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Robust MMSE Beamforming for Multiantenna Relay Networks

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Abstract

In this paper, we propose a robust minimum mean square error (MMSE) based beamforming technique for multiantenna relay broadcast channels, where a multi-antenna base station transmits signal to single antenna users with the help of a multiantenna relay. The signal transmission from the base station to the single antenna users is completed in two time slots, where the relay receives the signal from the base station in the first time slot and it then forwards the received signal to different users based on amplify and forward (AF) protocol. We propose a robust beamforming technique for sum-power minimization problem with imperfect channel state information (CSI) between the relay and the users. This robust scheme is developed based on the worst-case optimization framework and Nemirovski Lemma by incorporating uncertainties in the CSI. The original optimization problem is divided into three subproblems due to joint non-convexity in terms of beamforming vectors at the base station, the relay amplification matrix, and receiver coefficients. These subproblems are formulated into a convex optimization framework by exploiting Nemirovski Lemma, and an iterative algorithm is developed by alternatively optimizing each of them with channel uncertainties. In addition, we provide an optimization framework to evaluate the achievable worst-case mean square error (MSE) of each user for a given set of design parameters. Simulation results are provided to validate the convergence of the proposed algorithm.

Index Terms

Relay network; beamforming; AF relay; multiple-input-multiple-output; quality of service

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I. INTRODUCTION

The energy and spectral efficiencies are two important goals in the design of wireless networks. Wireless relay is considered to be an enabling technology for achieving these goals. For example, in Long-Term Evolution (LTE) Advanced systems, incorporation of relays has been proposed to increase data rate at the cell edges by improving the received signal-to-interference plus noise ratio (SINR) [1]. These relays enhance the quality of the wireless links influenced by multipath fading, shadowing, and path loss. Hence, relays have the potential to support the required quality of services (QoSs) at the destination by mitigating co-channel interference and improving the reliability of the links between the sources and destinations, while facilitating a better frequency re-usage and lower energy consumption [2].

A significant amount of research has been focused on amplify and forward (AF) based relay networks due to the benefits of their low complexity, less processing time, and easy implementation [3]- [9]. In [3], an optimal relay matrix design was proposed for an AF based single user MIMO multiantenna relay network. In an AF multihop relay network, a sum-rate duality was established between broadcast channel and multiple access channel with total power constraint and individual power constraints in [4], [5]. In [6], SINR based uplink-downlink duality was derived for a multihop AF based MIMO relay network. In [10], the majorization theory was exploited for the design of linear AF relay and source precoding matrices in an MIMO relay network. Later, a low complexity based linear and non-linear transceiver designs were proposed in [7], [11], [12]. Moreover, relay matrix design and power allocation techniques based on QoS requirements were investigated for an AF two-hop MIMO relay network in [11], [13]. In [14], beamforming vectors and relay amplification matrix were designed for a multiantenna relay broadcast channel to satisfy SINR target for each user. In [32], linear beamforming design has been considered for both uplink and downlink scenarios in AF relaying cellular networks based on minimum mean square error (MMSE) criteria, where novel iterative algorithms have been proposed to reveal their relationships with the existing conventional MIMO or multiuser beamforming designs. A unified framework for linear MMSE transceiver design has been presented for multiple MIMO system based on quadratic matrix programming in [31]. In [15], different algorithms were developed to solve MMSE based designs by exploiting convex optimization techniques and general mean square error (MSE) duality. Recently, a unified approach has been developed for precoder, equalizer, training sequences and radar waveform design in MIMO system based on matrix-monotonic optimization framework in [33], where the original matrix-variable optimization problems are simplified into vector variables ones based on the optimal structure.

A. Related work: MSE based Robust Designs

In most existing works, the required of QoS was satisfied for each user through AF based relays. This scenario may arise in a network having users with delay intolerant real-time services (real-time users), where the required QoS should be provided all the time. In order to provide the required QoSs, it is necessary to have the perfect channel state information (CSI) in a node, where optimization is implemented. In these QoS based designs, the perfect CSI was assumed in the optimizing node. In general, it is difficult to have the perfect CSI at the design nodes due to the channel estimation and quantization errors. Moreover, the designs implemented without considering the uncertainties will not be able to provide the required QoS for each user. To deal with these uncertainties, robust optimization is a well known approach, where the uncertainties are incorporated into the designs [16]–[20]. In [34], a joint robust design of linear relay forwarding matrix and equalizer at the destination is proposed based on Bayesian framework for dual-hop AF MIMO-OFDM relay systems, whereas robust design with Gaussian random channel uncertainties is investigated for dual-hop AF MIMO relay system in [35]. Then, this robust approach has been extended with Tomlinson-Harashima precoding for multi-hop AF MIMO relay systems in [30]. On the other hand, a general robust linear transceiver design has been proposed for multi-hop AF MIMO relaying system in [29], where different MSE based designs are unified into one matrix-variate optimization framework through majorization theory and matrix-monotone functions. In [36], an iterative algorithm based on alternate optimization approach has been proposed for an AF MIMO relay channel with direct link. A statistically robust design has been presented for linear AF MIMO relays for two imperfect CSI scenarios in [37], whereas this approach has been extended for the same network with a direct link in [38]. In [39], a robust joint relay precoder and destination receive filters design has been considered for an AF relay network with two models of CSI error, namely, stochastic and norm-bounded errors. A robust linear beamforming design has been presented for a point to point MIMO relay system for the same uncertainty models in [41] and extended for norm bounded error model with direct link in [42]. In [43], a robust transceiver design has been investigated for downlink multiuser MIMO AF relay system, where sum MSE and transmit power minimization problems have been solved through iterative approach. Here, we extend our previous work on a multiantenna relay broadcast channel by incorporating the channel uncertainties between the multiantenna relay and users [15]. This robust scheme will ensure that the required QoSs will be provided to each user regardless of the errors associated with the channels.

In this paper, we consider a multiantenna relay network as shown in Fig. 1, where the base station equipped with multiple antennas communicates with a number of single antenna users through a multiantenna relay. In this network, the base station transmits signal to a multiantenna relay in the first time-slot and the relay then forwards the received signal to different users in the second time-slot based on an AF protocol. The perfect CSI between the base station and the relay is available at the relay where the optimization is implemented. However, the relay has the imperfect CSI between the relay and the users. In this scenario, we solve the following robust optimization problem to provide the required QoS for each user.

We first formulate the optimization problem to minimize the total transmission power at the base station and the relay to achieve a predefined MSE threshold for each user. In order to satisfy the required MSE of each user, the design should incorporate the associated channel uncertainties between the relay and the users. This optimization problem with the imperfect CSI is not jointly convex in terms beamformers, relay amplification matrix, and receiver coefficients. Hence, it is difficult to find the globally optimal solution by incorporating the channel uncertainties. In order to circumvent this non-convexity issue, we divide the original problem into three subproblems. We then formulate each subproblem into a convex optimization framework by exploiting Nemirovski Lemma and incorporating imperfect CSI. Based on these subproblems, we develop an iterative robust sum-power minimization algorithm to satisfy the required MSE of each user regardless of the errors associated with the channels, where each design variable is optimized while the rest of the two variables are fixed. This optimization framework is developed based on the worst-case MSE of the users. The same problem has been solved in [43]. However, authors were not aware of this work, when this problem is independently solved by exploiting Nemirovski Lemma [27] to incorporate channel uncertainties in our previous work [15]. In addition, to validate the robustness of the proposed algorithm for all set of possible errors, we propose an optimization approach to obtain the achievable worst-case MSE of each user for a given set of design parameters. This worst-case MSE evaluation ensures that the target MSE of each user is satisfied all the time regardless of the channel uncertainties that has not been considered in [43].

The remainder of the paper is outlined as follows. The system model is described in Section II. The robust sum-power minimization problem is solved in Section III. Section IV provides the simulation results to validate the performance of the proposed robust scheme, followed by the conclusions in Section V.

The major notations used in this paper are defined as follows. We use the upper case boldface letters for matrices and lower case boldface letters for vectors. $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ denote the transpose, conjugate, and conjugate transpose, respectively. Tr$(\cdot)$ and $\mathbb{E}\{\cdot\}$ stand for the trace of a matrix and the statistical expectation for a random variable. Vec$(\mathbf{A})$ is the vector obtained by stacking the columns of $\mathbf{A}$ on top of one another, and $\otimes$ is the Kronecker product. $\mathbf{A} \succeq \mathbf{0}$ indicates that $\mathbf{A}$ is a positive semidefinite matrix, and $\mathbf{A} \succeq \mathbf{B}$ represents $\mathbf{A}-\mathbf{B} \succeq \mathbf{0}$, i.e., $\mathbf{A}-\mathbf{B}$ is a positive semidefinite matrix. $\mathbf{I}$ and $(\cdot)^{-1}$ denote the identity matrix with an appropriate size and the inverse of a matrix, respectively. $\|\cdot\|_2$ represents the Euclidean norm of a matrix. $|\cdot|$ and $\Re\{\cdot\}$ stand for the absolute value and real part of a complex number, respectively. The notation $diag\{\cdot\}$ represents a vector that consists of the diagonal elements of a matrix or a diagonal matrix, where the diagonal elements are from a vector. The notation $\succeq_K$ denotes the following generalized inequality:

$$\begin{bmatrix} a \\ \mathbf{b} \end{bmatrix} \succeq_K 0 \Longleftrightarrow \|\mathbf{b}\|_2 \leq a.$$

II. SYSTEM MODEL

Let us consider a multiantenna relay network as shown in Fig. 1, where a base station communicates with K single antenna users through a multiantenna relay. It is assumed that the base station and the relay are equipped with N_T and N_R antennas, respectively. In a multiantenna AF relay network, the maximum number of users to support is limited to $\min\{N_T, N_R\}$ and therefore the number of users are assumed to be $K \leq \min\{N_T, N_R\}$ [44]. The transmitted signal from the base station in the first time slot can be written as

$$\mathbf{x} = \sum_{k=1}^{K} \tilde{\mathbf{w}}_k s_k, \tag{1}$$

where $\tilde{\mathbf{w}}_k$ and $s_k(\mathbb{E}\{|s_k|^2\} = 1)$ are the beamforming vector and the transmitted symbol for the k^{th} user, respectively. The required transmission power at the base station is

$$P_T = \mathrm{Tr}\big\{\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\big\}, \tag{2}$$

where $\tilde{\mathbf{W}} = [\tilde{\mathbf{w}}_1 \cdots \tilde{\mathbf{w}}_K]$. The received signal at the relay can be written as

$$\mathbf{y}_r = \mathbf{H}_0\mathbf{x} + \mathbf{n}_r, \tag{3}$$

where $\mathbf{H}_0 \in \mathbb{C}^{N_R \times N_T}$ is the MIMO channel from the base station to the relay, and $\mathbf{n}_r$ is the noise vector at the relay with zero-mean and covariance matrix $\sigma^2\mathbf{I}$. The relay forwards the received signal to the users based on AF technique in the second time slot. The power consumed at the relay is

$$\begin{aligned} P_R &= \mathrm{Tr}\big\{\mathbb{E}\big\{\mathbf{x}_r\mathbf{x}_r^H\big\}\big\} \\ &= \mathrm{Tr}\big\{\mathbf{F}\big(\mathbf{H}_0\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\mathbf{H}_0^H + \sigma^2\mathbf{I}\big)\mathbf{F}^H\big\}, \end{aligned} \tag{4}$$

where $\mathbf{x}_r$ is the transmitted signal from the relay, and it can be written as

$$\mathbf{x}_r = \mathbf{F}\mathbf{y}_r, \tag{5}$$

where $\mathbf{F} \in \mathbb{C}^{N_R \times N_R}$ is the relay amplification matrix. The total transmission power at the base station and relay can be written as

$$P_T + P_R = \mathrm{Tr}\big\{\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\big\} + \mathrm{Tr}\big\{\mathbf{F}\big(\mathbf{H}_0\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\mathbf{H}_0^H + \sigma^2\mathbf{I}\big)\mathbf{F}^H\big\} \tag{6}$$

$$= \mathrm{Tr}\big\{\tilde{\mathbf{W}}^H\left(\mathbf{I} + \mathbf{H}_0^H\mathbf{F}^H\mathbf{F}\mathbf{H}_0\right)\tilde{\mathbf{W}}\big\} + \sigma^2\mathrm{Tr}\big\{\mathbf{F}\mathbf{F}^H\big\} \tag{7}$$

$$= \sum_{i=1}^{K} \big\|(\mathbf{H}_0^H\mathbf{F}^H\mathbf{F}\mathbf{H}_0 + \mathbf{I})^{1/2}\tilde{\mathbf{w}}_i\big\|_2^2 + \sigma^2\mathrm{Tr}\big\{\mathbf{F}\mathbf{F}^H\big\} \tag{8}$$

The MSE at the kth user can be written as

$$\begin{aligned}\varepsilon_k &= \mathbb{E}\big\{|\hat{s}_k - s_k|^2\big\} = \mathbb{E}\big\{\,(\hat{s}_k - s_k)\,(\hat{s}_k - s_k)^*\,\big\}\\ &= \Big[a_k\Big(\mathbf{h}_k^H\mathbf{F}\mathbf{H}_0\tilde{\mathbf{W}}\mathbf{s} + \mathbf{h}_k^H\mathbf{F}\mathbf{n}_r + n_k\Big) - s_k\Big]\Big[a_k\Big(\mathbf{h}_k^H\mathbf{F}\mathbf{H}_0\tilde{\mathbf{W}}\mathbf{s} + \mathbf{h}_k^H\mathbf{F}\mathbf{n}_r + n_k\Big) - s_k\Big]^* \\ &= 1 - 2\Re\big(a_k\mathbf{h}_k^H\mathbf{F}\mathbf{H}_0\tilde{\mathbf{w}}_k\big) + a_k\mathbf{h}_k^H\mathbf{F}\big(\mathbf{H}_0\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\mathbf{H}_0^H + \sigma^2\mathbf{I}\big)\mathbf{F}^H\mathbf{h}_k a_k^* + a_k a_k^*\sigma_k^2,\end{aligned} \tag{9}$$

where $\hat{s}_k$ and a_k are the estimated signal and the receiver coefficient at the kth user, respectively. $\mathbf{h}_k$ denotes the channel between the relay station and the kth user, whereas σ_k^2 is the noise variance at the kth user.

III. SUM-POWER MINIMIZATION WITH IMPERFECT CSI

In this section, the designs of the beamforming vectors at the base station, the relay amplification matrix and receiver coefficients are formulated into a sum-power minimization problem, where total transmission power required at the base station and the relay is minimized, while achieving the maximum tolerable MSE at each user. This sum-power minimization problem can be written as

$$\begin{aligned}&\min_{\tilde{\mathbf{W}},\mathbf{F},\mathbf{a}} \; P_T + P_R\\ &\text{s.t.} \quad \varepsilon_k \le \gamma_k, \quad k = 1, \cdots, K,\end{aligned} \tag{10}$$

where γ_k is the MSE threshold of the kth user, and $\mathbf{a}$ consists of all receiver coefficients, i.e., $\mathbf{a} = [a_1 \cdots a_K]^T$. The sum-power minimization problem defined in (10) is not jointly convex in terms of $\tilde{\mathbf{W}}, \mathbf{F}, \mathbf{a}$. Hence, it is difficult to find a globally optimal solution for this problem. Therefore, the original problem in (10) is divided into two subproblems, where the beamforming vectors and the relay amplification matrix are successively optimized using convex optimization techniques in [15].

In [15], it was assumed that the multiantenna relay has the perfect CSI between the relay and the users. However, there are practical difficulties to have these CSI at the transmitter due to the channel estimation and quantization errors. Therefore, we consider a robust optimization approach based on the worst-case MSE of the user, where the channel uncertainties between the multiantenna relay and the users are taken into account in the optimization framework, since the optimization is performed at the multiantenna relay. Note that this robust design can be extended to incorporate the channel uncertainties between the base station and multiantenna relay by ignoring the high order error terms (i.e., the second order error terms). However, the robust approach developed in this work incorporates only the channel uncertainties between the multiantenna relay and the users. The required QoS of the users might not be satisfied all the time due to these channel uncertainties. This scenario could arise in a network, where the users employ delay intolerant real-time services (real-time users), and these users' MSEs should not exceed certain thresholds all the time. Hence, the uncertainties in the channels should be incorporated into the optimization framework to satisfy the require QoS. In this work, we show that these robust optimization problems can be divided into subproblems, which can be formulated into a convex optimization framework by incorporating the channel uncertainties.

A. *Channel Uncertainty Model*

Here, we model the imperfect CSI based on the deterministic models [16], [18], [19]. The actual channels between the relay and the single-antenna users can be modeled as follows:

$$\mathbf{h}_k = \bar{\mathbf{h}}_k + \boldsymbol{\Delta}_k, \quad k = 1, \cdots, K, \tag{11}$$

where $\mathbf{h}_k$, $\bar{\mathbf{h}}_k$, and $\boldsymbol{\Delta}_k$ are the true channel, the channel with the error at the relay, and the error associated with the channel of the kth user, respectively. It is assumed that the Euclidean norms of channel errors are bounded by a set of thresholds, which are available at the relay. These channel errors can be expressed as

$$\begin{aligned} &\|\boldsymbol{\Delta}_k\|_2 \leq \rho_k, \quad k = 1, \cdots, K, \\ &\mathfrak{B}_k = \left\{\mathbf{h}_k \;\middle|\; \|\mathbf{h}_k - \bar{\mathbf{h}}_k\| \leq \rho_k\right\}, \;\; k = 1, \cdots, K, \end{aligned} \tag{12}$$

where ρ_k is the bounded threshold of the channel of the kth user from the relay, and $\mathfrak{B}_k$ is the set containing all possible sets of channel errors. The robust optimization techniques based on these channel error models have been studied in [18], [19], [21]–[24].

B. *Robust Sum-Power Minimization*

In this subsection, the robust sum-power minimization problem is formulated to satisfy the required MSEs of all users by incorporating the channel uncertainties between the multiantenna relay and the users. The MSE of the kth user can be written as

$$\varepsilon_k^{(e)} = 1 - 2\Re\left[a_k(\bar{\mathbf{h}}_k + \boldsymbol{\Delta}_k)^H \mathbf{F}\mathbf{H}_0\tilde{\mathbf{w}}_k\right] + a_k(\mathbf{h}_k + \boldsymbol{\Delta}_k)^H \mathbf{G}(\bar{\mathbf{h}}_k + \boldsymbol{\Delta}_k)a_k^* + a_k a_k^* \sigma_k^2, \tag{13}$$

where $\boldsymbol{\Delta}_k$ is the channel error associated with the channel $\mathbf{h}_k$, and

$$\mathbf{G} = \left[\mathbf{F}\mathbf{H}_0\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\mathbf{H}_0^H\mathbf{F}^H + \sigma^2\mathbf{F}\mathbf{F}^H\right]. \tag{14}$$

The robust sum-power minimization can be formulated by taking into account the channel uncertainties as

$$\begin{aligned}
&\min_{\tilde{\mathbf{W}},\mathbf{F},\mathbf{a}} \quad P_T + P_R, \\
&\text{s.t.} \quad \varepsilon_k^{(e)} \le \gamma_k, \quad k = 1, \cdots, K, \\
&\|\boldsymbol{\Delta}_k\|_2 \le \rho_k, \quad k = 1, \cdots, K,
\end{aligned} \tag{15}$$

where $\mathbf{a} = [a_1 \ a_2 \ \cdots \ a_K]$. The solution of the problem in (15) should satisfy the target MSEs (i.e., $\gamma_k, k = 1, \cdots, K$) of all users, regardless of the errors associated with the corresponding channels.

In addition, this optimization problem is not jointly convex in $\tilde{\mathbf{W}}, \mathbf{F}$, and $\mathbf{a}$ due to the non-convex objective function and the constraints. Hence, it is difficult to find a global optimal solution for this problem. In order to circumvent this non-convexity issue, the original problem in (15) is divided into three subproblems, where the beamforming vectors, the relay amplification matrix, and receiver coefficients are alternatively optimized, while the rest of the two design parameters are fixed. Note that the non-robust optimization framework developed in our previous work in [15] is divided into two subproblems by deriving the closed-form solution for the receiver coefficients. However, the closed-form expression of the receiver coefficients cannot be derived in the robust case with the channel uncertainties, which leads to the formulations of three subproblems in the robust approach. Through formulating these subproblems into convex optimization framework, an iterative algorithm is developed based on the worst-case MSE of each user. In addition, the proposed robust approach yields a sub-optimal solution by satisfying the required QoS at each user regardless of the channel uncertainties.

In order to incorporate channel uncertainties in the optimization framework, the following lemma is required.

Lemma 1: The constraints

$$\varepsilon_k^{(e)} \leq \gamma_k, \quad \|\mathbf{\Delta}_k\|_2 \leq \rho_k, \tag{16}$$

can be formulated into the following constraints as

$$\begin{bmatrix} \mathbf{D}_k - \mu_k \mathbf{C}_k^H \mathbf{C}_k & -\rho_k \mathbf{B}_k^H \\ -\rho_k \mathbf{B}_k & \mu_k \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \quad \mu_k \geq 0, \tag{17}$$

where we have

$$\begin{aligned} \mathbf{B}_k &= \left[\mathbf{0}_{N_R \times 1} \ \mathbf{F}\mathbf{A}_k^H \ \mathbf{0}_{N_R \times 1}\right], \\ \mathbf{C}_k &= \left[-1 \ \mathbf{0}_{1\times(K+N_R+1)}^H\right], \end{aligned} \tag{18}$$

$$\mathbf{D}_k = \begin{bmatrix} \gamma_k & \left[\tilde{\mathbf{h}}_k^H \mathbf{F}\mathbf{A}_k^H \ \sigma_k a_k\right] - \mathbf{e}_k^H \\ \left[\begin{pmatrix} \mathbf{A}_k \mathbf{F}^H \tilde{\mathbf{h}}_k \\ \sigma_k a_k \end{pmatrix}\right] - \mathbf{e}_k & \mathbf{I} \end{bmatrix}, \tag{19}$$

$\mathbf{D}_k \in \mathbb{C}^{(K+N_R+2)\times(K+N_R+2)}$ and

$$\mathbf{A}_k = \begin{bmatrix} a_k^* \tilde{\mathbf{W}}^H \mathbf{H}_0^H \\ \sigma a_k^* \mathbf{I} \end{bmatrix}_{(K+N_R)\times 1}. \tag{20}$$

Proof: Please refer to Appendix A. ■

C. Robust Beamforming Design at Base Station

Here, we present a convex optimization framework to design beamformers at the base station for fixed relay amplification matrix and receiver coefficients. For a fixed relay amplification matrix $\mathbf{F}$, the beamforming design to satisfy the target MSEs can be formulated by incorporating the uncertainties in the channels and dropping the constant term $\sigma^2 \mathbf{F}\mathbf{F}^H$ from the total transmission power in (8) as follows:

$$\begin{aligned} \min_{\tilde{\mathbf{W}}} \ & \sum_{i=1}^{K} \left\|(\mathbf{H}_0^H \mathbf{F}^H \mathbf{F} \mathbf{H}_0 + \mathbf{I})^{1/2} \tilde{\mathbf{w}}_i\right\|_2^2, \\ \text{s.t.} \ & \varepsilon_k^{(e)} \leq \gamma_k, \quad k = 1, \cdots, K, \\ & \|\mathbf{\Delta}_k\|_2 \leq \rho_k, \quad k = 1, \cdots, K. \end{aligned} \tag{21}$$

This optimization problem cannot be directly solved using convex optimization techniques. Hence, we reformulate this problem into a convex optimization framework by exploiting Nemirovski Lemma [25], [26].

Lemma 2: The original problem in (21) can be formulated into the following convex problem:

$$\begin{aligned}
&\min_{t_1,\cdots,t_K,\tilde{\mathbf{W}}} \sum_{i=1}^{K} t_i, \\
&\text{s.t.} \quad \begin{bmatrix} \sum_{i=1}^{K} t_i \\ [\|\mathbf{A}\tilde{\mathbf{w}}_1\|_2 \|\mathbf{A}\tilde{\mathbf{w}}_2\|_2 \cdots \|\mathbf{A}\tilde{\mathbf{w}}_K\|_2]^T \end{bmatrix} \succeq_K 0, \\
&\qquad \begin{bmatrix} \mathbf{D}_i - \mu_i \mathbf{C}_i^H \mathbf{C}_i & -\rho_i \mathbf{B}_i^H \\ -\rho_i \mathbf{B}_i & \mu_i \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \; i = 1, \cdots, K, \\
&\qquad t_i \geq 0, \quad i = 1, \cdots, K, \\
&\qquad \mu_i \geq 0, \quad i = 1, \cdots, K,
\end{aligned} \tag{22}$$

where

$$\mathbf{A} = \left[\mathbf{H}_0^H \mathbf{F}^H \mathbf{F} \mathbf{H}_0 + \mathbf{I}\right]^{1/2}. \tag{23}$$

Proof: Please refer to Appendix B. ■

The first and the second constraints in (22) represent the second order cone and semidefinite constraints, respectively, which define the convex sets in terms of optimization variables. Hence, the problem in (22) is convex.

The optimal beamformers obtained by solving the problem in (22) ensure that the target MSEs of all users are satisfied regardless of the channel uncertainties. This has been proved later by evaluating the achievable worst-case MSEs of all users for a given set of beamformers, relay amplification matrix, and receiver coefficients.

D. Robust Relay Matrix Design

The robust relay matrix design with fixed beamformers and receiver coefficients can be formulated by only considering the terms related to relay amplification matrix from the total power in (8) as

$$
\begin{aligned}
&\min_{\mathbf{F}} \ \mathrm{Tr}\{\mathbf{F}\mathbf{A}_0\mathbf{F}^H\}, \\
&\text{s.t.} \ \ 1 - 2\Re(a_k(\mathbf{h}_k + \mathbf{\Delta}_k)^H \mathbf{F}\mathbf{H}_0\tilde{\mathbf{w}}_k) + |a_k|^2(\mathbf{h}_k + \mathbf{\Delta}_k)^H \mathbf{F}\mathbf{A}_0\mathbf{F}^H(\mathbf{h}_k + \mathbf{\Delta}_k) \\
&\qquad + |a_k|^2\sigma_k^2 \le \gamma_k, \quad k = 1, \cdots, K. \\
&\qquad \|\mathbf{\Delta}_k\|_2 \le \rho_k, \quad k = 1, \cdots, K,
\end{aligned}
\tag{24}
$$

where

$$
\mathbf{A}_0 = \left[\mathbf{H}_0\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\mathbf{H}_0^H + \sigma^2\mathbf{I}\right]. \tag{25}
$$

The above problem can be formulated into a convex optimization framework by incorporating channel uncertainties.

Lemma 3: The original problem in (24) can be formulated into the following semidefinite programming framework through Nemirovski Lemma as follows:

$$
\begin{aligned}
&\min_{\mathbf{F}} \ \mathrm{Tr}\left[\mathbf{F}\mathbf{A}_0\mathbf{F}^H\right], \\
&\text{s.t.} \qquad \begin{bmatrix} \mathbf{D}_i - \mu_i\mathbf{C}_i^H\mathbf{C}_i & -\rho_i\mathbf{B}_i^H \\ -\rho_i\mathbf{B}_i & \mu_i\mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \quad i = 1, \cdots, K, \\
&\qquad \mu_i \ge 0, \quad i = 1, \cdots, K,
\end{aligned}
\tag{26}
$$

where $\mathbf{D}_i$, $\mathbf{C}_i$ and $\mathbf{B}_i$ are defined in (18) and (19).

Proof: Please refer to Appendix C. ■

The problem in (26) is convex and the robust relay amplification matrix can be obtained for a given set of beamformers and receiver coefficients.

E. Robust Receiver Coefficients Design

Here, we provide the problem formulation to obtain the optimal receiver coefficients for a given set of beamformers and relay amplification matrix. These robust receiver coefficients can be formulated into a convex optimization framework as in the robust beamformer and relay amplification matrix designs in (22) and (26).

$$\begin{aligned} \min_{t_0,\mathbf{a}} \quad & t_0, \\ \text{s.t.} \quad & \begin{bmatrix} \mathbf{D}_i - \mu_i \mathbf{C}_i^H \mathbf{C}_i & -\rho_i \mathbf{B}_i^H \\ -\rho_i \mathbf{B}_i & \mu_i \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \quad i = 1, \cdots, K, \\ & t_0 \geq 0, \quad \mu_i \geq 0, \quad i = 1, \cdots, K, \end{aligned} \tag{27}$$

Solving the above optimization problem, we see that the optimal receiver coefficients can be obtained for a fixed set of beamformers and relay amplification matrix.

An iterative algorithm is developed in Algorithm I by alternatively optimizing the beamformers, relay amplification matrix, and receiver coefficients. This algorithm ensures that the target MSEs of all users are satisfied for all possible set of errors defined by error bounds. However, the proposed algorithm yields a suboptimal solution due to the formulations of the subproblems which do not jointly solve the design parameters of the original problem in (15).

F. Convergence Analysis

Here, we analyze the convergence of the proposed robust sum-power minimization algorithm. The convergence of the proposed algorithm can be proved by analyzing the solution of each subproblem as follows. As mentioned earlier, the original problem in (15) is divided into three subproblems, where each design parameter is optimized while the other two are fixed.

Lemma 4: Assume that the problem in (22) is feasible for a given set of relay amplification matrix and receiver coefficients. Then, the proposed robust sum-power minimization algorithm will converge to a solution.

Proof: At the nth iteration, let $p_0^{(n)}$, $\tilde{\mathbf{W}}^{(n)}$, $\mathbf{F}^{(n)}$, and $\mathbf{a}^{(n)}$ be the total transmission power, beamformers from (22), relay amplification matrix from (26), and receiver coefficients from (27), respectively. The robust beamforming design in (22) will be feasible at the $(n+1)$th iteration for the given $\mathbf{F}^{(n)}$ and $\mathbf{a}^{(n)}$, since the beamformers obtained at the nth iteration (i.e.,$\tilde{\mathbf{W}}^{(n)}$) is a feasible solution for the problem (22) at the $(n+1)$th iteration. At the $(n+1)$th iteration, the problems in (26) and (27) are feasible with the similar argument. In addition, the transmission power at the $(n+1)$th iteration will be less than or equal to that from the nth iteration (i.e., $p_0^{(n+1)} \leq p_0^{(n)}$). Therefore, the proposed algorithm will result in a monotonically decreasing total transmission power with the iterations as observed in Figs. 2 and 3. On the other hand, the transmission power required to achieve a set of MSEs at each user is obviously lower bounded by a certain value. Hence, the proposed algorithm will converge to a certain amount of transmission power. This completes the proof of the convergence of the proposed algorithm. ■

G. Complexity Analysis

The complexity of the proposed algorithm is analyzed by evaluating computational complexity of each subproblem based on the complexity of the interior point methods [26]. This complexity can be defined by quantifying the required arithmetic operations in the worst-case at each iteration and the required number of iterations to achieve the solutions with certain accuracy.

The original robust sum-power minimization problem is divided into three subproblems, namely, beamformer, relay amplification matrix, and receiver coefficients designs. The beamformer design at the base station is formulated into a convex problem with the second order cone and semidefinite constraints. In addition, this problem consists of $KN_T + 2K$ variables and K semidefinite, as well as a second order cone constraints. In general, interior point method will require $\mathcal{O}[\sqrt{KN_T}\log(\frac{1}{\epsilon})]$ iterations to converge with ϵ solution accuracy at the termination of the algorithm. Each iteration requires at most $\mathcal{O}[(KN_T)^3 + K^2N_T]$ arithmetic operations in the worst-case [16], [26]. Similarly, the robust relay matrix design problem consists of N_R^2 variables with K semidefinite constraints. Therefore, interior point method will require $\mathcal{O}[\sqrt{N_R}\log(\frac{1}{\epsilon})]$ iterations with ϵ solution accuracy and at most $\mathcal{O}[N_R^3 + KN_R^2]$ arithmetic operations in the worst-case. The robust receiver coefficients design will require $\mathcal{O}[\sqrt{K}\log(\frac{1}{\epsilon})]$ iterations and $\mathcal{O}[K^3 + K^2]$ arithmetic operations in the worst-case. However, the actual complexity will be far less than this worst-case bound. Similarly, the complexity of the other two subproblems can be defined by evaluating the associated arithmetic operations and the number of iterations.

H. Worst-Case MSE Calculation

In this subsection, we formulate an optimization framework to evaluate the worst-case MSE for a given set of beamformers, relay amplification matrix, and receiver filter coefficients. This worst-case MSE of each user should satisfy the target MSE. In addition, this will ensure that the achievable MSE for all possible channel errors will satisfy the target MSEs of all users. The

$$\mathbf{M}_k \triangleq \begin{bmatrix} \hat{\mu}_k \mathbf{I} - \hat{a}_k^* \mathbf{G} \hat{a}_k, & \hat{a}_k^* \hat{\mathbf{F}} \mathbf{H}_0 \hat{\mathbf{u}}_k - \hat{a}_k^* \mathbf{G}^H \tilde{\mathbf{h}}_k \hat{a}_k \\ \hat{a}_k \hat{\mathbf{u}}_k^H \mathbf{H}_0^H \hat{\mathbf{F}}^H - \hat{a}_k^* \tilde{\mathbf{h}}_k^H \mathbf{G} \hat{a}_k, & -\mu_k \rho_k - 1 + 2\Re\left(\hat{a}_k \tilde{\mathbf{h}}_k^H \hat{\mathbf{F}} \hat{\mathbf{H}}_0 \hat{\mathbf{u}}_k\right) - a_k^* \tilde{\mathbf{h}}_k^H \mathbf{G} \hat{a}_k + t_k - \sigma_k^2 |\hat{a}_k|^2 \end{bmatrix}, \tag{31}$$

where

$$\mathbf{G} \triangleq \left[\hat{\mathbf{F}} \mathbf{H}_0 \hat{\mathbf{U}} \hat{\mathbf{U}}^H \mathbf{H}_0^H \hat{\mathbf{F}}^H + \sigma^2 \hat{\mathbf{F}} \hat{\mathbf{F}}^H\right], \tag{32}$$

and $\hat{\mathbf{U}}, \hat{\mathbf{F}}, \hat{\mathbf{u}}_k$, and $\hat{a}_k$ denote the solutions of $\tilde{\mathbf{W}}, \mathbf{F}, \tilde{\mathbf{w}}_k$, and a_k obtained from the proposed algorithm, respectively.

---

worst-case MSE of the kth user can be written as

$$\varepsilon_k^{(w)} = \max_{\mathbf{h}_k \in \mathfrak{B}_k} \varepsilon_k, \tag{28}$$

where $\varepsilon_k^{(w)}$ represents the worst-case MSE of the kth user and

$$\mathfrak{B}_k = \left\{\mathbf{h}_k \middle| \|\mathbf{h}_k - \tilde{\mathbf{h}}_k\| \leq \rho_k\right\}. \tag{29}$$

Lemma 5: The worst-case MSE evaluation of the kth user can be formulated into the following semidefinite programming:

$$\begin{aligned} \varepsilon_k^{(w)} \triangleq & \min_{t_k, \mu_k} \ t_k, \\ \text{s.t.} \ \ & \mathbf{M}_k \succeq \mathbf{0}, \\ & t_k \geq 0, \quad \mu_k \geq 0, \end{aligned} \tag{30}$$

where $\mathbf{M}_k$ is defined in (31) at the top of the next page.

Proof: Please refer to Appendix D. ■

By evaluating the achievable worst-case MSE for a given set of design parameters, the robustness of the algorithm is validated for all possible set of channel errors.

IV. SIMULATION RESULTS

In order to verify the proposed robust algorithm, let us consider a multiantenna relay network, where a base station equipped with multiple antennas communicates with its users through a multiantenna relay. Both the base station and the relay use five antennas. There are three users, and each user is equipped with a single antenna. The channel coefficients between the base station and the relay as well as those between the relay and the users are assumed to be known at the relay, and they have been generated using zero-mean circularly symmetric independent and identically distributed complex Gaussian random variables. However, the channel coefficients between the multiantenna relay and the single antenna users have the uncertainties, which are norm bounded by predefined thresholds. In addition, these bounds are available at the relay where the optimization is implemented. The noise power at the user terminals and noise covariance at the relay are assumed to be 0.075 and $0.075\mathbf{I}$, respectively.

First, we study the convergence behavior of the algorithm through simulation results for different sets of channels. In order to evaluate the convergence of the proposed robust sum-power minimization algorithm, the MSE threshold of each user has been set to 0.15. Here, the relay amplification matrix and receiver coefficients are initialized with an identity matrix and ones, respectively. Fig. 2 presents the convergence performance of the proposed algorithm for different sets of channels with the error bound of 0.05, whereas Fig. 3 depicts the convergence behavior of the algorithm for an error bound of 0.1. In addition, the required transmission power with perfect CSI scenario is also presented in Figs. 2 and 3. As observed in Figs. 2 and 3, these results confirm the convergence of the proposed algorithm as discussed in the convergence analysis. On the other hand, the robust scheme requires more transmission power than that of the perfect CSI scenarios to ensure the robustness against the channel uncertainties.

Next, we evaluate the robustness of the proposed algorithm in terms of achieved MSEs of each user. In order to validate the robustness of the algorithm, a random set of channel errors has been generated with error bounds of 0.05 and 0.1, respectively. Table I presents the achieved MSEs of the users for a random set of channels for the robust and the non-robust schemes with an error bound of 0.1 (i.e., $\rho_k = 0.1, \forall k$), whereas Table II provides the achieved MSEs of the users for another set of channels for the robust and the non-robust schemes with an error bound of 0.05 (i.e., $\rho_k = 0.05, \forall k$). In order to show the robustness of the algorithm, we compare the achieved MSEs of the users with the non-robust scheme proposed in [15] for the same set of channels. As validated by these achieved MSEs of all users with the robust and the non-robust schemes, the proposed robust scheme outperforms the non-robust scheme in terms of achieved MSEs with the error bounds of 0.1 and 0.05. In addition, these results ensure that the proposed robust scheme always satisfies the target MSEs regardless of the errors associated with the channels.

To demonstrate the robustness of the proposed scheme, we evaluate the average total transmission power against different channel error bounds ρ_k over 1000 random channel realizations, where the MSE threshold for each user has been set to 0.15. Fig. 4 depicts the required average total transmission power for the proposed robust scheme and the perfect CSI scenario with different error bounds. As seen in Fig. 4, the average total transmission power for the robust scheme increases with error bounds to achieve a predefined MSE threshold as it incorporates all possible set of errors in the design.

In order to validate that the proposed scheme satisfies the target MSEs of all users using (30) for all set of errors within the set of defined error bounds, we evaluate the achievable worst-case MSEs of all users for the beamformers, relay amplification matrix, and receiver coefficients obtained from the proposed robust algorithm. The achieved average worst-case MSEs were evaluated over 1000 random channel realization. Fig. 5 depicts the average worst-case MSEs for the proposed robust, non-robust and perfect CSI schemes. As seen in Fig. 5, the achieved worst-case MSEs for the robust scheme were below 0.15 with all possible sets of errors, whereas the predefined MSE thresholds in non-robust scheme were not satisfied all the times. These achieved worst-case MSEs confirm that the proposed robust scheme satisfies the target MSEs regardless of the errors associated with the channels and validate the effectiveness of the proposed robust algorithm on the channel uncertainties between the relay and the users.

V. CONCLUSIONS

In this paper, we proposed a robust MMSE based beamforming scheme for an AF based multiantenna relay network. The original problem was not convex in terms of beamformers, relay amplification matrix, and receiver coefficients. To tackle this issue, we divided the original problem into subproblems and formulated these subproblems into a convex optimization framework by incorporating channel uncertainties between relay and users. For these subproblems, we developed an iterative algorithm to obtain a robust solution for the sum-power minimization problem based on alternative optimization framework and the worst-case MSEs of all users. In addition, we presented the convergence and complexity analysis of the proposed algorithm. In order to show the robustness of the proposed algorithm, we evaluated the achievable worst-case

MSEs of the users for a given set of beamformers, relay amplification matrix, and receiver coefficients. Simulation results validated the performance of the proposed algorithm. In addition, it is confirmed that the proposed scheme outperforms the non-robust scheme in terms of achievable MSEs, regardless of the errors associated with the channels between the relay and users.

APPENDIX A
PROOF OF LEMMA 1

Taking into account the corresponding channel error, we can write the MSE of the kth user as

$$\begin{aligned}
\varepsilon_k &= 1 - 2\Re(a_k \mathbf{h}_k^H \mathbf{F}\mathbf{H}_0 \tilde{\mathbf{w}}_k) + a_k \mathbf{h}_k^H \mathbf{F}(\mathbf{H}_0 \tilde{\mathbf{W}}\tilde{\mathbf{W}}^H \mathbf{H}_0^H + \sigma^2 \mathbf{I})\mathbf{F}^H \mathbf{h}_k a_k^* + a_k a_k^* \sigma_k^2 \\
&= |a_k^* \tilde{\mathbf{w}}_k^H \mathbf{H}_0^H \mathbf{F}^H \mathbf{h}_k - 1|^2 + \left\| a_k^* \tilde{\mathbf{W}}_{-k}^H \mathbf{H}_0^H \mathbf{F}^H \mathbf{h}_k \right\|_2^2 + \sigma^2 \left\| a_k^* \mathbf{F}^H \mathbf{h}_k \right\|_2^2 + \sigma_k^2 |a_k|^2 \\
&= \left\| \begin{bmatrix} a_k^* \tilde{\mathbf{W}}^H \mathbf{H}_0^H \mathbf{F}^H \mathbf{h}_k \\ \sigma a_k^* \mathbf{F}^H \mathbf{h}_k \\ \sigma_k a_k \end{bmatrix} - \mathbf{e}_k \right\|_2^2 \\
&= \left\| \begin{bmatrix} \mathbf{A}_k \mathbf{h}_k \\ \sigma_k a_k \end{bmatrix} - \mathbf{e}_k \right\|_2^2 = \|\boldsymbol{\mu}_k\|_2^2 ,
\end{aligned} \tag{A.1}$$

where $\mathbf{e}_k$ and $\mathbf{A}_k$ are defined respectively as

$$\mathbf{e}_k = \begin{bmatrix} \overbrace{0 \cdots 0}^{(k-1)\ \text{zeros}} & 1 & \overbrace{0 \cdots 0}^{(K+N_R+1-k)\ \text{zeros}} \end{bmatrix}^T , \tag{A.2}$$

and

$$\mathbf{A}_k = \begin{bmatrix} a_k^* \tilde{\mathbf{W}} \mathbf{H}_0^H \mathbf{F}^H \\ \sigma a_k^* \mathbf{F}^H \end{bmatrix} . \tag{A.3}$$

In addition, $\tilde{\mathbf{W}}_{-k}$ is the matrix $\tilde{\mathbf{W}}_k$ without the column $\tilde{\mathbf{w}}_k$. The MSE constraint of the kth user can be written into a semidefinite constraint as

$$\varepsilon_k = \|\boldsymbol{\mu}_k\|_2^2 \leq \gamma_k \quad \Longrightarrow \quad \begin{bmatrix} \gamma_k & \boldsymbol{\mu}_k^H \\ \boldsymbol{\mu}_k & \mathbf{I} \end{bmatrix} \succeq \mathbf{0}. \tag{A.4}$$

The above semidefinite constraint can be written in terms of channel uncertainties as

$$\begin{bmatrix} \gamma_k & \tilde{\boldsymbol{\mu}}_k^H + \boldsymbol{\varsigma}_k \\ \tilde{\boldsymbol{\mu}}_k + \boldsymbol{\varsigma}_k & \mathbf{I} \end{bmatrix} \succeq \mathbf{0} \tag{A.5}$$

$$\Longrightarrow \quad \begin{bmatrix} \gamma_k & \tilde{\boldsymbol{\mu}}_k^H \\ \tilde{\boldsymbol{\mu}}_k & \mathbf{I} \end{bmatrix} \succeq \begin{bmatrix} 0 & -\boldsymbol{\varsigma}_k^H \\ -\boldsymbol{\varsigma}_k & \mathbf{0} \end{bmatrix}, \tag{A.6}$$

where we have

$$\tilde{\boldsymbol{\mu}}_k = \begin{bmatrix} \mathbf{A}_k \tilde{\mathbf{h}}_k \\ a_k \sigma_k \end{bmatrix} - \mathbf{e}_k \quad \text{and} \quad \boldsymbol{\varsigma}_k = \begin{bmatrix} \mathbf{A}_k \boldsymbol{\Delta}_k \\ 0 \end{bmatrix}, \tag{A.7}$$

and the vectors $\tilde{\mathbf{h}}_k$ and $\boldsymbol{\Delta}_k$ represent the estimated channel of the kth user and the multiantenna relay, as well as the error of the channel, respectively. In the optimization framework, the robust beamformer designs should incorporate the following constraints to provide the required target MSEs, or

$$\begin{aligned} &\begin{bmatrix} \gamma_k & \tilde{\boldsymbol{\mu}}_k^H \\ \tilde{\boldsymbol{\mu}}_k & \mathbf{I} \end{bmatrix} \succeq \begin{bmatrix} 0 & -\boldsymbol{\varsigma}_k^H \\ -\boldsymbol{\varsigma}_k & \mathbf{0} \end{bmatrix}, \\ &\|\boldsymbol{\Delta}_k\|_2 \leq \rho, \end{aligned} \tag{A.8}$$

where the channel uncertainties (i.e., $\boldsymbol{\Delta}_k$, $k = 1, \cdots, K$) are norm-bounded. The right hand side of the first constraint in (A.8) can be written as

$$\begin{aligned} \begin{bmatrix} 0 & -\boldsymbol{\varsigma}_k^H \\ -\boldsymbol{\varsigma}_k & \mathbf{0} \end{bmatrix} &= \begin{bmatrix} 0 & \begin{bmatrix} \boldsymbol{\Delta}_k^H \mathbf{F} \mathbf{A}_k^H & 0 \end{bmatrix} \\ \begin{bmatrix} \mathbf{A}_k \mathbf{F}^H \boldsymbol{\Delta}_k \\ 0 \end{bmatrix} & \mathbf{0} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{0}^H \\ \mathbf{A}_k \mathbf{F}^H \\ \mathbf{0}^H \end{bmatrix} \boldsymbol{\Delta}_k \begin{bmatrix} -1 & \mathbf{0}^H \end{bmatrix} + \begin{bmatrix} -1 \\ \mathbf{0} \end{bmatrix} \boldsymbol{\Delta}_k^H \begin{bmatrix} \mathbf{0} & \mathbf{F} \mathbf{A}_k^H & \mathbf{0} \end{bmatrix}. \end{aligned} \tag{A.9}$$

In order to incorporate the channel uncertainties in the robust optimization framework, let us consider the following lemma.

Lemma 7 (Nemirovski lemma) [27]: For a given set of matrices $\mathbf{A} = \mathbf{A}^H$ and $\mathbf{B},\ \mathbf{C}$, the following linear matrix inequality is satisfied,

$$
\begin{aligned}
&\mathbf{A} - \mathbf{B}^H \mathbf{X} \mathbf{C} - \mathbf{C}^H \mathbf{X}^H \mathbf{B} \succeq \mathbf{0}, \\
&\|\mathbf{X}\| \leq \rho,
\end{aligned} \tag{A.10}
$$

if and only if there exist non-negative real numbers μ such that

$$
\begin{bmatrix} \mathbf{A} - \mu \mathbf{C}^H \mathbf{C} & -\rho \mathbf{B}^H \\ -\rho \mathbf{B} & \mu \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \tag{A.11}
$$

The both constraints in (A.8) can be written as

$$
\begin{aligned}
&\begin{bmatrix} \gamma_k & \tilde{\boldsymbol{\mu}}_k^H \\ \tilde{\boldsymbol{\mu}}_k & \mathbf{I} \end{bmatrix} \succeq \begin{bmatrix} \mathbf{0}^H \\ \mathbf{A}_k \mathbf{F}^H \\ \mathbf{0}^H \end{bmatrix} \boldsymbol{\Delta}_k \begin{bmatrix} -1 & \mathbf{0}^H \end{bmatrix} + \begin{bmatrix} -1 \\ \mathbf{0} \end{bmatrix} \boldsymbol{\Delta}_k^H \begin{bmatrix} \mathbf{0} & \mathbf{F}\mathbf{A}_k^H & \mathbf{0} \end{bmatrix}, \\
&\|\boldsymbol{\Delta}_k\|_2 \leq \rho.
\end{aligned} \tag{A.12}
$$

Exploiting the Nemirovski Lemma, we have that the channel uncertainties can be incorporated in the following semidefinite constraint:

$$
\begin{bmatrix} \mathbf{D}_k - \mu_k \mathbf{C}_k^H \mathbf{C}_k & -\rho_k \mathbf{B}_k^H \\ -\rho_k \mathbf{B}_k & \mu_k \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \tag{A.13}
$$

where $\mathbf{B}_k = \left[\mathbf{0}_{N_R \times 1} \ \mathbf{F}\mathbf{A}_k^H \ \mathbf{0}_{N_R \times 1}\right]$, $\mathbf{C}_k = \left[-1 \ \mathbf{0}^H_{1 \times (K+N_R+1)}\right]$,

$$
\mathbf{D}_k = \begin{bmatrix} \gamma_k & \left[\tilde{\mathbf{h}}_k^H \mathbf{F} \mathbf{A}_k^H \ \sigma_k a_k\right] - \mathbf{e}_k^H \\ \left[\begin{pmatrix} \mathbf{A}_k \mathbf{F}^H \tilde{\mathbf{h}}_k \\ \sigma_k a_k \end{pmatrix}\right] - \mathbf{e}_k & \mathbf{I} \end{bmatrix}, \tag{A.14}
$$

and

$$
\mathbf{A}_k = \begin{bmatrix} a_k^* \tilde{\mathbf{W}}^H \mathbf{H}_0^H \\ \sigma a_k^* \mathbf{I} \end{bmatrix}_{(K+N_R) \times 1} \tag{A.15}
$$

This completes the proof of *Lemma 1*. ■

APPENDIX B
PROOF OF LEMMA 2

Using slack variables, we can express the robust beamformer design in (21) as

$$\min_{\tilde{\mathbf{w}}} \quad \sum_{i=1}^{K} t_i, \tag{B.1}$$

$$\begin{aligned}
\text{s.t.} \quad & \left\| (\mathbf{H}_0^H \mathbf{F}^H \mathbf{F} \mathbf{H}_0 + \mathbf{I})^{1/2} \tilde{\mathbf{w}}_k \right\|_2^2 \leq t_k, \quad k = 1, \cdots, K, \\
& \varepsilon_k^{(e)} \leq \gamma_k, \quad k = 1, \cdots, K, \quad t_k \geq 0, \quad k = 1, \cdots, K, \\
& \|\mathbf{\Delta}_k\|_2 \leq \rho_k, \quad k = 1, \cdots, K.
\end{aligned}$$

The first set of constraints in (B.1) can be written into the second order cone constraint as

$$\begin{aligned}
& \left\| [\|\mathbf{A}\tilde{\mathbf{w}}_1\|_2 \ \|\mathbf{A}\tilde{\mathbf{w}}_2\|_2 \cdots \|\mathbf{A}\tilde{\mathbf{w}}_K\|_2]^T \right\|_2 \leq \sum_{i=1}^{K} t_i \\
\Longrightarrow & \begin{bmatrix} \sum_{i=1}^{K} t_i \\ [\|\mathbf{A}\tilde{\mathbf{w}}_1\|_2 \|\mathbf{A}\tilde{\mathbf{w}}_2\|_2 \cdots \|\mathbf{A}\tilde{\mathbf{w}}_K\|_2]^T \end{bmatrix} \succeq_K 0.
\end{aligned} \tag{B.2}$$

Based on *Lemma 1*, the rest of the two constraints in (B.1) can be written as

$$\begin{aligned}
& \begin{bmatrix} \mathbf{D}_k - \mu_k \mathbf{C}_k^H \mathbf{C}_k & -\rho_k \mathbf{B}_k^H \\ -\rho_k \mathbf{B}_k & \mu_k \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \\
& \mu_k \geq 0,
\end{aligned} \tag{B.3}$$

where $\mathbf{D}_k, \mathbf{C}_k$, and $\mathbf{B}_k$ are defined in (18) and (19). Hence, the robust beamforming design can be formulated into a convex optimization framework as in (22). ■

APPENDIX C
PROOF OF LEMMA 3

The original robust relay design in (24) can be formulated into the following constrained quadratic programming (QCQP) problem [15]:

$$\begin{aligned} &\min_{\mathbf{f},\alpha} \quad \alpha, \\ &\text{s.t.} \quad \mathbf{f}^H\mathbf{B}\mathbf{f} \leq \alpha, \\ &\qquad 1 - 2\Re(\mathbf{g}_k^T\mathbf{f}) + \mathbf{f}^H\mathbf{D}_k\mathbf{f} + |a_k|^2\sigma_k^2 \leq \gamma_k, \ k = 1, 2, \ldots, K, \\ &\qquad \alpha \geq 0, \end{aligned} \tag{C.1}$$

where we have

$$\begin{aligned} \mathbf{f} &= \text{Vec}(\mathbf{F}), \\ \mathbf{B} &= \left[\mathbf{R}_r^{1/2} \otimes \mathbf{I}\right]^T \left[\mathbf{R}_r^{T/2} \otimes \mathbf{I}\right], \\ \mathbf{R}_r &= \mathbf{H}_0\tilde{\mathbf{W}}\tilde{\mathbf{W}}^H\mathbf{H}_0^H + \sigma^2\mathbf{I}, \\ \mathbf{D}_k &= \left[\mathbf{R}_r^{1/2} \otimes a_k^*\mathbf{h}_k^T\right]^T \left[\mathbf{R}_r^{T/2} \otimes a_k\mathbf{h}_k^H\right], \\ \mathbf{g}_k &= \text{Vec}\Big(a_k\mathbf{h}_k^*\tilde{\mathbf{w}}_k^T\mathbf{H}_0^T\Big). \end{aligned} \tag{C.2}$$

The following matrix identities are used in formulating the QCQP in (C.1):

$$\begin{aligned} \text{Vec}(\mathbf{AXB}) &= (\mathbf{B}^T \otimes \mathbf{A})\text{Vec}(\mathbf{X}), \\ \text{Tr}(\mathbf{A}^T\mathbf{B}) &= \text{Vec}(\mathbf{A})^T\text{Vec}(\mathbf{B}). \end{aligned} \tag{C.3}$$

However, the channel uncertainties cannot be incorporated directly into the design defined in (C.1). Hence, we formulate the robust relay amplification matrix design by exploiting Nemirovski Lemma as

$$\begin{aligned} &\min_{\mathbf{F}} \quad \text{Tr}\left[\mathbf{F}\mathbf{A}_0\mathbf{F}^H\right], \\ &\text{s.t.} \quad \begin{bmatrix} \mathbf{D}_i - \mu_i\mathbf{C}_i^H\mathbf{C}_i & -\rho_i\mathbf{B}_i^H \\ -\rho_i\mathbf{B}_i & \mu_i\mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \quad i = 1, \cdots, K, \\ &\qquad \mu_i \geq 0, \quad i = 1, \cdots, K, \end{aligned} \tag{C.4}$$

where $\mathbf{D}_i$, $\mathbf{C}_i$, and $\mathbf{B}_i$ are defined in (18) and (19). This formulation is similar to that of the robust beamforming design in (22). ■

Appendix D
Proof of Lemma 5

Here, we provide a problem formulation to calculate achievable worst-case MSE of the kth user. The MSE of the kth user with the solutions obtained from the proposed algorithm by incorporating the channel estimation errors can be written as

$$\varepsilon_k = 1 - 2\Re(\hat{a}_k \hat{\mathbf{h}}_k^H \hat{\mathbf{F}} \mathbf{H}_0 \hat{\mathbf{u}}_k) + \hat{a}_k \hat{\mathbf{h}}_k^H \hat{\mathbf{G}} \hat{\mathbf{h}}_k \hat{a}_k^* + \hat{a}_k \hat{a}_k^* \sigma_k^2, \tag{D.1}$$

where

$$\hat{\mathbf{G}} = \hat{\mathbf{F}} \mathbf{H}_0 \hat{\mathbf{U}} \hat{\mathbf{U}}^H \mathbf{H}_0^H \hat{\mathbf{F}}^H + \sigma^2 \hat{\mathbf{F}} \hat{\mathbf{F}}^H, \tag{D.2}$$

$\hat{\mathbf{h}}_k = \bar{\mathbf{h}}_k + \mathbf{\Delta}_k$ and $\hat{\mathbf{U}}, \hat{\mathbf{F}}, \hat{\mathbf{u}}_k$ as well as $\hat{a}_k$ denote the solutions $\tilde{\mathbf{W}}, \mathbf{F}, \tilde{\mathbf{w}}_k$ and a_k obtained from the proposed algorithm, respectively. The worst-case MSE calculation can be formulated as

$$\varepsilon_k^{(w)} = \max_{\mathbf{h}_k \in \mathfrak{B}_k} \varepsilon_k, \tag{D.3}$$

where $\varepsilon_k^{(w)}$ denotes the worst-case MSE of the kth user. This problem can be cast into the following optimization problem:

$$\begin{aligned} \min_{t_k} \quad & t_k, \\ \text{s.t.} \quad & 1 - 2\Re(\hat{a}_k (\bar{\mathbf{h}}_k + \mathbf{\Delta}_k)^H \hat{\mathbf{F}} \mathbf{H}_0 \hat{\mathbf{u}}_k) + \hat{a}_k (\bar{\mathbf{h}}_k + \mathbf{\Delta}_k)^H \hat{\mathbf{G}} (\bar{\mathbf{h}}_k + \mathbf{\Delta}_k) \hat{a}_k^* + \hat{a}_k \hat{a}_k^* \sigma_k^2 \leq t_k, \\ & t_k \geq 0, \quad \|\mathbf{\Delta}_k\|_2^2 \leq \rho_k^2. \end{aligned} \tag{D.4}$$

In order to incorporate the channel uncertainties in the worst-case MSE calculation framework, we consider the following lemma:

Lemma 8 (S-Procedure) [28]: Let $f_1(\mathbf{x}) = \mathbf{x}^H \mathbf{A}_1 \mathbf{x} + 2\Re\left\{\mathbf{b}_1^H \mathbf{x}\right\} + c_1$ and $f_2(\mathbf{x}) = \mathbf{x}^H \mathbf{A}_2 \mathbf{x} + 2\Re\left\{\mathbf{b}_2^H \mathbf{x}\right\} + c_2$ be two quadratic functions of $\mathbf{x}$, where $\mathbf{A}_1 = \mathbf{A}_1^H \in \mathbb{C}^{n \times n}$ and $\mathbf{A}_2 = \mathbf{A}_2^H \in \mathbb{C}^{n \times n}$,$\mathbf{b}_k \in \mathbb{C}^n$ and $c_k \in \mathbb{R}$. There exists a $\tilde{\mathbf{x}}$ satisfying $f_1(\tilde{\mathbf{x}}) > 0$. Then

$$f_1(\mathbf{x}) \geq 0 \Longrightarrow f_2(\mathbf{x}) \geq 0 \tag{D.5}$$

holds true if only if there exists a $\mu \geq 0$ such that

$$\begin{bmatrix} \mathbf{A}_2 & \mathbf{b}_2 \\ \mathbf{b}_2^H & c_2 \end{bmatrix} - \mu \begin{bmatrix} \mathbf{A}_1 & \mathbf{b}_1 \\ \mathbf{b}_1^H & c_1 \end{bmatrix} \succeq \mathbf{0}. \tag{D.6}$$

By exploiting the *S-Procedure*, the constraints in (D.4) can be written in terms of channel estimation error as

$$\begin{aligned}&1-2\Re(\hat{a}_k\bar{\mathbf{h}}_k^H\hat{\mathbf{F}}\mathbf{H}_0\hat{\mathbf{u}}_k)-2\Re\left\{\hat{a}_k^*\hat{\mathbf{u}}_k^H\mathbf{H}_0^H\hat{\mathbf{F}}^H\boldsymbol{\Delta}_k\right\}\\&-2\Re\left\{\hat{a}_k\hat{a}_k^*\tilde{\mathbf{h}}_k^H\hat{\mathbf{G}}\boldsymbol{\Delta}_k\right\}+\hat{a}_k^*\bar{\mathbf{h}}_k^H\hat{\mathbf{G}}\bar{\mathbf{h}}_k\hat{a}_k+\hat{a}_k^*\boldsymbol{\Delta}_k^H\hat{\mathbf{G}}\boldsymbol{\Delta}_k\hat{a}_k+\hat{a}_k\hat{a}_k^*\sigma_k^2\leq t_k,\\&\boldsymbol{\Delta}_k^H\boldsymbol{\Delta}_k\leq\rho_k^2.\end{aligned}\tag{D.7}$$

Both of these constraints can be formulated into a semidefinite constraint using *S-Procedure* as

$$\mathbf{M}_k\succeq\mathbf{0},\quad \mu_k\geq 0,\tag{D.8}$$

where $\mathbf{M}_k$ is defined in (31). Hence, the worst-case MSE calculation can be formulated into the following optimization problem:

$$\begin{aligned}\varepsilon_k^{(w)}\triangleq\min_{t_k,\mu_k}&\quad t_k,\\ \text{s.t.}\quad \mathbf{M}_k&\succeq\mathbf{0},\\ t_k\geq 0,&\quad\mu_k\geq 0.\end{aligned}\tag{D.9}$$

This completes the formulation of the worst-case MSE calculation for a given set of beamformers, relay amplification matrix, and receiver filter coefficients. ■

REFERENCES

[1] Y. Yang, H. Hu, J. Xu, and G. Mao, "Relay technologies for WiMAX and LTE-Advanced Mobile Systems," IEEE Commun. Mag., pp. 100- 105, Oct. 2009.

[2] S. Peters and J. R. W. Heath, "The future of WiMAX: Multihop relaying with IEEE 802.16j," IEEE Commun. Mag., pp. 104-111, Jan. 2009.

[3] X. Tang and Y. Hua, "Optimal design of non-regenerative MIMO wireless relays," IEEE Trans. Wireless Commun., vol. 6, no. 4, pp. 1398-1407, Apr. 2007.

[4] S. A. Jafar, K. S. Gomadam, and C. Huang, "Duality and rate optimization for multiple access and broadcast channels with amplify-and- forward relays," IEEE Trans. Inf. Theory, vol. 53, no. 10, pp. 3350- 3370, Oct. 2007.

[5] K. S. Gomadam and S. A. Jafar, "Duality of MIMO multiple access channel and broadcast channel with amplify-and-forward relays," IEEE Trans. Commun., vol. 58, no. 11, pp. 211-217, Jan. 2010.

[6] Y. Rong and M. R. A. Khandaker, "On uplink-downlink duality of multi-hop MIMO relay channel," IEEE Trans. Wireless Commun., vol. 10, no. 6, pp. 1923-1931, Jun. 2011.

[7] O. Munoz-Medina, J. Vidal, and A. Agustin, "Linear transceiver design in nonregenerative relays with channel state information," IEEE Trans. Signal Process., vol. 55, no. 6, pp. 2593-2604, Jun. 2007.

[8] J. Kim and D. Kim, "Performance of dual-hop amplify and forward beamforming and its equivalent systems in Rayleigh fading channels," IEEE Trans. Commun., vol. 58, no. 3, pp. 729-732, Mar. 2010.

[9] R. Krishna, K. Cumanan, Z. Xiong and S. Lambotharan, "A novel cooperative relaying strategy for wireless networks with signal quantization," IEEE Trans. Veh. Technol., vol. 59, no. 1, pp. 485-489, Jan., 2010.

[10] Y. Rong, "Multihop nonregenerative MIMO relays: QoS considerations," IEEE Trans. Signal Process., vol. 59, no. 1, pp. 290-303, Jan. 2011.

[11] L. Sanguinetti and A. A. D'Amico, "Power allocation in two-hop amplify-and-forward MIMO relay systems with QoS requirements," IEEE Trans. Signal Process., vol. 60, no. 5, pp. 2494-2507, May 2012.

[12] W. Guan, H. Luo, and W. Chen, "Linear relaying scheme for MIMO relay system with QoS requirements," IEEE Signal Process. Lett., vol. 15, pp. 697-700, 2008.

[13] Y. Fu, L. Yang, W. P. Zhu, and C. Liu, "Optimum linear design of two- hop MIMO relay networks with QoS requirements," IEEE Trans. Signal Process., vol. 59, no. 5, pp. 2257-2269, May 2011.

[14] R. Zhang, C. C. Chai, and Y. C. Liang, "Joint beamforming and power control for multiantenna relay broadcast channel with QoS constraints," IEEE Trans. Signal Process., vol. 57, no. 2, pp. 726-737, Feb. 2009.

[15] K. Cumanan, Y. Rahulamathavan, S. Lambotharan, and Z. Ding, "MMSE based beamforming techniques for relay broadcast channels," IEEE Trans. Veh. Technol., vol. 62, no. 8, pp. 4045-4051, Oct., 2013.

[16] S. Vorobyov, A. Gershman, and Z.-Q. Luo, "Robust adaptive beam- forming using worst-case performance optimization: A solution to the signal mismatch problem," IEEE Trans. Signal Process., vol. 51, no. 2, pp. 313-324, Feb. 2003.

[17] R. Krishna, K. Cumanan, V. Sharma and S. Lambotharan, "A robust cooperative relaying strategy for wireless networks using semidefinite constraints and worst-case performance optimization," in Proc. ISITA 2008, Auckland, New Zealand, pp. 1-5, Dec. 2008.

[18] R. Lorenz and S. Boyd, "Robust minimum variance beamforming," IEEE Trans. Signal Process., vol. 53, no. 5, pp. 1684-1696, May 2005.

[19] A. Abdel-Samad, T. Davidson, and A. Gershman, "Robust transmit eigen beamforming based on imperfect channel state information," IEEE Trans. Signal Process., vol. 54, no. 2, pp. 1596-1609, May 2006.

[20] K. Cumanan, R. Krishna, V. Sharma, and S. Lambotharan, "A robust beamforming based interference control technique and its performance for cognitive radios," in Proc. ISCIT 2008, Lao, pp. 9-13, Oct. 2008.

[21] Y. Guo and B. Levy, "Worst-case MSE precoder design for imperfectly known MIMO communication channels," IEEE Trans. Signal Process., vol. 53, no. 8, pp. 2918-2930, Aug. 2005.

[22] A. Pascual-Iserte, D. Palomar, A. Pérez-Neria, and M. Lagunas, "A robust maximin approach for MIMO communications with partial channel state information based on convex optimization," IEEE Trans. Signal Process., vol. 54, no. 1, pp. 346-360, Jan. 2006.

[23] K. Cumanan, R. Krishna, V. Sharma, and S. Lambotharan, "Robust interference control techniques for multi-user cognitive radios using worst-case performance optimization," in Proc. Asilomar Conf. Sign., Syst. and Comp., Pacific Grove, CA, pp. 378-382, Oct. 2008.

[24] K. Cumanan, R. Krishna, V. Sharma, and S. Lambotharan, "Robust interference control techniques for cognitive radios using worst-case performance optimization," in Proc. ISITA 2008, Auckland, New Zealand, pp. 1-5, Dec. 2008.

[25] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge, UK: Cambridge University Press, 2004.

[26] Y. Ye, Interior Point Algorithms. Theory and Analysis. John Wiley & Sons, 1997.

[27] C. Y. Eldar, A. Ben-Tal, and A. Nemirovski, "Robust mean-squared error estimation in the presence of model uncertainities," IEEE Trans. Signal Process., vol. 53, no. 1, pp. 168-181, Jan. 2005.

[28] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory. Philadelphia PA: SIAM Studies in Applied Mathematics, 1994.

[29] C. Xing, S. Ma, Z. Fei, Y-C. Wu and H. V. Poor, "A general robust linear transceiver design for multi-hop amplify-and-forward MIMO relaying systems," IEEE Trans. Signal Process., vol. 61, no. 5, pp. 1196-1209, Mar. 2013.

[30] C. Xing, M. Xia, F. Gao and Y-C. Wu, "Robust transceiver with Tomlinson-Harashima precoding amplify-and-forward MIMO relaying systems," IEEE J. Sel. Areas Commun., vol. 30, no. 8, pp. 1370-1382, Sep. 2012.

[31] C. Xing, S. Li, Z. Fei and J. Kuang, "How to understand linear minimum mean-square-error transceiver design for multiple-input-multiple-output systems from quadratic matrix programming," IET Commun., vol. 7, no. 12, pp. 1231-1242, Aug. 2013.

[32] C. Xing, S. Ma, M. Xia and Y-C. Wu, "Cooperative beamforming for dual-hop amplify-and-forward multi-antenna relaying cellular networks," Signal Process., vol. 92, no. 11, pp. 2689-2699, Nov. 2012.

[33] C. Xing, S. Ma and Y. Zhou, "Matrix-monotonic optimization for MIMO systems," IEEE Trans. Signal Process., vol. 63, no. 2, pp. 334-348, Jan. 2015.

[34] C. Xing, S. Ma, Y.-C. Wu and T.-S. Ng , "Transceiver design for dual-hop nonregenerative MIMO OFDM relay systems under channel uncertainties," IEEE Trans. Signal Process., vol. 58, no. 12, pp. 6325-6339, Dec. 2010.

[35] C. Xing, S. Ma and Y.-C. Wu, ”Robust joint design of linear relay precoder and destination equalizer for dual-hop amplify-and-forward MIMO relay systems,” IEEE Trans. Signal Process., vol. 58, no. 4, pp. 2273-2283, Apr. 2010.

[36] B. K. Chalise, and L. Vandendorpe, ”Joint linear processing for and amplify-and-forward MIMO relay channel with imperfect channel state information,” Eurasip Journal on Advances in Signal Process., vol. 2010.

[37] Y. Rong, ”Robust design of linear non-regenerative MIMO relays with imperfect channel state information,” IEEE Trans. Signal Process., vol. 59, no. 5, pp. 2455-2460, May 2011.

[38] Z. He, W. Jiang and Y. Rong, ”Robust design for amplify-and-forward MIMO relay systems with direct links and imperfect channel information,” IEEE Trans. Wireless Commun., vol. 14, no. 1, pp. 353-363, Jan. 2015.

[39] P. Ubaidulla and A. Chockalingam, ”Relay precoder optimization in MIMO-relay networks with imperfect CSI,” IEEE Trans. Signal Process., vol. 59, no. 11, pp. 5473-5484, Nov. 2011.

[40] L. Zhang, H. Chen and M. He, ”Worst-case-based robust beamforming for wirless cooperative networks,” Eurasip Journal on Advances in Signal Process., vol. 2014.

[41] E. A. Gharavol, Y. C. Liang and K. Mouthann, ”Robust linear beamforming for MIMO relay with imperfect channel state information,” in Proc. IEEE Personal and Indoor and Mobile Radio Communications (PIMRC) Symp., Istanbul, Turkey, pp. 510-514, Sept. 2010.

[42] E. A. Gharavol and E. G. Larsson, ”Robust joint optimization of nonregenerative MIMO relay channels with imperfect CSI,” in Proc. Asilomar Conf. Sign., Syst. and Comp., Pacific Grove, CA, pp. 1589-1593, Nov. 2011.

[43] J. Liu, F. Gao and Z. Qiu, ”Robust transceiver design for downlink multiuser MIMO AF relay systems,” IEEE Trans. Wireless Commun., vol. 14, no. 4, pp. 2218-2231, Apr. 2015.

[44] Q. H. Spencer, A. L. Swindlehurst and M. Haardt, ”Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels,” IEEE Trans. Signal Process., vol. 52, no. 2, pp. 461-471, Feb. 2004.

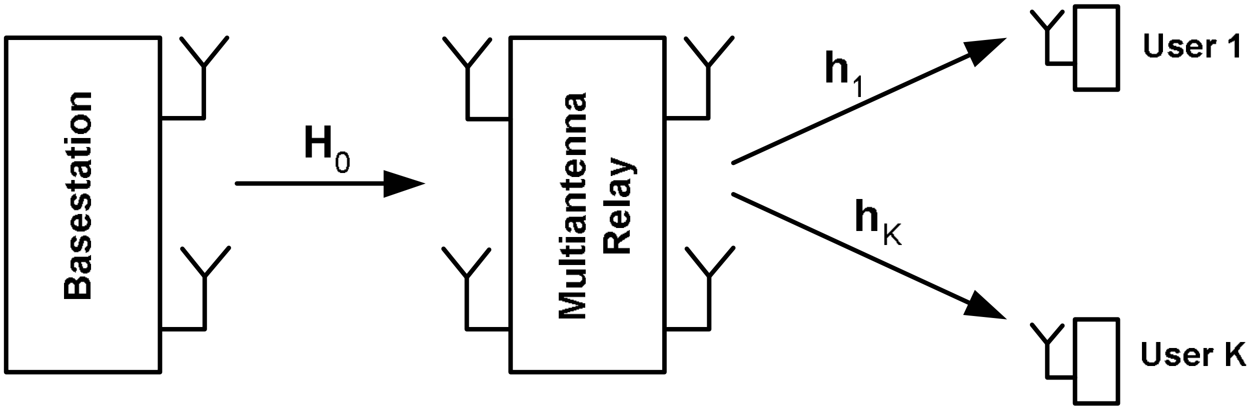

Fig. 1. A multiantenna relay network with multiple users.

Algorithm I: Robust sum-power minimization algorithm.

1) Initialize: $\mathbf{F} = \mathbf{F}_0$ and $\mathbf{a} = \mathbf{a}_0$.
2) **Repeat**
   a) Solve the problem in (22) for a fixed relay amplification matrix $\mathbf{F}$, and receiver coefficients $\mathbf{a}$. Obtain optimal beamformers $\tilde{\mathbf{W}}$.
   b) Solve the problem in (26) for a fixed set of beamformers $\tilde{\mathbf{W}}$ and receiver coefficients $\mathbf{a}$. Obtain the optimal relay amplification matrix $\mathbf{F}$ using (26).
   c) Solve the problem in (27) for a fixed set of beamformers $\tilde{\mathbf{W}}$ and relay amplification matrix $\mathbf{F}$. Obtain the optimal receiver coefficients $\mathbf{a}$ using (27).
3) **Until** the required accuracy is met.

TABLE I

COMPARISON OF ACHIEVED MSES OF ALL USERS FOR THE ROBUST AND THE NON-ROBUST SCHEMES FOR DIFFERENT CHANNELS WITH AN ERROR BOUND OF 0.1 ($\rho_k = 0.1, \forall k$).

	Robust Scheme			Non-Robust Scheme		
Channels	Achieved MSE of User 1	Achieved MSE of User 2	Achieved MSE of User 3	Achieved MSE of User 1	Achieved MSE of User 2	Achieved MSE of User 3
Channel 1	0.1398	0.1397	0.1437	0.1617	0.1527	0.1573
Channel 2	0.1350	0.1390	0.1342	0.1613	0.1564	0.1607
Channel 3	0.1296	0.1319	0.1331	0.1618	0.1440	0.1563
Channel 4	0.1381	0.1356	0.1305	0.1452	0.1614	0.1558
Channel 5	0.1324	0.1370	0.1422	0.1609	0.1579	0.1626

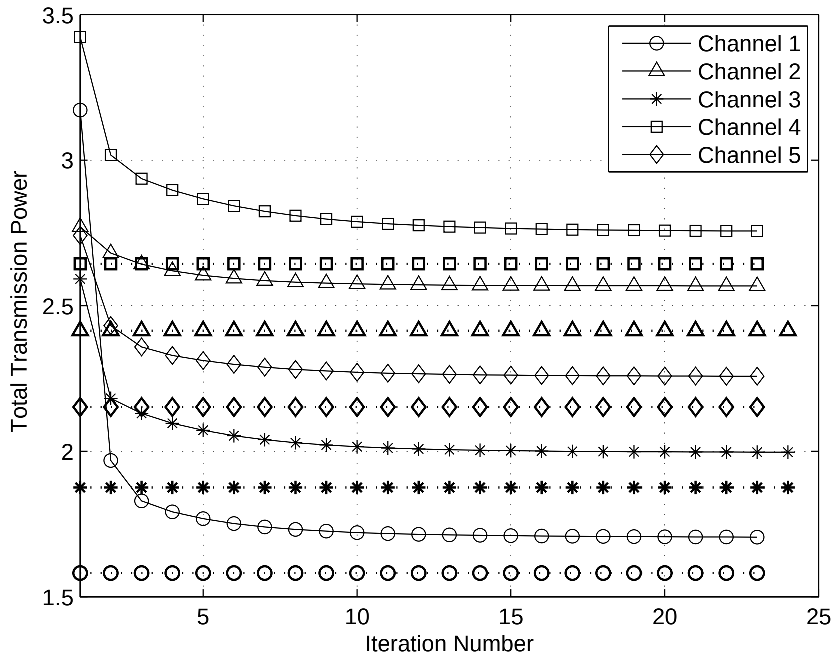

Fig. 2. The convergence of the proposed robust sum-power minimization algorithm for different sets of channels with an error bound of 0.1. The dotted and solid lines denote the perfect CSI scenario and the robust scheme for different sets of channels.

TABLE II
COMPARISON OF ACHIEVED MSEs OF ALL USERS FOR THE ROBUST AND THE NON-ROBUST SCHEMES FOR DIFFERENT CHANNELS WITH AN ERROR BOUND OF 0.05 ($\rho_k = 0.05, \forall k$).

	Robust Scheme			Non-Robust Scheme		
Channels	Achieved MSE of User 1	Achieved MSE of User 2	Achieved MSE of User 3	Achieved MSE of User 1	Achieved MSE of User 2	Achieved MSE of User 3
Channel 6	0.1453	0.1439	0.1438	0.1528	0.1506	0.1529
Channel 7	0.1402	0.1426	0.1415	0.1522	0.1540	0.1557
Channel 8	0.1453	0.1436	0.1444	0.1540	0.1522	0.1506
Channel 9	0.1392	0.1395	0.1392	0.1533	0.1506	0.1535
Channel 10	0.1432	0.1383	0.1458	0.1530	0.1494	0.1521

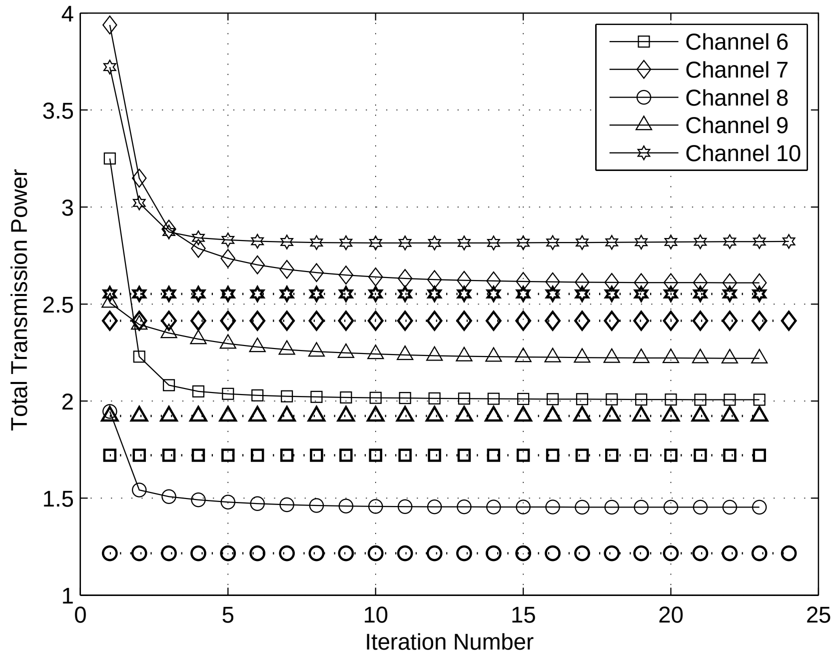

Fig. 3. The convergence of the proposed robust sum-power minimization algorithm for different sets of channels with an error bound of 0.05. The dotted and solid lines denote the perfect CSI scenario and the robust scheme for different sets of channels.

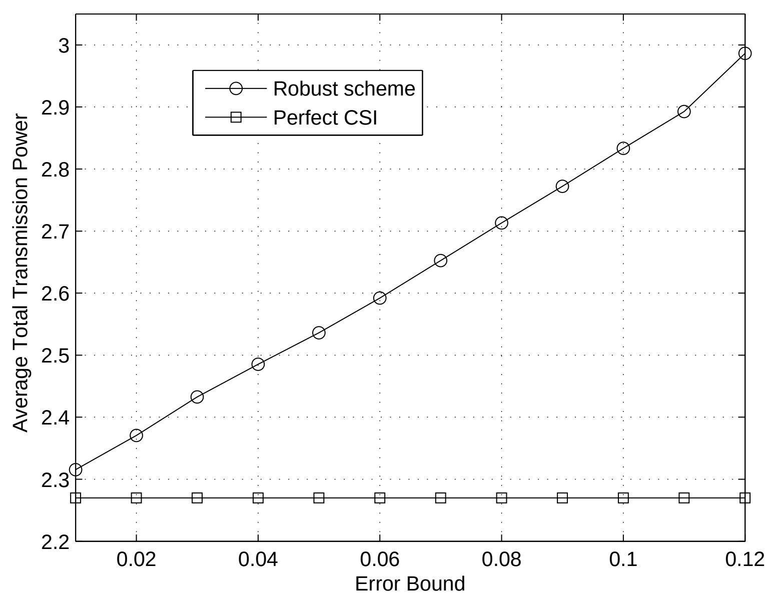

Fig. 4. The required average total transmission power for the proposed robust scheme and the perfect CSI scenario against different error bounds. The MSE threshold at each user is set to 0.15.

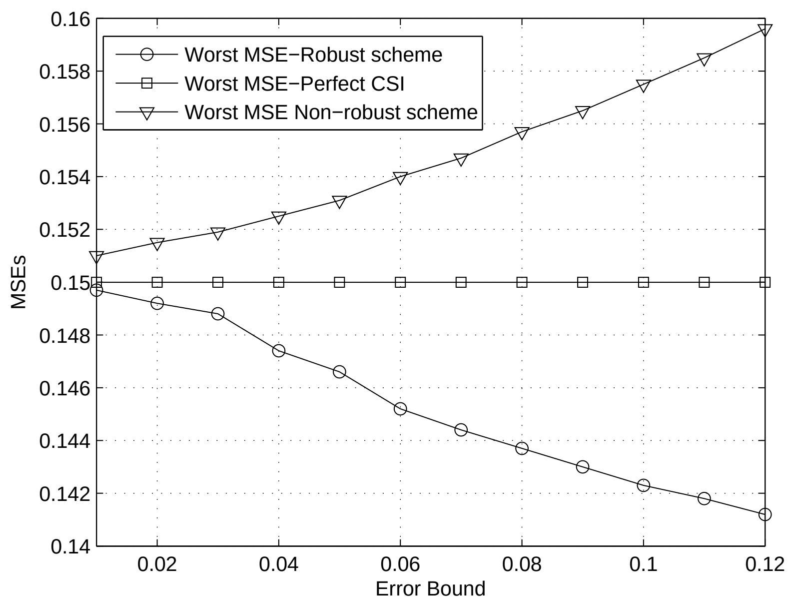

Fig. 5. The achieved average worst-MSEs between different users for the proposed robust scheme, non-robust scheme and the perfect CSI scenario against different error bounds. The MSE threshold at each user is set to 0.15.