Effects of Inflation on Output Gap: A MRS-IV Approach

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In this paper, we propose an analytical framework to explore the level and volatility effects of inflation on the output gap. Using quarterly US data over 1977:q2-2009:q4, we examine the empirical implications of the model by implementing an instrumental variables Markov regime switching approach. We show that inflation uncertainty has a negative and regime dependent impact on the output gap but the level of inflation does not have any such effect. Our empirical investigation also provides evidence that the US economy was moving towards a period of turmoil before the recent financial crisis was imminent. The results are robust to the use of alternative measures of inflation uncertainty.

Keywords: Output gap; inflation uncertainty; Markov-switching modeling; instrumental variables; endogeneity.

JEL Classifications: E31, E32

1 Introduction

A major objective of a central bank is to achieve low and stable inflation. This is relevant because scarce resources can be channelled towards their best use in a stable economic environment.1 To that end Beaudry et al. (2001) argue that in periods of low inflation uncertainty firm managers can direct funds towards high return projects as they can forecast

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1 See, for instance, Friedman (1977).
relative prices of goods and services more accurately.\textsuperscript{2} It is also well acknowledged that during periods of high uncertainty, external funds become prohibitively expensive as a result of asymmetric information, causing managers to delay or cancel fixed investment projects. Lower investment, in return, hinders output growth. However, a careful examination of the empirical literature yields rather mixed results on the linkages between inflation uncertainty and economic growth. While some studies provide evidence that inflation uncertainty has a negative impact on economic growth some others show that the effect can be positive or insignificant.

Sifting through the literature regarding the role of inflation uncertainty on output one can observe that a vast majority of the empirical studies examine reduced form models that are based on empirical regularities rather than an analytical framework. One would also observe that empirical results tend to depend on the choice of inflation uncertainty measure which could be based on the standard deviation of a series, or on survey data, or on a variant of ARCH/GARCH methodology. In general, those studies that use a standard deviation based measure fail to provide a significant link between inflation uncertainty and the economic activity.\textsuperscript{3} Empirical studies that use a survey based measure tend to support the view that an increase in inflation uncertainty dampens the economic growth.\textsuperscript{4} Similarly, researchers using a variant of ARCH/GARCH methodology conclude that inflation uncertainty exerts a negative impact on output growth.\textsuperscript{5}

However, each uncertainty measure is criticized on various grounds. For instance, Cukierman and Wachtel (1979) and Cukierman (1983) argue that survey based inflation uncertainty measures display high correlation with the actual standard deviation of inflation. Jansen (1989) and Grier and Perry (2000) argue against the use of an inflation uncertainty measure based on the standard deviation approach stating that this approach induces a positive bias. Despite its attractiveness, uncertainty measures obtained from ARCH/GARCH models are also criticized on the grounds that these models may not be appropriate to construct an uncertainty measure if the underlying macroeconomic and financial series exhibit structural breaks. In such cases, it has been shown that the standard GARCH models may overstate the persistence in the conditional variance.\textsuperscript{6}

\textsuperscript{2} A deep literature examines the linkages between inflation and price stability (variability). For instance see Becker and Nautz (2012), and Caglayan et al. (2008) and the references therein.

\textsuperscript{3} See Barro (1996) and Clark (1997).


\textsuperscript{5} For instance see Grier et al. (2004), Fountas et al. 2006) and Mallik and Chowdury (2011).

In this paper, different from the literature, we first propose an analytical framework to explore the level and volatility effects of inflation on output gap assuming that central banks have asymmetric preferences over the business cycle. Next, we test the model’s prediction using quarterly US data covering the period between 1977:q2–2009:q4. In constructing the analytical model, we assume that the central bank uses a linear exponential (linex) loss function to entertain the possibility that the policy maker (i.e. the central bank) weighs positive and negative deviations of inflation and the output gap from their respective targets differently. This framework shows that both the level and the volatility of inflation affect the output gap.

In estimating the level and volatility effects of inflation on the output gap, we implement a two stage approach. In the first stage, to generate a measure of inflation uncertainty, we use a Markov switching GARCH model as suggested in Gray (1996). In the second stage, we implement a Markov regime switching instrumental variables approach to overcome the endogeneity between the output gap and inflation while we examine the impact of inflation uncertainty on the output gap. This strategy overcomes several issues that were raised against the earlier studies including those on structural breaks in the underlying series (inflation, output gap), generated regressor and endogeneity problems.

Our results can be summarized as follows. We show that inflation uncertainty has a regime dependent impact on the output gap. In particular, we show that the impact of an increase in inflation uncertainty is negative and significant during periods of high output gap volatility, and negative but not significant during periods of low output gap volatility. We also provide evidence that changes in inflation level do not have a significant impact on the output gap. More interestingly, we show that the US economy was moving towards a period of turmoil long before the approaching 2007/08 financial crisis. This finding suggests that the period of great moderation has come to an end. We check the robustness of our findings by using several additional measures of inflation uncertainty which were used in the empirical literature including those measures based on i) a GARCH(1,1) model, ii) the standard deviation of inflation, and iii) survey data.

The paper is organized as follows. Section 2 lays out our theoretical framework which will guide us in our empirical investigation. Section 3 provides information about the data and the empirical methodology. Section 4 discusses our findings while Section 5 concludes the study.

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8 To guard against the generated regressor problem, some researchers, see for example Wilson (2006), use a bivariate GARCH model. However, Harvey et al. (1994) argue that this approach is subject to identification problem and the results are difficult to interpret.
2 The Model

In this section we present a model to establish a link between the output gap and the level and volatility of inflation. To derive this relationship, we assume that the central bank has an asymmetric loss function with respect to inflation where output gap and inflation are subject to regime changes. In developing the model, we describe the dynamics of the supply and demand curves using Svensson (1997). Here, the state dependent inflation and output gap equations take the following form:

\[ \pi_{t+1} = \pi_t + a_t(S_t) y_t + \sigma^{\pi}_{(S_t)} u_{t+1} \]  
\[ y_{t+1} = \beta_1(S_t) y_t + \beta_2(S_t)(i_t - \pi_t) + \sigma^y_{(S_t)} \varepsilon_{t+1} \]

with \( u_{t+1} \sim i.i.d. N(0,1) \) and \( \varepsilon_{t+1} \sim i.i.d. N(0,1) \)

where \( \pi_t \) denotes inflation at time \( t \), \( y_t \) is the output gap, \( i_t \) is the nominal interest rate, \( u_t \) and \( \varepsilon_t \) denote supply and demand shocks, respectively. Note that all the parameters of the model including the variances associated with equations (1) and (2) are state dependent. Here, \( S_j, j \in \{1,2,...,N\} \) depicts a vector of unobserved-state variable which follows a Markov process. The transition between regimes is characterized by a transition probability matrix \( P = p_{jk} \) with \( j,k =1,2,...,N \). Note that the elements of \( p_{jk} \) gives the transition probability that regime \( j \) will be followed by regime \( k \) and the sum of each column of \( P \) is equal to unity.

Defining the probability that the unobserved state at time \( t \) is in regime \( j \) given the available information, \( \Psi_{t-1} \) as \( \hat{\xi}_{t|t-1} = P(S_t = j|\Psi_{t-1}) \), it follows that \( E(\hat{\xi}_{t|t-1} | \xi_t, \Psi_t) = P \hat{\xi}_{t|t} \). For a given starting value \( \hat{\xi}_{1|1} \), Hamilton (1989) shows that an optimal estimate of the unknown state probability can be derived by iterating the following two equations

\[ \hat{\xi}_{t+1|t} = \frac{\hat{\xi}_{t|t-1} \odot \kappa_t}{1^{\prime} (\hat{\xi}_{t|t-1} \odot \kappa_t)} \] 

(3)

\[ \hat{\xi}_{t+1|t} = P \hat{\xi}_{t|t} \] 

(4)

where \( \kappa_t = f(\pi_t|S_t = j, \Psi_{t-1}, \theta) \), \( \theta = \left[a_t(S_t), \beta_1(S_t), \beta_2(S_t), \sigma^{\pi}_{(S_t)}, \sigma^y_{(S_t)} \right] \) and the operator \( \odot \) represents the Hadamard (element-by-element) product.

An important aspect of the monetary policy is that the central bank chooses the interest rate before observing the demand and supply shocks based on the information which is available at the end of the previous period. This is captured by the intertemporal loss function.
where $\delta$ is the discount factor. Here, following Nobay and Peel (2003), Ruge-Murcia (2003) and Surico (2007; 2008), we assume that the central bank has an asymmetric lineex loss function, $L_t(\pi)$, with respect to inflation

$$E_t, L(\pi_{t+1}) = E_t, \exp[\mu(\pi_{t+1} - \pi^*)] - \mu E_t, (\pi_{t+1} - \pi^*) - 1$$

where $\mu$ is the asymmetry parameter. When this parameter is greater than zero, $\mu > 0$, positive deviations from the inflation target will be biting more than negative deviations. This is so because the exponential component $\exp[\mu(\pi_{t+1} - \pi^*)]$ will rule over the linear component. In this case the central bank will be more concerned about inflation exceeding the set target level $\pi^*$ since the cost of high inflation exceeds that of low inflation. Thus, positive deviations from the inflation target will dominate over negative deviations. When this parameter is less than zero, $\mu < 0$, the converse is true. In case $\mu = 0$ the loss function becomes quadratic.

Note that the right-hand side of (1) shows that $\pi_{t+1}$ is state-dependent. Thus, using the conditional normality of $u_t$ and (4) we can write equation (6) as

$$L_t = \frac{\hat{\epsilon}_t}{\sigma^2} E_t, \exp[\mu(\pi_{t+1} - \pi^*)] - \mu \frac{\hat{\epsilon}_t}{\sigma^2} E_t, \pi_{t+1} + \mu \pi^* - 1$$

After taking the expected value of (7) and using the result that, if $\pi_{t+1} - N(\pi_{t+1}, \sigma^2)$ then $E_t, \exp(\mu \pi_{t+1}) = \exp(\mu \pi_{t+1} + \mu^2 \sigma^2 / 2)$, one obtains:

$$L_t = \frac{\hat{\epsilon}_t}{\sigma^2} \exp[\mu(\pi_{t+1} - \pi^*) + \mu^2 \sigma^2 / 2] - \mu \frac{\hat{\epsilon}_t}{\sigma^2} \exp \pi_{t+1} + \mu \pi^* - 1$$

Substituting (1) into (8), we can rewrite it as:

$$L_t = \frac{\hat{\epsilon}_t}{\sigma^2} \exp \left[ \mu \pi_t + a_t(S_t) y_t, -\pi^* \right] + \mu^2 \sigma^2 / 2 - \mu \frac{\hat{\epsilon}_t}{\sigma^2} \exp \pi_{t+1} + \mu \pi^* - 1$$

Taking the first order condition of (9) with respect to $\pi_t$ and organizing, we can show that:
Finally, taking the logarithm of (10) and solving the resulting equation with respect to the output gap we arrive at:

\[ y_t = \left( \frac{1}{\alpha_t(S_t)} \right) \left( \pi_t^* - \pi_t - \frac{\mu^2}{2} \sigma_{\pi(S_t)}^2 \right) \]  (11)

This equation implies that the output gap is negatively related to the first and the second moments of inflation while the size of their impact is regime dependent. That is both the level and volatility of inflation exerts a negative impact on the output gap whose size depends on the regime.

3 Data and Econometric Methodology

3.1 Data

In our empirical investigation, we use quarterly consumer price index (CPI) and GDP for the United States. Data are obtained from the International Financial Statistics of the International Monetary Fund and span the period 1977:q2–2009:q4. \(^9\) We measure the output gap using the Hodrick-Prescott filter with a smoothing parameter 1600. Researchers have used various filters to compute output gap series and all filters have various shortcomings (see Mitchell et al. (2003)). In our study we use the Hodrick-Prescott filter as it is widely used by researchers (see, among others, Martin and Milas (2013), Lanne and Luoto (2014)) and it is shown that it outperforms other filters (see, Nilsson and Gyomai (2011)). \(^10\)

We compute the inflation rate \(\pi_t\) as the first difference of the log of consumer price index \(\pi_t = \log(CPI_t / CPI_{t-1})\). We check for the presence of GARCH effects in the inflation series by applying the Lagrange Multiplier test. This test reveals significant GARCH effects. We then estimate a GARCH(1,1) model for inflation where the conditional variance

\(^9\) Our analysis starts as of 1977 with the start of the University of Michigan inflation expectation (MICH) series with which we examine the robustness of our findings. Our sample period ends in 2009 as we aim to examine the effects of inflation and inflation uncertainty on the output gap up to the period right after the recent financial crisis. As the last trough point for the US economy is 2009:q2 and the stance of monetary policy not change after our sample ended we do not expect that our empirical results would change considerably in the case of extension of the data.

follows $h_t = \alpha_0 + \alpha_1 e_t^2 + \alpha_2 h_{t-1}$. As the sum of ARCH coefficients and GARCH terms $(\alpha_1 + \alpha_2)$ from this model are very close to one, we suspect that the effects of past shocks on current variance is very strong; i.e. the persistence of volatility shocks is strong.\(^{11}\) Hence, we model inflation and inflation uncertainty implementing the generalized regime switching (GRS) GARCH model introduced by Gray (1996). In doing so we model the conditional mean and the conditional variance of the inflation process allowing the series to switch between high- and low-inflation regimes. This model is superior to the standard (G)ARCH model for its GARCH term can capture the persistence parsimoniously as it takes into account the regime shifts in the series. The model is estimated using the maximum likelihood methodology. The details are available from the authors upon request.

### 3.2 Empirical Implementation

We examine the impact of inflation uncertainty on the output gap by augmenting equation (11) with several lags of the dependent variable and inflation. We do so to guard against the possibility of misspecification of demand and supply curves given by equations (1) and (2).\(^{12}\)

The specification for our baseline model takes the following form:

$$y_t = \phi + \sum_{j=1}^{m} \beta_{j} y_{t-j} + \sum_{j=0}^{l} \varphi_{j} \pi_{t-j} + \delta_{k} \sigma_{\pi} + \epsilon_t$$

$$\epsilon_t | \Psi_{t-1} \sim N(0, \sigma^2_{\epsilon_t}) \text{ for } k = 1, 2 \text{ regimes when } S_t = k$$

where $y_t$ is the output gap at time $t$ and $\pi_t$ and $\sigma_{\pi}$ denotes the level and volatility of inflation, respectively.

Given that the above model and its counterparts in the literature contain a proxy for inflation uncertainty, one must guard against the bias that would be introduced on the estimated coefficients and the standard deviation due to measurement errors. According to Pagan (1984) although such problems can be accounted for by using an instrumental variables approach, the use of lagged observations as instruments may not be possible when an endogenous variable is a function of the entire history of the available data. Under such circumstances, Pagan and Ullah (1988) suggest testing the validity of the underlying assumptions of the model that generates the proxy. For instance, Ruge-Murcia (2003) follows

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\(^{11}\) Lamoureux and Lastrapes (1990) and Gray (1996) point out that the high volatility persistence may be due to regime shifts in the conditional variance.

\(^{12}\) For example, Canova (2007) shows that an omitted variable will induce highly autocorrelated residuals.
these suggestions and uses lagged conditional volatility of unemployment obtained from a GARCH(1,1) model as an own instrument after checking for any remaining heteroscedasticity in the standardized residuals. Here, we too follow a similar route. We generate our inflation volatility measure implementing the GRS-GARCH(1,1) model and check whether the model is well specified and whether there is any neglected heteroscedasticity. We then use the lags of this proxy as an instrument when we investigate the impact of inflation uncertainty on the output gap.\(^{13}\)

It is also possible that the inflation uncertainty measure used in equation (12) could respond to an exogenous shock to inflation or to the output gap where the causation between inflation uncertainty and the output gap is not totally clear. This is so because a negative demand or supply shock would increase uncertainty, reduce output while the behavior of the level of inflation depends on the type of the shock. So an unobservable shock can increase the correlation between the output gap and inflation uncertainty due to the presence of endogeneity between inflation uncertainty, inflation and output. Although, the lags of a proper inflation uncertainty proxy can be used as an own instrument there is still the endogeneity problem between output and inflation which one has to account for. In this context, Kim (2004) and Spagnolo et al. (2005) note that the maximum likelihood estimation of a Markov-switching model based on the Hamilton filter yields inconsistent parameter estimates in the presence of endogenous variables. These two studies get around the endogeneity problem by implementing a two-step model. In the first step both studies use an instrumenting equation to generate a proxy of the endogenous variables and in the second step they estimate a Markov switching model using the proxy generated from the first stage.\(^{14}\) We follow an approach similar to that in Spagnolo et al. (2005) which we explain below.

### 3.3 MRS with Instrumental Variables

This section presents an instrumental variable Markov regime switching model to overcome the endogeneity between the output gap and the level and the volatility of inflation where the reduced-form equations for the endogenous regressors also have state-dependent parameters. In particular, we estimate the following system of equations for the output gap and the instrumenting equation for inflation:

\(^{13}\) We also use a GARCH(1,1) model to generate a second measure of uncertainty to check for the robustness of our findings. The same principles are applied prior to proceeding with the estimation.

\(^{14}\) Kim (2004) uses a linear OLS regression to model the endogenous variables while Spagnolo et al. (2005) allow the instrumenting equation to have state-dependent parameters.
\[ y_t = \phi_k + \sum_{p=1}^{m} \beta_{pk} y_{t-p} + \sum_{p=0}^{l} \varphi_{pk} \hat{\pi}_{t-p} + \delta_k \hat{\sigma}_\pi + \epsilon_t \]  \hspace{1cm} (13) 

\[ \pi_t = \rho_k + \sum_{p=1}^{l} \alpha_{pk} y_{t-p} + \sum_{p=1}^{N} \eta_{pk} \pi_{t-p} + u_t \]  \hspace{1cm} (14) 

where \( k = 1, 2 \) indicates the state and \( \epsilon_t \mid \Psi_{t-1} \sim N\left(0, \sigma^2_{\epsilon_t}\right) \) and \( u_t \mid \Psi_{t-1} \sim N\left(0, \sigma^2_{u_t}\right) \).

The first equation models the output gap \((y_t)\), and the second equation models the inflation \((\pi_t)\) while the coefficients of all explanatory variables depend upon the state of the economy. The output gap equation includes the lagged dependent variable, inflation rate, \( \hat{\pi}_t \), and the inflation uncertainty proxy, \( \hat{\sigma}_\pi \). Here, \( \hat{\pi}_t = E[\pi_t \mid S_t, \psi_t] \) is the fitted value of inflation rate obtained from equation (14) where \( S_t \) is the state variable and \( \psi_t \) is the information set available at time \( t \). Equation (14) is a reduced-form model for the endogenous regressor, \( \pi_t \), which is assumed to respond asymmetrically as a function of the lagged output and lagged dependent variable. The state variable, \( S_t \), is a homogenous first order Markov process with the transition probabilities \( Q = Pr[S_t = 1 \mid S_{t-1} = 1] \) and \( P = Pr[S_t = 2 \mid S_{t-1} = 1] \).

To estimate such a model within the Markov regime switching framework while using lags of inflation and output gap as instruments, Spagnolo et al. (2005) suggest implementing a recursive algorithm as in Hamilton (1994). This process yields a likelihood function which can be maximized with respect to \( \nu = (\phi_k, \beta_{pk}, \varphi_{pk}, \delta_k, \rho_k, \alpha_{pk}, \eta_{pk}) \). The conditional probability density function of the data \( w_t = (y_t, \pi_t) \) given the state \( S_t \) and the history of the system can be written as follows:

\[ pdf \left( w_t \mid w_{t-1}, \ldots, w_1; \nu \right) = \frac{1}{\sqrt{2\pi\sigma_{\epsilon_k}}} \exp \left[-\frac{1}{2} \left( \frac{y_t - \phi_k - \sum_{p=1}^{m} \beta_{pk} y_{t-p} - \sum_{p=0}^{l} \varphi_{pk} \hat{\pi}_{t-p} - \delta_k \hat{\sigma}_\pi}{\sigma_{\epsilon}} \right)^2 \right] \]

\[ \times \frac{1}{\sqrt{2\pi\sigma_{u_k}}} \exp \left[-\frac{1}{2} \left( \frac{\pi_t - \rho_k - \sum_{p=1}^{l} \alpha_{pk} y_{t-p} - \sum_{p=1}^{N} \eta_{pk} \pi_{t-p}}{\sigma_{u}} \right)^2 \right] \]  \hspace{1cm} (15)

Here \( \hat{\pi}_t = \rho_k + \sum_{p=1}^{l} \alpha_{pk} y_{t-p} + \sum_{p=1}^{N} \eta_{pk} \pi_{t-p} \) is the state-dependent instrumenting equation for \( \pi_t \) where parameters are estimated from equation (14).
4 Empirical Findings

In this section, we present our empirical observations obtained from the system of equations (13) and (14). Our main set of results are based on the uncertainty measure constructed using Gray’s (1996) model. To establish the robustness of our results, we then present three additional sets of results based on alternative uncertainty measures. These additional uncertainty measures are based on a GARCH(1,1) model, forecasters’ survey and rolling standard deviation of inflation. The state of the economy in each model is determined by the underlying volatility of output gap ($\sigma_{\gamma_k}$, where $k = 1, 2$). Thus, state 1 (2) is defined as the low (high) volatility period since $\sigma_{\gamma_1}$ is smaller than $\sigma_{\gamma_2}$. Overall, the results are similar and can be summarized as follows.

Inflation uncertainty has a regime dependent impact on the output gap. We observe a negative and significant impact of inflation uncertainty on the output gap during the high volatility regime while its impact is insignificant during the low volatility regime. The level of inflation does not play a significant role in either state of the economy. We should note that in all cases the average state dependent conditional output gap is zero and insignificant for all models. This finding is verified by the simple descriptive statistics of output gap computed for each state.15 Interestingly, the model provides evidence that the US economy will go through a period of high volatility much earlier than the approaching financial crises.

4.1 Main Results

Table 1 presents the results for the system of equations (13) and (14) where we use an inflation uncertainty measure computed from a Markov switching GARCH model. Based on state dependent volatility estimates ($\sigma_{\gamma_1} < \sigma_{\gamma_2}$), state 1 is identified as the low volatility regime and state 2 is identified as the high volatility regime. Estimates of the transition probabilities $Q$ and $P$ imply that there is high persistence for both regimes but the low volatility regime is more persistent than the high volatility regime as expected.

Results shown in Table 1 suggest that inflation uncertainty has a negative and significant impact ($\delta_1$) on output gap during the high volatility regime while this effect ($\delta_2$) is positive but insignificant during the low volatility regime. Given the point estimates, our model suggests that a one percentage point increase in inflation uncertainty during a high volatility regime leads to a reduction of 0.325 percentage point in output gap. This is a substantial effect. However, inspecting the impact of inflation on the output gap, we find that its effect is insignificant in both regimes ($\phi_{\delta1}$ and $\phi_{\delta2}$). This is expected as the policies implemented by the FED accords well with Taylor rule.

15 The sample average of output gap is 0.00046. The average output gap during high and low volatility states are also very close to zero (-0.00033 and 0.0028, respectively). None of these figures are found to be significantly different from zero.
Table 1: Estimates of Parameters of the Model for Output Gap and Inflation (based on an MRS GARCH(1,1) uncertainty measure)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
</tr>
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<tr>
<td>$\phi_1$</td>
<td>-0.003</td>
<td>0.002</td>
<td>$\rho_1$</td>
<td>0.007</td>
<td>*** 0.001</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>0.900</td>
<td>*** 0.106</td>
<td>$\alpha_{11}$</td>
<td>0.945</td>
<td>*** 0.197</td>
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<tr>
<td>$\beta_{21}$</td>
<td>0.118</td>
<td>0.141</td>
<td>$\alpha_{21}$</td>
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<td>0.260</td>
</tr>
<tr>
<td>$\beta_{31}$</td>
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<td>** 0.159</td>
<td>$\alpha_{31}$</td>
<td>0.192</td>
<td>0.251</td>
</tr>
<tr>
<td>$\beta_{41}$</td>
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<td>$\alpha_{41}$</td>
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<tr>
<td>$\varphi_{01}$</td>
<td>0.396</td>
<td>0.253</td>
<td>$\eta_{11}$</td>
<td>0.326</td>
<td>*** 0.076</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.016</td>
<td>0.052</td>
<td>$\eta_{21}$</td>
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<td>** 0.071</td>
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<td>$\phi_2$</td>
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<td>$\beta_{12}$</td>
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<td>*** 0.267</td>
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<tr>
<td>$\delta_2$</td>
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<td>** 0.160</td>
<td>$\eta_{22}$</td>
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<td>*** 0.000</td>
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<td>$Q$</td>
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<td>$P$</td>
<td>0.951</td>
<td>*** 0.041</td>
</tr>
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</table>

Log likelihood = 990.370

Notes: The estimates on the left hand side of the table are for the model

$$y_t = \phi_k + \sum_{p=1}^m \beta_{pk} y_{t-p} + \sum_{p=0}^l \varphi_{pk} \hat{\pi}_{t-p} + \delta_k \hat{\sigma}_\pi + \epsilon_t.$$  The estimates on the right hand side of the table are for the model

$$\pi_t = \rho_k + \sum_{p=1}^l \alpha_{pk} y_{t-p} + \sum_{p=1}^n \eta_{pk} \pi_{t-p} + u_t,$$

where $\epsilon_t \sim N(0, \sigma_{\epsilon_k}^2)$ and $u_t \sim N(0, \sigma^2_{\pi_k})$. Regimes are indexed by $k = 1, 2$. Significance at the 10%, 5% and 1% are denoted by *, **, ***.
The results on the impact of inflation uncertainty on output gap make sense. In state 1, when the volatility of output gap is low, monetary policy authorities can achieve low and stable inflation as they can effectively control inflation. Because in periods of low inflation its volatility will also be lower, one would not expect to observe a significant impact of inflation volatility on output gap \((\delta_1)\). In contrast, during periods of high output gap volatility the central bank will be uncertain about the impact of monetary policy on the economic activity. As a consequence, the changes in monetary policy may be carried out gradually. In such circumstances small interest rate changes will accommodate rather than fighting inflation leading to higher inflation volatility, which in return affects output gap negatively \((\delta_2)\). This is consistent with the estimates of inflation equation presented in the second column of each table where the inflation volatility is higher in the high volatility regime \((i.e., \sigma_{u_2} > \sigma_{u_1})\).\(^{16}\)

Figure 1 plots the filter probabilities of state 1 (low volatility regime). The shaded areas in the Figure depict the recessions acknowledged by the NBER for our sample. We observe that, except for the 2001 recession, the model successfully captures the economic downturns announced by the NBER. It is worth noting that the filter probability drops to low levels between the second quarter of 1978 and the third quarter of 1983. In this period the US economy experienced a deep recession as the FED adopted a new monetary policy framework to fight inflation which is documented by a large number of studies. For example, Clarida et al. (2000) and Lubik and Shorfeide (2004) demonstrate that the way monetary policy was conducted changed significantly with the appointment of Paul Volcker as the FED Chairman at the third quarter of 1979. Providing further support for the above argument, Bernanke and Mihov (1998) show that the operating procedures of the FED shifted during the period 1979:q4-1982:q3 from Federal Funds rate to nonborrowed reserves targeting.

The filter probability drops down to low levels for a second time in the fourth quarter of 2006, long before the 2007/2008 financial crisis, and stays low until the end of the third quarter of 2009. This observation and the information in Figure 2 show that inflation volatility is on the rise as of 1999 onwards reaching unprecedented levels between 2006-2009 signal the approaching rough economic conditions ahead. This evidence along with the fact that the policy makers have been implementing similar policies throughout the period before the financial crises suggest that the period of great moderation may be a result of good-luck rather than good-policy.\(^{17}\) In Figure 2 we also observe that during the periods of high inflation and

\(^{16}\) The implications of our results are consistent with Ball’s (1992) model where uncertainty about the type of central bank leads to high inflation volatility.

\(^{17}\) See for instance including Clarida et al. (2000) and Benati and Surico (2009) who argue in favor of good policy versus Stock and Watson (2002), Sims and Zha (2006) and Gambetti et al. (2008) who support the good-luck hypothesis.
inflation uncertainty the output growth has a dramatic downward trend, while inflation and its volatility are likely to move together.

Figure 1: The Filter Probabilities of State 1- Low Volatility Regime

Figure 2: Output Gap, Inflation and Inflation Uncertainty
4.2 Robustness

To check for the robustness of the results that we present in Table 1, we estimate three additional sets of models while we proxy inflation uncertainty based on i) a GARCH(1,1) model, ii) the standard deviation of inflation and iii) the standard deviation of inflation forecasts obtained from the University of Michigan’s monthly survey data on inflation expectations. Table 2 gives the results when inflation uncertainty is measured by the conditional variance of inflation obtained from a GARCH(1,1) model. Table 3 provides results for the case when we use the standard deviation of the inflation forecasts based on the survey data. Table 4 presents the results for the case of the standard deviation of inflation series.

As noted for our earlier Tables, for all three cases, state 1 is identified as the low output gap volatility regime and state 2 is identified as the high output gap volatility regime. In each table, the point estimate of the impact of inflation uncertainty is always negative and statistically significant in state 2, the high output gap volatility regime. The filter probabilities associated with each proxy, which are available upon request from the authors, are similar to that provided in Figure 1. What is more interesting is that in all cases we observe a drop in the filter probability in the last quarter of 2006 long before the financial crises in 2007/08. This observation suggests that the US economy is entering a period of high volatility before the start of the 2007/08 financial crises. Our findings which are common across all uncertainty proxies show that the results are robust.

5 Conclusions

In this paper we examine the impact of inflation uncertainty on output using US quarterly data. Our contribution to the literature is twofold. We, first, provide an analytical framework to guide us in our empirical investigation. We, then, empirically examine regime dependent effects of inflation as well as inflation uncertainty on the output gap. The investigation is carried out using quarterly US data over the period 1977:q2–2009:q4.

In our empirical investigation we follow a two stage modeling approach. In the first stage we construct an inflation uncertainty proxy using a Markov switching GARCH model as suggested in Gray (1996). In the second stage, we use a Markov switching instrumental variables approach to estimate the impact of inflation uncertainty on output while the economy transits between high and low volatility regimes. This strategy provides a basis where we examine whether inflation uncertainty has regime dependent impact on the output gap. In doing so, we also overcome the endogeneity problem between output and inflation by instrumenting the endogenous variables. We check for the robustness of our findings using three additional inflation uncertainty proxies obtained from a GARCH(1,1) model, forecasters’ survey and rolling standard deviation of inflation.
Table 2: Estimates of Parameters of the Model for Output Gap and Inflation (based on a GARCH(1,1) uncertainty measure)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>-0.003</td>
<td>0.003</td>
<td>$\rho_1$</td>
<td>0.007</td>
<td><strong>0.001</strong></td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>0.900 ***</td>
<td>0.106</td>
<td>$\alpha_{11}$</td>
<td>0.041</td>
<td>0.112</td>
</tr>
<tr>
<td>$\beta_{21}$</td>
<td>0.122</td>
<td>0.132</td>
<td>$\alpha_{21}$</td>
<td>0.123</td>
<td>0.129</td>
</tr>
<tr>
<td>$\beta_{31}$</td>
<td>-0.407 ***</td>
<td>0.133</td>
<td>$\alpha_{31}$</td>
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<td>0.162</td>
</tr>
<tr>
<td>$\beta_{41}$</td>
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<td>0.094</td>
<td>$\alpha_{41}$</td>
<td>-0.009</td>
<td>0.135</td>
</tr>
<tr>
<td>$\varphi_{01}$</td>
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<td>0.324</td>
<td>$\eta_{11}$</td>
<td>0.326 ***</td>
<td>0.079</td>
</tr>
<tr>
<td>$\delta_1$</td>
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<td>0.156</td>
<td>$\eta_{21}$</td>
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<td>0.073</td>
</tr>
<tr>
<td>$\phi_2$</td>
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<td>$\rho_2$</td>
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<td>0.005</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>0.942 ***</td>
<td>0.197</td>
<td>$\alpha_{12}$</td>
<td>0.142</td>
<td>0.223</td>
</tr>
<tr>
<td>$\beta_{22}$</td>
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<td>$\alpha_{22}$</td>
<td>-0.256</td>
<td>0.300</td>
</tr>
<tr>
<td>$\beta_{32}$</td>
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<td>0.276</td>
<td>$\alpha_{32}$</td>
<td>0.274</td>
<td>0.289</td>
</tr>
<tr>
<td>$\beta_{42}$</td>
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<td>0.240</td>
<td>$\alpha_{42}$</td>
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<td>0.262</td>
</tr>
<tr>
<td>$\varphi_{02}$</td>
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<td>0.280</td>
<td>$\eta_{12}$</td>
<td>0.751 **</td>
<td>0.322</td>
</tr>
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<td>$\delta_2$</td>
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<td>0.336</td>
<td>$\eta_{22}$</td>
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<tr>
<td>$\sigma_{e1}$</td>
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<td>0.000</td>
<td>$\sigma_{u1}$</td>
<td>0.003 ***</td>
<td>0.000</td>
</tr>
<tr>
<td>$\sigma_{e2}$</td>
<td>0.010 ***</td>
<td>0.001</td>
<td>$\sigma_{u2}$</td>
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<td>0.001</td>
</tr>
<tr>
<td>$Q$</td>
<td>0.980 ***</td>
<td>0.015</td>
<td>$P$</td>
<td>0.951 ***</td>
<td>0.041</td>
</tr>
</tbody>
</table>

Log likelihood = 990.320

Notes: The estimates on the left hand side of the table are for the model

$$y_t = \phi + \sum_{p=1}^{\alpha} \beta_{pk} y_{t-p} + \sum_{p=0}^{\alpha} \varphi_{pk} \hat{\pi}_{t-p} + \delta_k \hat{\sigma}_{\pi} + \epsilon_t.$$  

The estimates on the right hand side of the table are for the model

$$\pi_t = \rho + \sum_{p=0}^{\alpha} \alpha_{pk} y_{t-p} + \sum_{p=0}^{\alpha} \eta_{pk} \pi_{t-p} + \epsilon_t,$$

where $\epsilon_t \mid \Psi_{t-1} \sim N(0, \sigma_{\epsilon}^2)$ and $\psi_t \mid \Psi_{t-1} \sim N(0, \sigma_{\psi}^2)$. Regimes are indexed by $k = 1,2$. Significance at the 10%, 5% and 1% are denoted by *, **, ***.
Table 3: Estimates of Parameters of the Model for Output Gap and Inflation (based on survey data)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>-0.003</td>
<td>0.002</td>
<td>$\rho_1$</td>
<td>0.007 ***</td>
<td>0.001</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
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<td>0.105</td>
<td>$\alpha_{11}$</td>
<td>0.035</td>
<td>0.087</td>
</tr>
<tr>
<td>$\beta_{21}$</td>
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<td>$\alpha_{21}$</td>
<td>0.131</td>
<td>0.122</td>
</tr>
<tr>
<td>$\beta_{31}$</td>
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<td>0.132</td>
<td>$\alpha_{31}$</td>
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<td>0.092</td>
</tr>
<tr>
<td>$\beta_{41}$</td>
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<td>0.087</td>
<td>$\alpha_{41}$</td>
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<td>0.048</td>
</tr>
<tr>
<td>$\varphi_{01}$</td>
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<td>0.252</td>
<td>$\eta_{11}$</td>
<td>0.329 ***</td>
<td>0.076</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.040</td>
<td>0.497</td>
<td>$\eta_{21}$</td>
<td>-0.185 ***</td>
<td>0.070</td>
</tr>
<tr>
<td>$\phi_2$</td>
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<td>0.004</td>
<td>$\rho_2$</td>
<td>0.004</td>
<td>0.004</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
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<td>0.192</td>
<td>$\alpha_{12}$</td>
<td>0.145</td>
<td>0.206</td>
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<tr>
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<td>0.256</td>
<td>$\alpha_{22}$</td>
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<tr>
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<td>0.251</td>
<td>$\alpha_{32}$</td>
<td>0.274</td>
<td>0.286</td>
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<tr>
<td>$\beta_{42}$</td>
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<td>0.212</td>
<td>$\alpha_{42}$</td>
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<td>0.245</td>
</tr>
<tr>
<td>$\varphi_{02}$</td>
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<td>$\eta_{12}$</td>
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<td>0.268</td>
</tr>
<tr>
<td>$\delta_2$</td>
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<td>0.703</td>
<td>$\eta_{22}$</td>
<td>-0.084</td>
<td>0.262</td>
</tr>
<tr>
<td>$\sigma_{\epsilon 1}$</td>
<td>0.004 ***</td>
<td>0.000</td>
<td>$\sigma_{u1}$</td>
<td>0.003 ***</td>
<td>0.000</td>
</tr>
<tr>
<td>$\sigma_{\epsilon 2}$</td>
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<td>0.001</td>
<td>$\sigma_{u2}$</td>
<td>0.011 ***</td>
<td>0.001</td>
</tr>
<tr>
<td>Q</td>
<td>0.980 ***</td>
<td>0.015</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>0.952 ***</td>
<td>0.040</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Log likelihood = 990.050

Notes: The estimates on the left hand side of the table are for the model

$$y_t = \phi_k + \sum_{p=1}^n \beta_{p,k}y_{t-p} + \sum_{p=0}^l \varphi_{p,k}\hat{r}_{t-p} + \delta_k\hat{\sigma}_{\pi} + \epsilon_t.$$  

The estimates on the right hand side of the table are for the model

$$\pi_t = \rho_k + \sum_{p=1}^L \alpha_{p,k}y_{t-p} + \sum_{p=1}^N \eta_{p,k}\pi_{t-p} + u_t,$$

where

$$\epsilon_t | \Psi_{t-1} \sim N\left(0, \sigma_{\epsilon_t}^2\right) \text{ and } u_t | \Psi_{t-1} \sim N\left(0, \sigma_{u_t}^2\right).$$

Regimes are indexed by $k = 1, 2$. Significance at the 10%, 5% and 1% are denoted by *, **, ***.
Table 4: Estimates of Parameters of the Model for Output Gap and Inflation (based on a standard deviation uncertainty measure)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>-0.003</td>
<td>0.002</td>
<td>$\rho_1$</td>
<td>0.007***</td>
<td>0.001</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
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<td>0.195</td>
<td>$\alpha_{11}$</td>
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<td>0.408</td>
</tr>
<tr>
<td>$\beta_{21}$</td>
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<td>0.150</td>
<td>$\alpha_{21}$</td>
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<td>0.374</td>
</tr>
<tr>
<td>$\beta_{31}$</td>
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<td>0.201</td>
<td>$\alpha_{31}$</td>
<td>-0.098</td>
<td>0.187</td>
</tr>
<tr>
<td>$\beta_{41}$</td>
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<td>0.156</td>
<td>$\alpha_{41}$</td>
<td>-0.009</td>
<td>0.267</td>
</tr>
<tr>
<td>$\varphi_{01}$</td>
<td>0.403</td>
<td>0.372</td>
<td>$\eta_{11}$</td>
<td>0.320**</td>
<td>0.158</td>
</tr>
<tr>
<td>$\delta_1$</td>
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<td>0.830</td>
<td>$\eta_{21}$</td>
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<td>0.090</td>
</tr>
<tr>
<td>$\phi_2$</td>
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<td>0.012</td>
<td>$\rho_2$</td>
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<td>0.005</td>
</tr>
<tr>
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<td>0.320</td>
<td>$\alpha_{12}$</td>
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</tr>
<tr>
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<tr>
<td>$\beta_{32}$</td>
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<td>0.283</td>
<td>$\alpha_{32}$</td>
<td>0.280</td>
<td>0.281</td>
</tr>
<tr>
<td>$\beta_{42}$</td>
<td>-0.273</td>
<td>0.245</td>
<td>$\alpha_{42}$</td>
<td>-0.168</td>
<td>0.297</td>
</tr>
<tr>
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<td>$\sigma_{e1}$</td>
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<td>0.000</td>
<td>$\sigma_{u1}$</td>
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<td>0.000</td>
</tr>
<tr>
<td>$\sigma_{e2}$</td>
<td>0.010***</td>
<td>0.001</td>
<td>$\sigma_{u2}$</td>
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<tr>
<td>$Q$</td>
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<td>0.018</td>
<td>$P$</td>
<td>0.947***</td>
<td>0.046</td>
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</tbody>
</table>

Log likelihood = 990.430

Notes: The estimates on the left hand side of the table are for the model

$$y_t = \phi_k + \sum_{p=1}^{n} \beta_{pk} y_{t-p} + \sum_{p=0}^{l} \varphi_{p} \hat{\pi}_{t-p} + \delta_x \sigma_x + \epsilon_t.$$  

The estimates on the right hand side of the table are for the model

$$\pi_t = \rho_k + \sum_{p=1}^{l} \alpha_{pk} y_{t-p} + \sum_{p=0}^{N} \eta_{p} \pi_{t-p} + u_t,$$

where

$$\epsilon_t | \Psi_{t-1} \sim N(0, \sigma_{e_k}^2)$$

and

$$u_t | \Psi_{t-1} \sim N(0, \sigma_u^2).$$

Regimes are indexed by $k = 1, 2$. Significance at the 10%, 5% and 1% are denoted by *, **, ***.
Results from all four proxies yield similar observations and can be summarized as follows. Inflation uncertainty has a regime dependent impact on output gap: inflation uncertainty has a significant negative impact on output gap during the high volatility regime, but no significant effect during the low volatility regime. The level of inflation does not have a significant impact on output gap in either states. Last but not the least we show that the US economy was heading towards a period of high volatility long before the onset of the recent financial crisis. This last finding when evaluated along with the increasing inflation uncertainty in the US as of 1999 suggests that the period of great moderation may be a result of good-luck rather than good-policy as the policy makers have been implementing similar policies throughout the period before the financial crisis which closely match the so called Taylor principle. More research along this line is warranted.

References


