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Experimental evaluation of environmental effects on a polymer-coated aluminium structure: a time-series analysis and pattern recognition approach

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Abstract

Temperature variation is an important issue that needs to be considered when trying to develop a reliable Structural Health Monitoring (SHM) strategy. In the case that a data-based approach is chosen for damage detection, environmental fluctuations could be erroneously regarded as an abnormal condition of the structure and could mask the presence of damage. One of the objectives of the current work is to examine a statistical pattern recognition approach for novelty detection under different temperature conditions. A second important issue that could hinder the reliability of a SHM strategy is any kind of nonlinear behaviour, not associated with damage, in a system. For the purposes of this paper, the dynamic behaviour of a polymer-coated aluminium structure with ribs fixed with bolts is examined. The autoregressive parameters are the damage sensitive features and later, it is performed Principal Component Analysis (PCA) for robust novelty detection that takes into account the temperature variation.

1 Introduction

The main objective of Structural Health Monitoring (SHM) strategies is to assess the performance and/or health state of a structure in aerospace, civil and mechanical engineering context. These strategies are developed to evaluate whether a structure needs to be repaired or replaced in order to reduce associated maintenance costs in SHM applications. Damage can be seen as a change of the material and/or the geometrical or structural-dynamical properties of the structure which is detrimental to performance. Normal conditions, on the other hand, refer to the common behaviour of the system under different operational and environmental conditions when the structure is known to be undamaged [1,2]. Damage identification techniques can be divided into two main groups: model-based and data-driven approaches. The former approach is usually based on a reliable physics-based model of the structure, while the latter is based on analysing the acquired data via the statistical point of view. Using data-driven methods, the indication of damage could be directly determined by a comparison between a baseline and the data collected during the operational life of the structure. The procedure does not need any kind of physics-based model to interpret the reality, and when a system is complex it becomes difficult to realise a reliable physics-based model. The core of the process is based on a robust statistical analysis of the data. A feature is defined as some characteristic of the measured response that is extracted via signal processing or parameter estimation which is sensitive to the presence of damage or otherwise. Feature extraction transforms “data” into “information”. It is desirable to have examples of the features from both damaged and undamaged structures to infer more information about the damage, e.g. the severity of the damage [3]. However, to ensure the reliability of the analysis the reference database has to include as many healthy

scenarios as possible. In real SHM applications, operational and environmental effects can mask damage-related changes in the features, especially if the damage is small. For this reason, a wide description of all the possible combinations of environmental and operational conditions is useful in order to confidently uncover damage. Damage often changes the structural dynamics; therefore, it can be potentially detected through any vibration data recorded, as long as the derived features are damage-sensitive. This strategy belongs to the vibration-based methods. A change in the dynamics may be also caused by varying environmental conditions and uncertainty. Continuous monitoring systems make use of sensors permanently installed on the structure to extract the current state of the structure. As a result, the collected structural responses are analysed and compared to baseline patterns in order to detect abnormal behaviours and define the structural integrity. There are several useful techniques available in literature [4-15] whose main effort is to extract significant features from data. In many scientific fields, including SHM, the number of measured variables can be large. Principal Component Analysis (PCA) provides ways for reducing a complex data set into a lower dimension and revealing some hidden and simplified patterns linked to specific scenarios. PCA is a classical linear technique of multivariate statistics for mapping multidimensional data into a lower dimension with minimal loss of information [16]. In SHM, PCA has been applied in the past to vibration signals to remove the influences of the environmental effects from the vibration data, to extract structural damage sensitive features and to discriminate features from damaged and undamaged structures [17]. In this paper, PCA is applied to the vibration data collected from a plate and it is used to identify the different conditions on which the structure is exposed. Autoregressive (AR) models are used to extract damage-sensitive features from time-series measured by accelerometers when the structure is subjected to different environment temperatures and structural state conditions. The main contribution of this work is the performance evaluation of PCA-based algorithms applied to the parameters of autoregressive models based on experimental data in the presence of variations in the environmental conditions. This work is organised as follows. Section 2 gives an explanation about the AR model and the PCA background theory. A description of the test structure, the simulated operational and environmental variability, and a summary of the data sets is provided in Section 3. In Section 4, the results and the discussion of the application of the PCA on the AR parameters are carried out. Section 5 report the conclusion of this work.

2 Background theory

2.1 AR models

In time series analysis, one of the most useful representations of a time series process is via autoregressive (AR) modelling. It is a stochastic finite linear model and a brief description is given here; for a detailed analysis of the AR model, please refer to [18]. If Z is a generic stationary process, one can estimate the value of Z at time t just basing the evaluation on its past values plus white noise. Let one fix the value of a process at equally spaced times $t, t-1, t-2, \dots$ by $z_t, z_{t-1}, z_{t-2}, \dots$. Also let $\tilde{z}_t, \tilde{z}_{t-1}, \tilde{z}_{t-2}, \dots$ be the deviation from the process mean value μ (assumed stationary); for example, $\tilde{z}_t = z_t - \mu$. Then the value \tilde{z}_t can be written as:

$$\tilde{z}_t = \Phi_1 \tilde{z}_{t-1} + \Phi_2 \tilde{z}_{t-2} + \dots + \Phi_p \tilde{z}_{t-p} + a_t \quad (1)$$

where p is the order of the model, Φ_i are the (constant) coefficients of the AR model and a_t is white noise. a_t is a sequence of uncorrelated random values from a fixed distribution with constant mean $E(a_t)$, (usually assumed to be 0) and constant variance $\text{Var}(a_t) = \sigma_a^2$.

If one defines an AR operator of order p by the following equation:

$$\Phi(B) = 1 - \Phi_1 B - \Phi_2 B^2 - \dots - \Phi_p B^p \quad (2)$$

where B is the backward-shift operator, then the AR model may be written as:

$$\Phi(B)\tilde{z}_t = a_t \quad (3)$$

A key point in using an AR model is to establish its order. Many criteria for model order estimation have been introduced and are described in the literature for a proper model selection; in this work, the Bayesian Information Criterion (BIC) is used to identify the most appropriate model order.

2.2 Principal Component Analysis

Principal component analysis (PCA) is a useful technique of multivariable analysis that reduces a complex multivariate data set to a lower dimension and can thus reveal hidden and simplified patterns. The theory of PCA can be found in many books and articles, e.g. [16], [19]; here, just a brief description is presented. PCA can be defined as the orthogonal projection of the given data onto a lower dimensional linear space, known as the principal subspace, such that the variance of the projected data is maximised. Consider a data set of observations $\{x_n\}$ where $n = 1, \dots, N$, and x_n is a Euclidean variable with dimensionality D . Our goal is to project the data onto a space having dimensionality $M < D$ while maximizing the variance of the projected data.

The sample mean of the data is:

$$\bar{x} = \frac{1}{N} \sum_{n=1}^N x_n \quad (4)$$

The covariance matrix is defined by

$$S = \frac{1}{N} \sum_{n=1}^N (x_n - \bar{x})(x_n - \bar{x})^T \quad (5)$$

One now maximises the projected variance $u^T S u$ with respect to u . Clearly, this has to be a constrained maximization to prevent $\|u\| \rightarrow \infty$. The appropriate constraint comes from the normalisation condition $u^T u = 1$. To enforce the constraint, one introduces a Lagrange multiplier, denoted here by λ , and then makes an unconstrained maximisation of

$$u^T S u + \lambda (1 - u^T u) \quad (6)$$

By setting the derivative with respect to u equal to zero, one sees that this quantity will have a stationary point when

$$S u = \lambda u \quad (7)$$

which says that u must be an eigenvector of S . If one left-multiplies by u^T and makes use of $u^T u = 1$, one sees that the variance is given by

$$u^T S u = \lambda \quad (8)$$

Therefore, the variance will be a maximum when one sets u equal to the eigenvector having the largest eigenvalue λ .

If one considers the general case of an M -dimensional projection space, the optimal linear projection for which the variance of the projected data is maximised is now defined by the M eigenvectors $\mathbf{u}_1, \dots, \mathbf{u}_M$ of the data covariance matrix S corresponding to the M largest eigenvalues $\lambda_1, \dots, \lambda_M$. [19]. The projected variables are referred to as principal component scores.

In order to develop PCA, since the data collected, in general, have different magnitudes and scales, here each data point is scaled using the mean and the standard deviation of all measurements.

3 Design of the experiment and data collected

The test rig is made from a rectangular plate with dimension of 250x375 mm, 3 mm width, and with four ribs to increase the strength. Two ribs are on the short side of the plate and they are C ribs; the others two are in the centre and they are L ribs. All ribs are fixed by bolts.

The plate and the ribs are of aluminium, but there is also a thin layer of a polymer (3mm) on the bottom of the plate. The plate is suspended by four springs to provide free-free boundary condition. Six accelerometers are used to collect the vibration data of the plate, and their location is illustrated in Fig. 1.

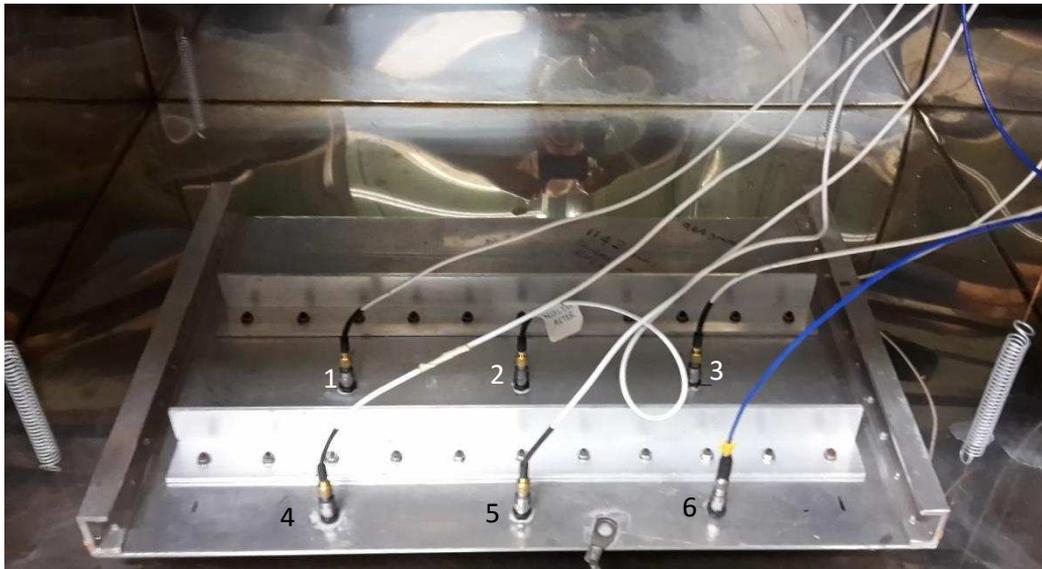


Figure 1: Plate with the indication of the position of the accelerometers

The plate is excited by a shaker (Fig. 2) and a load cell is placed under the plate (Fig. 3), connected directly to the shaker, in order to measure the force.



Figure 2: Shaker

Figure 3: Force transducer

The hardware used to perform the data acquisition and to set the force given to the plate is LMS Scadas Lab. The sampling frequency chosen was 2560 Hz, because the maximum frequency of interest was taken at around 1000 Hz. The spectral resolution was 0.15 Hz, requiring time histories of 6.4 s duration each. The selected input given to the plate was white noise, and in order to reduce the noise associated to the measures, an average of 50 spectra was performed to obtain the Frequency Response Functions (FRFs). The amplitude of the input was set at three different levels, of rms value: 22 N, 44 N and 110 N. The plate was placed inside an environmental chamber to test its behaviour in different environmental conditions (Fig. 4).



Figure 4: Plate inside the chamber

The temperature range selected was from -20°C up to 80°C . In this way, it was possible to investigate the three different states of the epoxy: glassy, transitional and rubbery. A thermocouple was fixed to plate to check when the specimen reached the selected temperature before performing the vibration tests. In order to perform a damage detection, a number of abnormal conditions of the plate were considered, as the temperature changed. Nine levels of damage were chosen. The abnormal condition was represented by removing the bolts from a selected stringer. The first level of the damage was to remove just one bolt while the ninth consisted of removing nine bolts from the selected stringer, as shown in Tab. 1.

Level of Damage	Number of bolts removed	ID Damage
1	1	D1
2	2	D2
3	3	D3
4	4	D4
5	5	D5
6	6	D6
7	7	D7
8	8	D8
9	9	D9

Table 1: Description of damage level

The bolts are clamped through a dynamometric Allen key to ensure the same torque moment is given to the bolts. For M3 bolts, the maximum recommended tightening torque for a low quality screw is 0.57 Nm, so it was decided to set it here at 0.6 Nm.

The experiment has been divided in four phases:

- i. normal condition.
- ii. abnormal conditions.
- iii. normal condition with temperature variations.
- iv. abnormal conditions with temperature variations.

The first stage in the testing was for characterising the dynamic behaviour of the plate in the normal condition by collecting vibration data from the six accelerometers under a white noise excitation. The normal condition is specified by the temperature at 20° C for the structure without removing any bolts. The second test stage acquired vibration data related to each abnormal condition, removing one bolt to nine bolts. For each acquisition case, time histories of 5 minutes duration (50 consecutive time histories of 6.4 s) were taken for the three different rms levels of amplitude chosen for the shaker (22 N, 44 N and 110 N). All these amplitudes were guaranteed to force equally all the modes of the plate in the range of frequency considered. Using three different level of amplitude allowed investigation of the role of different input intensities in the pattern recognition. The tests were conducted starting with all the bolts on the plate, and they were then removed one by one, until the test cycle was complete. To assure the repeatability of the experiment, when the test related to the removal of one more bolt is finished, all the others bolts are replaced and it is checked that the normal condition is repeated, before passing to the next test. The third test stage was to give a picture of the behaviour of the plate when only subjected to temperature variations, from -20° C to 80° C with steps of 10° C. The target temperature was set into the environmental chamber and then the dynamic tests were performed after the plate reached the thermodynamic equilibrium with the environment, checking the thermocouple applied on the upper surface of the plate. Finally, in the last test stage, the vibration data were collected for all the mixed cases of abnormal conditions and temperature variations. Also in this stage, to guarantee the repeatability, after complete a temperature cycle for a considered abnormal condition, all the removed bolts are replaced, and the normal condition (20° C and all the bolts on) is checked, before proceeding to the next abnormal condition. Fig. 5 shows the Frequency Response Function (FRF) of the plate in its normal condition, which is the reference.

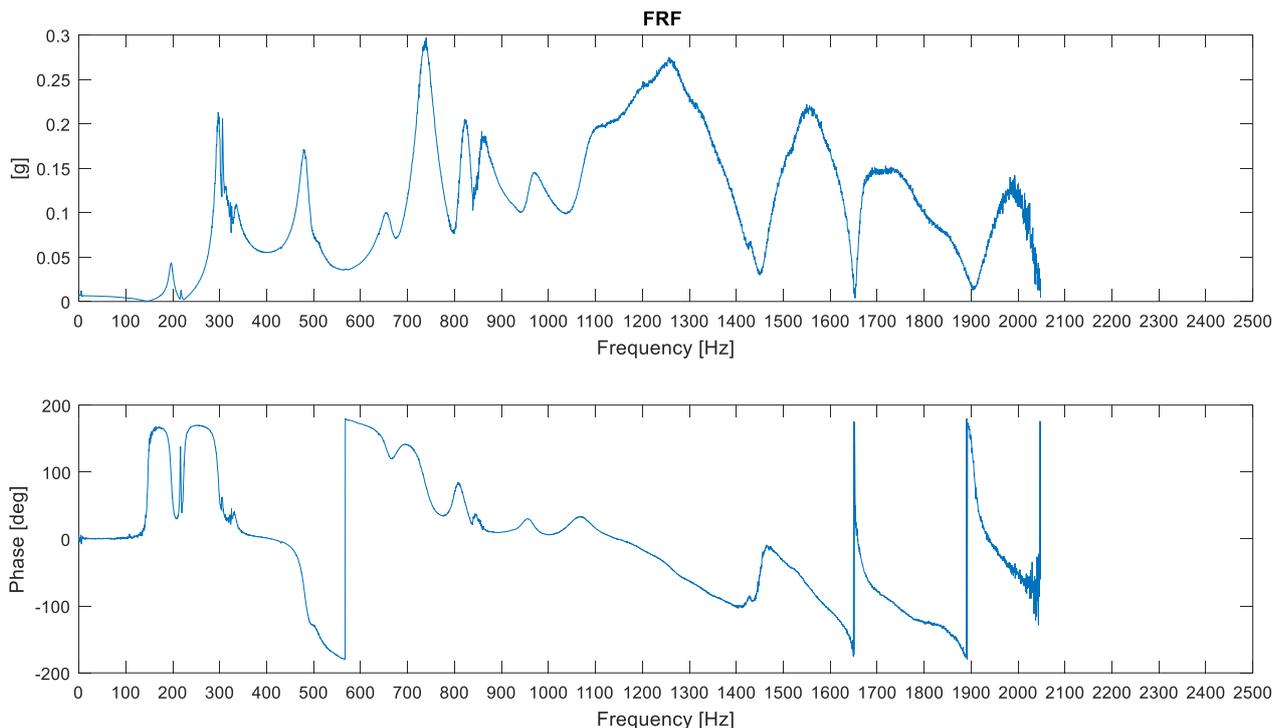


Figure 5: FRF of the plate in the normal condition

The time histories obtained for each case were filtered by an eighth-order Butterworth filter, setting the cut-off frequency at 630 Hz; in this way, the first four modes of the plate were considered.

The aim of this work was to try to use the AR parameters as a representation of the data and use them in combination with PCA to perform a damage detection despite the environmental changes, which usually mask damage. Therefore, the effort is to separate the effect of an abnormal conditions from the effect of the changes in temperature. Moreover, it is possible to understand how the transition of the polymer status influences the data. Another interesting point is to observe if the amplitude of the input gives a variation in the first two principal components of the AR model.

4 Results and discussion

In this section, the results provided by only one accelerometer are discussed. The accelerometer is the one placed in position 2, as shown in the Fig. 1. Normal condition is made by all the bolts screwed in and temperature at 20° C. During the tests, generally, it was difficult to reach a very precise temperature. Therefore, some tests related to the normal condition are conducted at 18° C and others at 22° C. In this way, it is possible to check if a small fluctuation around the nominal temperature makes a great difference in the results or not. The time histories acquired to check the restoration of normal condition between one test and the other are also considered in this work. Overall, the normal condition is represented by 280 time histories acquired in different trials. Nine time histories are considered for each temperature cluster. For a fixed nominal temperature, three different trials were performed in different stages of the test. Each trial consisted of using three different amplitude levels of the input. The same procedure was used for each investigated damage scenario and the mixed cases of temperature variation and damage. Nine time histories were considered for each different condition of the plate, in terms of damage and/or temperature.

The AR model order was checked using the Bayesian Information Criterion (BIC) and it was found to have the best order at 35 parameters. The parameters of the AR model are computed using the ordinary least square approach.

The first step was to normalise these parameters by removing the mean and dividing by the standard deviation of the parameters themselves. The PCA was applied to the normalised matrix and the results are plotted just along the first two principal components (PC) of the AR parameters. The main effect is the reduction of dimensions from 35 to 2. The variance associated with the first two principal components preserves 99% of that of the original multivariate system.

Since there are several combinations of different scenarios, it is important to analyse the data gradually. At first, only the damaged scenarios without any variation in temperature is considered, which is fixed around 20° C. Figure 6 shows the first two principal components of the AR parameters related to the normal and the damaged condition together. As explained in Section 3, the damaged scenarios are divided into 9 levels where each one corresponds to the number of removed bolts. Therefore, the first level of damage means only one bolt is removed, and so on until the ninth is removed.

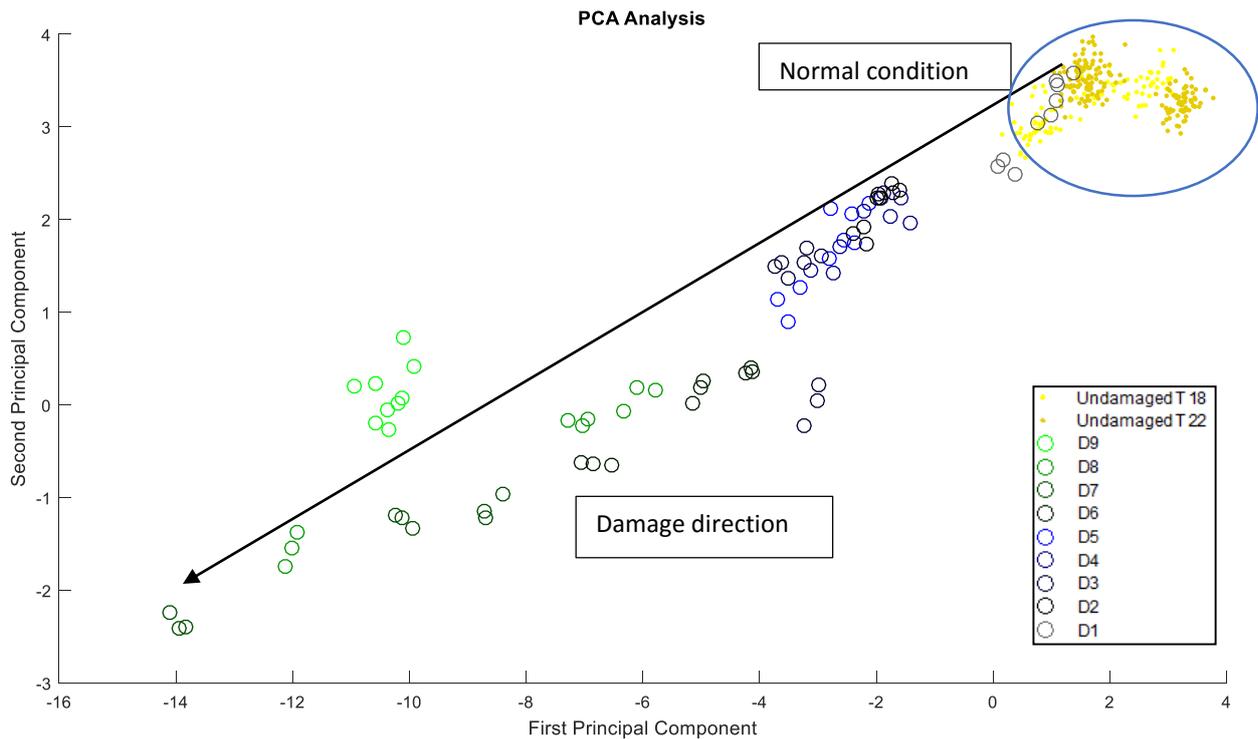


Figure 6: First two principal component scores of the AR parameters related to damage scenarios

As can be seen from the plot, the pattern of the principal component scores is almost linear among different scenarios. The lowest damage is not visible because it is overlapped with the normal condition, which is common when damage is too small to be separated from the operational variation. As the intensity of the damage increases, the principal component scores are far from the normal condition. The damage trend could be summarised as a linear increase monotonic from the normal condition to the 9th level of damage. It can be seen that the different amplitude levels of the input do not have a great influence on the data, because for each different condition the first two PC scores are very close to each other.

Small variations of the temperature have negligible effect on the normal behaviour of the structure. Looking at the data inside the blue circle, even if they are related to two different (but close) temperatures (18° C and 22° C), they are more or less in the same portion of the graph and are very well superimposed.

Then the effects of temperature can be taken into account; Fig. 7 shows the distribution of the first two principal component scores when all the bolts are fixed and temperature is free to change with a step of 10° C, from -20°C up to 80°C. These data represent the effect of temperature on the undamaged scenario.

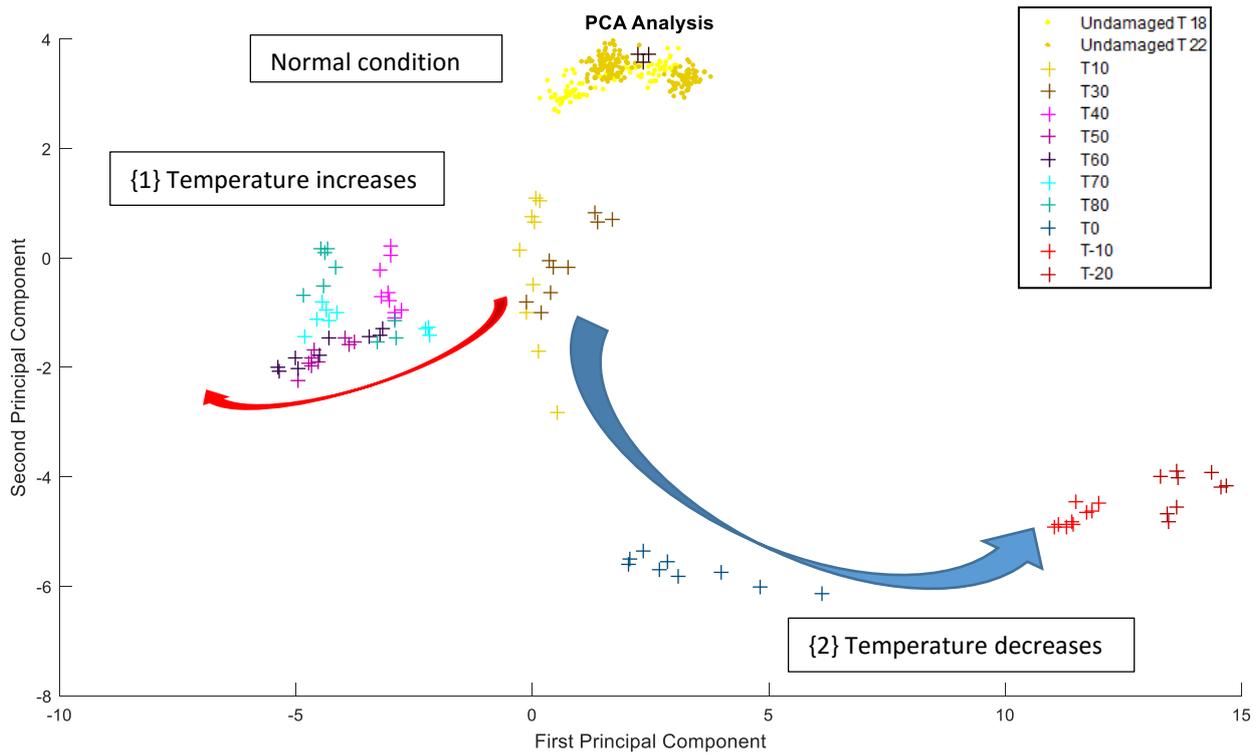


Figure 7: First two principal component scores of the AR parameters related to temperature variations

Fig.7 shows the changes of the system when the temperature variation is more significant and two trends can be identified, as described in Table 2.

ID Trend	Condition
{1}	Temperature increases
{2}	Temperature decreases

Table 2: Identified trends due to temperature variations

Both the identified trends are nonlinear. The main factor in the nonlinearity of the structure is related to the presence of the polymer layer. At different temperatures, the state of the polymer is different, causing it to shift its damping behaviour. Figure 8 shows the influence of temperature on the damping ratio of the polymer at frequencies of 200 Hz and 600 Hz. Between these two frequencies, there are the first four modes of the plate, and the damping behaviour of the plate is quite completely described by the Fig. 8.

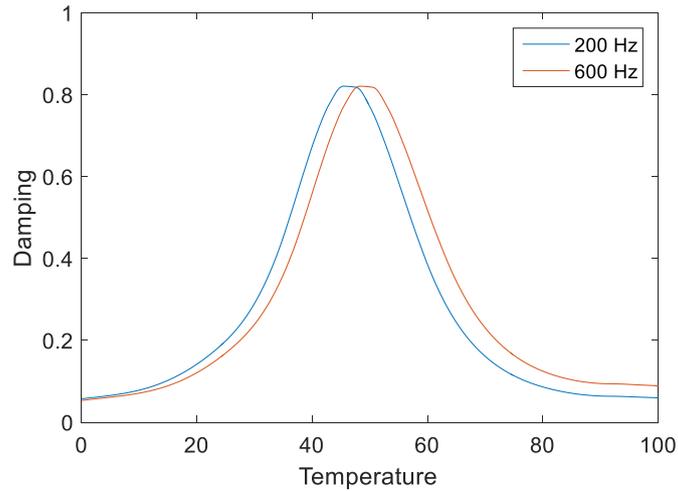


Figure 8: Damping ratio of the polymer

Generally, the temperature and the frequency greatly influence the damping ratio of a polymer. Here, Fig. 8 shows that, even if the frequency changes a lot, the temperature has a greater impact on the variation of the damping ratio of the polymeric material.

Here again, the effect of different input amplitudes is negligible. To make a complete overview of the results presented, the next figure (Fig. 9) reports the first two principal components related to all the possible combinations shown so far: both damaged scenarios and undamaged scenarios with temperature variations.

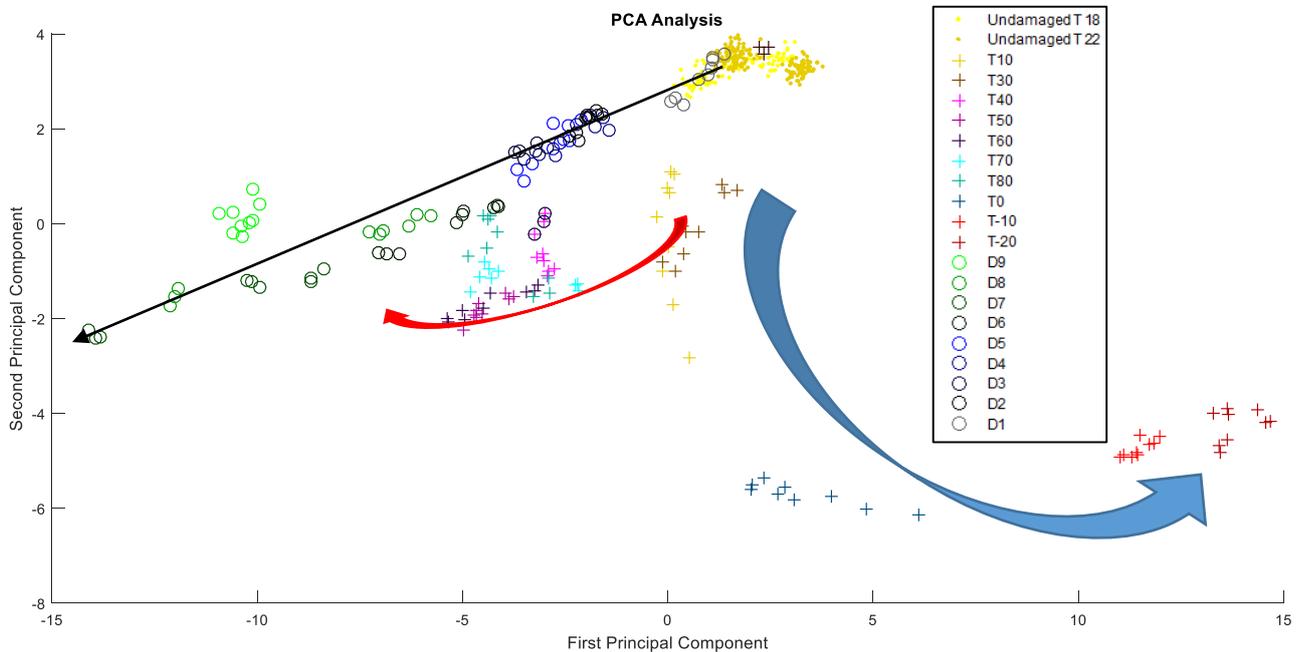


Figure 9: First two principal component scores of the AR parameters related to temperature variations and damage scenarios

It can be seen that damage and temperature have two different effects on the first two principal component scores of the AR parameters; so the variations in the data related to damage are well distinguished from those related to temperature.

The last stage to the analysis was to consider the joined effect of temperature and damage together. For the sake of simplicity, just the results of one level of damage are reported here. The results are analogous for different levels of damage. For instance, in Fig. 10 are superimposed, the normal condition, the temperature variation and the cases of temperature variation with a damage level of 3 (means 3 bolts are removed from the ribs).

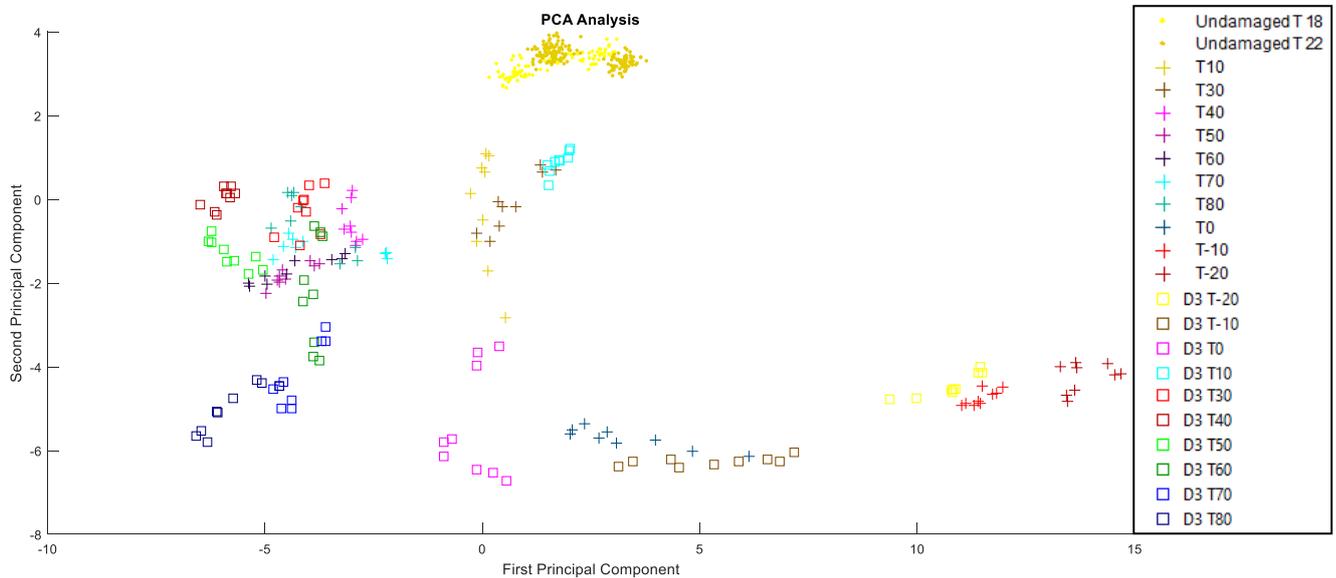


Figure 10: First two principal component of the AR parameters related to temperature variations and 3rd level of damage plus temperature variations

The data related to temperature variation superimposed on damage are mapped near the temperature scenarios without damage and not near the damage scenarios without temperature variation. Moreover, the disposition of the principal component scores are not linear. Therefore, in this case, the temperature variation has greater effect than the damage on the first two principal components.

5 Summary and conclusions

PCA has been used in the SHM field in the past. Here it is used in order to detect and discern damages in structures by analysing the vibration responses across several scenarios. The problem of vibration-based damage detection under varying environmental conditions was considered by employing data records from the healthy and damaged states of the given structure under various environmental conditions. The information from the state of the structure has been used to verify the quality of the classification. The approach proposed in this paper reveals that environmental variations can manifest themselves completely differently compared to fault observations. The next step of the work could be the analysis of the data related to the mixed case (i.e. the combination of damage and temperature variations at the same time) in more depth. An idea to deal with these cases is to consider also the third principal component and represent the data by the principal curves, e.g. as in Fig. 11.

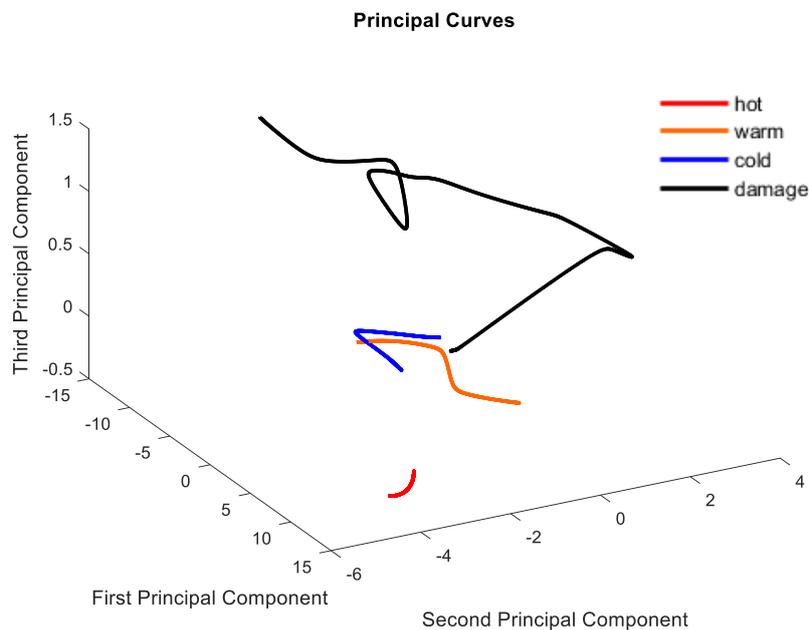


Figure 11: Principal curves of damage and temperature conditions

Detailed study of this aspect is left for a future work.

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