. Two graphs are said to be isomorphic if they have the same basic structure, regardless of how different their representations may look. More formally, let G = (V, E) and G' = (V', E'). G and G' are isomorphic if there exists a bijection $\phi: V \rightarrow V'$ with $xy \in E \leftrightarrow \phi(x)\phi(y) \in E'$, for all $x, y \in V$. Hence isomorphisms are edge-preserving. Note that two graphs can be isomorphic even if their vertices have completely different labels.

If the above bijection maps a graph to itself, it is an automorphism. It is easy to see that all graphs have at least one automorphism- the trivial automorphism. This is the bijection that maps every vertex to itself and leaves the edges unchanged. Due to its trivial status, this is regarded as uninteresting. If there is a non-trivial automorphism of a graph, then the graph is symmetric. If there are no such non-trivial automorphisms, the graph is asymmetric. The graph G_1, given as our earlier example, is symmetric. It has five non-trivial automorphisms: rotating the diagram clockwise by 120 and 240 degrees visually represents two of these; the other three correspond to the three

[3] This is exactly the result that we want since all of a graph's structural properties are determined by its adjacency relations. See West (2001, 10).

[4] See both Ladyman (2007) and Leitgeb and Ladyman (2008).

reflections of an equilateral triangle. As a point of interest, the smallest asymmetric graphs (other than the trivial graph of a single vertex) have six vertices; there are no smaller ones. An example of an asymmetric graph is given in figure 2.

A note regarding the above definitions: Ladyman and Leitgeb avoid describing the issue in set theoretic terms, whereas De Clercq chooses to set up the problem via these standard definitions. There is certainly a thorny issue here regarding how best to understand the nature of graphs and whether they are, in some fundamental way, set theoretic. While I have no desire to attempt to settle this matter here, it is De Clercq's argument that I shall be criticising and so, in the spirit of charity, it is his framing of the issue that I will adopt.

Fortunately, the objection raised in Leitgeb and Ladyman has a straightforward equivalent if graphs are indeed set theoretic. The vertices of a graph, as elements of a set, must possess their identities regardless of whether the PII holds within graph theory (whether this identity is primitive or not is another matter). But we can still ask whether there are any graph theoretic properties that will discern the vertices. If there are no such properties in the language of graph theory, then the PII fails within that particular mathematical domain.[5]

The proposed graph-theoretic counterexamples that I wish to consider are discussed in Ladyman (2007), and Leitgeb and Ladyman (2008). Before giving them, however, something should be said regarding an initial point that Ladyman makes. Let a labelled graph be a graph in which vertices are assigned linguistic labels.[6] Then the claim is that labelled graphs cannot give us counterexamples to the PII, as every vertex is distinguishable by means of reference to its label, no matter what the graph is like. Assigning names to what might otherwise be indistinguishable objects is thought to be problematic in the case of Black's spheres (otherwise its status as a putative counterexample would be easily dismissed), as using it to fix identity appears to beg the question. Ladyman (2007, 33) tells us that the use of labels in graph theory is assumed to be possible, marking a difference between the two cases. I assume that he believes this because of the practices of mathematicians.

Even if we do not adopt the set-theoretic construal of graph theory, this point strikes me as only partially true. That mathematicians assign labels to vertices without first checking for discernibility tells us little about the metaphysical status of vertices and labels. Instead, it may reveal that many working mathematicians care little for this sort of philosophical puzzle. The very fact that labels can be assigned to vertices does not reveal whether the identities of the vertices must be fixed before labels can be applied or not. It is consistent with mathematical practice that some means of discerning the vertices is available before labels are assigned.

Returning to the set-theoretic construal, using labelled graphs will not trivialise the PII. Properties like being the vertex a have already been ruled out due to the fact that differing with respect to them is nothing more than differing numerically. That they are object-involving is further reason to be suspicious of them. The mere fact that vertices are assigned names in labelled graphs does not mean that we are thereby obliged to accept a form of the PII which takes object-involving properties to be permissible. In what follows, I shall therefore be discussing the graph-theoretic counterexamples using labelled graphs; more on unlabelled graphs will be said in the next section.

Let us now consider the counterexamples (Ib., 34-35). Let $G_3 = (V_3, E_3)$, where $V_3 = \{a, b\}$ and $E_3 = \{\{a, b\}\}$. This is the graph of two vertices connected by an edge (see figure 3). In fact, this is the graph corresponding to the usual putative counterexamples to the PII, such as Black's spheres. The two vertices, being connected by an edge, are weakly discernible from one another. Now consider $G_4 = (V_4, E_4)$, where $V_4 = \{a, b\}$ and $E_4 = \{\}$. This is the graph consisting of two vertices and no edges- we can get G_4 by deleting the edge from G_3 (see figure 4).

Both G_3 and G_4 are symmetric graphs: permuting the vertices gives rise to a non-trivial

[5] I would like to thank an anonymous referee for pushing for clarity on this definitional issue.

[6] This talk of vertices having labels is somewhat misleading: if we take seriously the graph theoretic definitions, then the vertices simply are elements of the vertex set such as a, b and c; there is no need to assign further labels to them. However, talk of labels is standard practice in graph theory and so I persist with it.

automorphism. In the case of the former, the presence of an edge allows us to weakly discern the vertices. However, there does not appear to be any way of discerning the vertices in G_4. There are no edges and we have already seen that labels alone cannot be used to distinguish them. There does not appear to be any other way in which this might be done. As Ladyman points out, G_4 is hardly unique in this respect- it is very easy to find other graphs that do not have the required relations to discern all of their vertices. One other example would be $G_5 = (V_5, E_5)$, where $V_5 = \{a, b, c, d\}$ and $E_5 = \{\{a, b\}, \{c, d\}\}$. This is the graph of two unconnected components, each one isomorphic to G_3 (see figure 5). Yet the two components are not discernible, for there are no relations between them to ground the discernibility.

The obvious conclusion from this is that the PII fails to apply generally in graph theory. Ladyman takes this to mean that identity here should be ungrounded- that is, primitive (Ib., 36-37).

3. De Clercq's Defence of the PII

De Clercq attempts to save the PII in graph theory through a distinction between intrastructural properties and relations (those that can be found in a particular graph) and interstructural ones (those that can be found holding between graphs). The crucial question then is whether we are restricted to the former, or whether we can appeal to any non-trivialising graph-theoretic relations. He believes that we ought to be less restrictive and hence the PII is preserved: we can discern vertices via the properties they have in other graphs.

We have already seen that allowing only intrastructural properties will cause the PII to fail; simply recall the graph in figure 4 that Ladyman raises as a counterexample. There are no properties within that graph that will allow us to discern the vertices, even if we allow for weak discernibility. As De Clercq notes, the PII could be read as being more or less restrictive. Since the more restrictive version fails, let us consider the more inclusive reading.

We need to examine the consequences of allowing for intergraph relations. De Clercq's proposal is, in short, that two vertices might satisfy the PII because they bear different relations in different graphs. For this to work, of course, it must be possible for a vertex to have cross-graph identity. Exactly how this is possible is a related, but separate, issue. Unfortunately, I know of no detailed philosophical or mathematical account of this. However, since mathematicians do talk as if it is possible, let us merely note that, eventually, the details ought to be fleshed out if one was to accept De Clercq's defence.

Recall graph G_4 given in figure 4. It will be instructive to use this in an example of how De Clercq's proposal works. The graph consists of two vertices, a and b, which also appear in graphs like $G_6 = (V_6, E_6)$, where $V_6 = \{a, b, c\}$ and $E_6 = \{\{a, c\}\}$ (see figure 6). In G_6, a and b are absolutely discernible from one another (although a is not absolutely discernible from c). The two vertices bear different properties: for example, vertex b is such that it is an isolated vertex, while vertex a is not. So the vertices are not counterexamples to the PII in G_6. Since we have shown that they are discernible within graph theory generally, and they also appear in G_4, a and b are not counterexamples in G_4 (even though we lack a way of discerning them in G_4).

The method is simple, but one might worry that it only works for labelled graphs. Without labels attached, there does not appear to be a way of finding which vertices feature in which graphs. Worse, there might simply be no fact of the matter as to when cross-graph identity occurs. Was Ladyman right to focus on unlabelled graphs when seeking counterexamples?

De Clercq's response to this is to point out that this relies on unlabelled graphs actually being graphs. He provides strong evidence that many graph theorists believe unlabelled graphs are, more strictly speaking, isomorphism classes of graphs. Unlabelled graph diagrams are then attempts at representing those members of the associated isomorphism class independently of their labels. Cross-graph identity of unlabelled vertices cannot be an issue for this interpretation as, again strictly speaking, there are no such objects in need of distinctness.

Consider again the graph G_4 in figure 4, but disregard the labels. This does not constitute a counterexample to the PII because it is actually a representation of an isomorphism class that contains the labelled graph G_4 as an element. We have already seen how the vertices a and b in G_4 are can be discerned within graph theory. Hence the PII is saved.

It is worth emphasising at this juncture exactly what the putative counterexamples are taken to be. Ladyman takes them to be found in the unlabelled graphs due to concerns that labelled vertices can be distinguished by appeal to their labels. I follow De Clercq in not taking unlabelled graphs to actually be graphs at all, hence they cannot be counterexamples to the PII. We need not worry about labelled graphs introducing names that will trivialise the PII since the version we are interested in counts such properties as impermissible. Therefore any counterexamples will be found in the labelled graphs.

4. The Problem of Unrestricted Graphs

De Clercq's defence of the PII using intergraph relations is a very interesting one, but it faces a problem in its details. To tease this out, let us consider exactly how vertices like b are supposed to be discerned. Recall that the suggestion is that they are absolutely discernible on account of graphs like G_6. To begin with, take the property is isolated in another graph. Vertex b has this property on account of its position in G_6, where b is isolated, whereas vertex a is not isolated in G_6. The property is not trivialising, makes no appeal to object names and is of the right form to absolutely discern a vertex.

Yet this property will not save the PII. The problem is that this property will not uniquely pick out vertex b. For every graph G_6, there is a corresponding graph $G_7 = (V_7, E_7)$, where $V_7 = \{a, b, c\}$ and $E_7 = \{\{b, c\}\}$ (see figure 7). This is just the graph resulting from swapping a and b around in G_6. But while b is isolated in G_6, it is a which is isolated in G_7. So both vertices bear the property is isolated in another graph. Hence this property will not absolutely discern them.

Of course, the problem is not specific to these graphs. Take any graph in which a has some property that b does not and permute the vertices. We now have a graph in which it is b which has this property while a does not. A natural thought here is that this problem only arises because the property above was poorly chosen. In particular, it says nothing about which other graph the vertex is isolated in. A more specific property may fare better.

The most specific non-object-involving property would be is isolated in a graph with three vertices and an edge between two of them. As an aside, the non-reductive nature of the PII we are considering is clear here: this property is non-trivialising despite being identity-involving on account of its appeal to a particular number of distinct vertices. Yet even this property will not absolutely discern vertices a and b. The problem is the same as before: both vertices bear the property since it describes both G_6 and G_7.

In order to avoid the problem posed to De Clercq by the plurality of graphs, the property appealed to must be specific enough to single out a single graph. A property like is isolated in a graph with vertices a, b and c, and an edge between b and c would do this by completely describing the relevant graph. However, in order to do this, it must utilise object names which, following Muller, were banned in the first section.

While this ban was motivated by appeal to the equivalence of weak discernibility in a language without object names to all forms of discernibility in a language with names, a defender of De Clercq might try to run the argument the other way. That is, the existence of object-involving properties like the previous one entails the existence of an acceptable weakly discerning relation that does not make use of object names.

But such a relation has not yet been provided. And neither can I see what it might be. The burden of proof here lies upon De Clercq’s account to offer a relation for consideration. Only at that point will we be in a position to tell whether it truly is trivialising or not.

As an alternative way of narrowing the number of graphs under consideration, we might note the similarity between interstructural properties and modal ones. Both are concerned with how an object features in contexts other than the actual one. Perhaps this might be expanded upon to give us the required account. In particular, the notion of the closest possible world has proven itself to be a useful one – the analogue would be of the closest graph.

For this to work, there would need to be some way of determining how close one graph is to another. This, at least, does look possible. Erdős and Rényi (1963) use the notion of the degree of asymmetry of a graph: this is the number of 'moves' it takes to turn an asymmetric graph into a symmetric one. By 'move' I mean the graph theoretic operations of addition or deletion of an edge. While they then go on to prove several interesting facts, the details do not concern us here. The important point is that it is easy to see how a parallel notion to a graph's degree of asymmetry might help De Clercq. Let the degree of difference of a graph G relative to a graph G' be the minimum number of edges and vertices that must be added or deleted to transform G into G'. We can then say that one graph is closer to G than another if its degree of difference is lower. Then we could interpret De Clercq's method as considering only the closest graphs to our problematic ones.

Setting aside the issue of justification for the moment (as noting that there is a possible analogy is hardly sufficient), we must ask whether this method will do what it is needed to. Unfortunately, the answer is that it will not. Consider G_4, G_6 and G_7 once more (figures 4, 6 and 7 respectively). The degree of difference of G_6 relative to G_4 is 1. But the degree of difference of G_7 relative to G_4 is also 1. While this suggestion will restrict those graphs that can be considered, it is not restrictive enough: for any degree of difference, there will be many graphs that have that value relative to G_4. Nor is there any sense in looking for graphs with a lower degree of difference than 1. Clearly the degree of difference must be a positive integer, or 0. But the only graphs with a degree of difference relative to G_4 of 0 are those isomorphic to it, in which case the problem of discernibility will still arise.

Hence we must conclude that this defence will not succeed, despite the analogy. In the absence of any other defence being offered or of any weakly discerning relations being found, it is reasonable to conclude that De Clercq's method will not save the PII in graph theory. Further, the current lack of any account of cross-graph identity that does not beg the question makes the prospects of a defence reliant on inter-graph properties dim at best.[*]

REFERENCES

BLACK, M. 1952, "The Identity of Indiscernibles", Mind, **61**, pp. 153-164.

BONDY, J. A. and MURTY, U. S. R. 2007, Graph Theory, London: Springer.

CAULTON, A. and BUTTERFIELD, J. 2012, "On Kinds of Indiscernibility in Logic and Metaphysics", British Journal for the Philosophy of Science, **63**, pp. 27-84.

DE CLERCQ, R. 2012, "On some putative graph-theoretic counterexamples to the Principle of the Identity of Indiscernibles", Synthese, **187**, pp. 661-672.

DIESTEL, R. 2010, Graph Theory, London: Springer.

ERDÖS, P. and RÉNYI, A. 1963, "Asymmetric Graphs", Acta Mathematica Academiae Scientarum Hungaricae, **14**, pp. 295-315.

LADYMAN, J. 2007, "On the identity and diversity of objects in a structure", Proceedings of the Aristotelian Society Supplementary Volume LXXXI, pp. 23-43.

LADYMAN, J., LINNEBO, Ø. and PETTIGREW, R. 2012, "Identity and indiscernibility in philosophy and logic", Review of Symbolic Logic, **121**, pp. 162-186.

LEITGEB, H. and LADYMAN, J. 2008, "Criteria of identity and structuralist ontology", Philosophia Mathematica, **16**, pp. 388-96.

MULLER, F. A. 2015, "The Rise of Relationals", Mind, **124**, pp. 201-237.

QUINE, W. V. O. 1976, "Grades of discriminability", Journal of Philosophy, **73**, pp. 113-116.

RODRIGUEZ-PEREYRA, G. 2006, "How Not to Trivialize the Identity of Indiscernibles", in: P. F.

[*] I would like to thank James Ladyman, Juha Saatsi and Robbie Williams for helpful discussion on the issues raised in this paper. I am particularly indebted to Steven French for his feedback on several earlier drafts. Finally, I am grateful to three anonymous referees for this journal whose comments have significantly improved this material.

Strawson and A. Chakrabarti, eds, Concepts, Properties and Qualities, Aldershot: Ashgate, pp. 205-223.
SAUNDERS, S. 2003, "Physics and Leibniz's Principles", in: K. Brading and E. Castellani, eds, Symmetries in Physics: Philosophical Reflections, Cambridge: Cambridge University Press, pp. 289-308.
SHAPIRO, S. 2008, "Identity, indiscernibility, and ante rem structuralism: the tale of i and –i", Philosophia Mathematica, **16**, pp. 285-309.
WEST, D. 2001, Introduction to Graph Theory 2nd Ed., Upper Saddle River, NJ: Pearson.

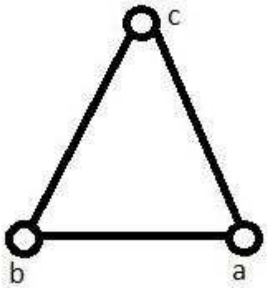

Figure 1

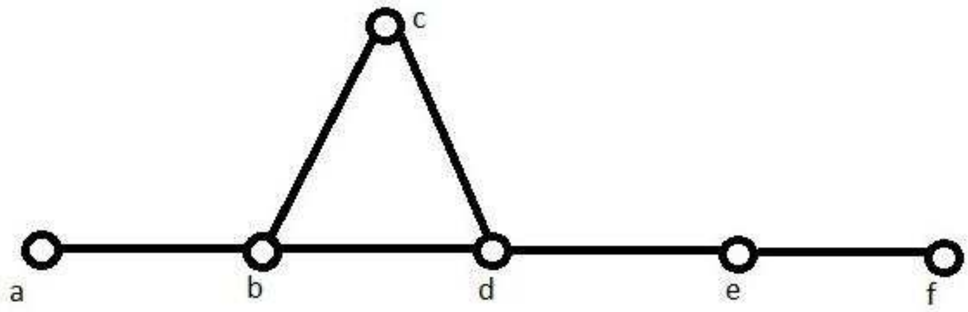

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7