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Low-complexity Lattice Reduction Aided Detection for Generalised Spatial Modulation

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Abstract—Generalised spatial modulation (GSM) was first introduced with the maximum-likelihood (ML) optimum decoder. However, ML decoder may be infeasible for practical implementation due to its exponential complexity especially when the number of antennas or the constellation size is large. Lattice reduction (LR) aided linear decoders are known to have much lower complexity while achieving near-optimal bit-error-rate (BER) performance in MIMO V-BLAST systems. In this paper, LR-aided linear decoders are applied to GSM systems for the first time, but the simulation results demonstrate unsatisfactory BER performances. Thereby, two improved LR-aided linear decoders are proposed in this work. The proposed schemes achieve significant BER performance enhancement compared to that of conventional LR-aided linear decoders as well as linear decoders including zero forcing (ZF) detection and minimum mean square error (MMSE) detection. Compared to the ML decoder, the proposed schemes can provide fairly lower complexities with small BER performance degradation.

Keywords—Generalised spatial modulation; Lattice Reduction; linear detection; lattice reduction aided precoding;

I. INTRODUCTION

Spatial modulation (SM) is a MIMO transmission technology to increase spectral efficiency (SE) by transmitting extra information using antenna index compared to single input multiple output (SIMO) systems [1]. SM mitigates inter-channel interference (ICI) [1], reduces implementation complexity [19] and energy consumption [10] by activating only a single antenna to convey information in each symbol period. In SM, the input data bits are divided into two groups, one of which is used to select active antenna and the other determines the transmitted symbol. Therefore, a total SE of \(\log_2 N_t + \log_2 M\) is achieved, where \(N_t\) and \(M\) are the number of transmit antennas and modulation order respectively.

However, SM has its limitations. The number of transmit antennas \(N_t\) has to be a power of two and the logarithm increase in spectral efficiency requires a large number of transmit antennas due to its sub-optimality in SE [11]. Generalised spatial modulation (GSM) [2] is an extension of SM to overcome the limitation in \(N_t\) and continues to offer higher SE by activating more than one antennas in each symbol period to simultaneously transmit data symbols. Research in [12] shows that GSM increases the achievable SE while maintaining all the advantages of SM. Therefore, GSM is considered as a promising candidate for future MIMO systems [12]-[14].

A number of detection schemes have been studied for GSM. The maximum likelihood (ML) decoder achieves optimal performance but requires extremely high complexity which increases exponentially with the number of transmit antennas. Low-complexity linear decoders can be used to detect GSM, but their performances are not comparable to that of the ML decoder. Considering that a linear equalizer is optimal for an orthogonal channel matrix, the Lattice Reduction (LR) technique is utilised to improve the channel orthogonality and LR-aided linear equalizers are proposed for V-BLAST systems in [8],[13] where simulation results demonstrated near-optimal performance with low-complexity.

For the first time, this paper studies the applicability of utilising the low-complexity LR-aided linear decoders in GSM systems. Firstly, the conventional LR-aided linear equalizers are applied to GSM, but simulation results show unsatisfactory BER performance which is due to the noise enhancement at the receiver. Therefore, the improved novel LR-aided linear decoders are proposed for GSM in this work. Unlike the conventional LR-aided linear decoders, the proposed LR-aided linear decoders can avoid the noise enhancement at the receiver by employing a simple LR-aided precoding at the transmitter. With the help of this precoding, LR-aided linear decoders can achieve near-optimal BER performance with lower complexity in GSM systems.

II. GSM SYSTEM MODEL

Consider a GSM system equipped with \(N_t\) transmit antennas and \(N_r\) receive antennas. GSM activates only \(n_t\) (\(1 \leq n_t \leq N_t\)) from \(N_t\) transmit antennas to convey the same complex symbol while the other antennas remain idle in each symbol period. Alternatively, the \(n_t\) active antennas in GSM also can be designed to transmit different data symbols to increase the SE. Among \(\binom{N_t}{n_t}\) possible combinations of activating \(n_t\) from \(N_t\) transmit antennas, only \(N_c = 2^m\) combinations can be used, where \(m_t = \lceil \log_2 \left( \binom{N_t}{n_t} \right) \rceil\) and \(\lceil \cdot \rceil\) is the floor operation. In this paper, only the first \(N_c\) active antenna combinations are legitimate, and the research about active antenna combination selection will be presented in our future work.

In GSM, the transmitted data bits are divided into groups containing \(m = m_t + m_2\) bits in each of them, where \(m_2 = \log_2 M\). The first \(m_t\) bits are used to select \(n_t\) active antennas.
The remaining \( m_k \) bits are mapped to a conventional modulation symbol chosen from the constellation diagram of M-QAM modulation. Thus, the incoming data bits are modulated to:

\[
X = e_i b_i ,
\]

where \( e_i (e_i \in \mathbb{C}^{N_t \times n_t}) \) consists of \( n_t \) columns chosen from the \( N_t \times N_t \) identity matrix, and the ordinals of the chosen columns correspond to the antenna indices in each active antenna combination. \( e_o \) contains all possible \( e_i \). \( B \) is the set of all possible data symbol vectors, where each vector \( b_i \) has \( n_t \) same complex-valued symbols chosen from the constellation points of the conventional modulation schemes (e.g. 4-QAM, \( n_t = 2 \)). \( b_i = [1 + j, 1 + j]^T, b_i \in B \).

The modulated signal is then transmitted through a \( N_t \times N_t \) MIMO flat-fading channel \( H \) with complex independent and identically distributed (i.i.d) entries according to CN \((0,1)\). The received vector is given by:

\[
y = HX + n ,
\]

where \( n \in \mathbb{C}^{N_r \times 1} \) represents the additive white Gaussian noise (AWGN) vector with complex i.i.d entries according to CN \((0,1)\). After the receiver, the joint ML decoder for GSM is denoted as:

\[
[	ilde{i}, 	ilde{b}] = \arg\min \{||y - HX||^2 \} ,
\]

where \( \tilde{b} \) is the estimated value of each symbol in \( b_i \), and \( \tilde{i} \) is the set of the indices of the active antennas. Substitute \( X = e_i b_i \) into (3), then it can be simplified to \([23]\):

\[
[	ilde{i}, 	ilde{b}] = \arg\min_{\tilde{H} \in \mathbb{C}^{N_r \times N_t}} \{||y - \tilde{H} b_i||^2 \}
= \arg\min_{\tilde{H} \in \mathbb{C}^{N_r \times N_t}} \{\sum_{r=1}^{N_r} |y_r - \tilde{H}^r b_i|^2 \} .
\]

where \( \tilde{H} = H e_i = [h_{i_1}, h_{i_2}, \ldots, h_{i_{n_t}}] \in \mathbb{C}^{N_r \times n_t} \) is the sub-channel matrix containing \( n_t \) columns chosen from the channel matrix \( H \), \( h_{i_r} \) is the \( r \)-th row of the channel matrix \( H \). \( H^r \) is the \( r \)-th row of \( H \). \( h_o \) is the set of all possible \( H^r \). Furthermore, \( y_r \) is the \( r \)-th entry of the received signal \( y \), and \( b_i \) is as defined in (1).

III. LATTICE REDUCTION

Based on the fact that the channel matrices are inherently complex-valued, we only introduce the concept of complex lattice in this paper. If we interpret \( A = [a_1, a_2, \ldots, a_n], a_i \in \mathbb{C}^{m \times n} (n \leq m) \) as a basis, then a complex lattice spanned by this basis is given by \([6,7]\):

\[
\mathcal{L}(a_1, a_2, \ldots, a_n) = \{\sum_{i=1}^{n} \lambda_i a_i | \lambda_i \in \mathbb{G} \} ,
\]

where \( \lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n]^T \) is the co-efficient vector constituted by Gaussian integer weights, \( \mathbb{G} \) is the set of Gaussian integers \( \mathbb{G} = \mathbb{Z} + j \mathbb{Z}, j = \sqrt{-1} \).

As can be seen from (1), the transmit vector \( X \) is drawn from Gaussian integer space \( \mathbb{Z}[j] \) (e.g. QAM constellation). Given the system model in (2), if we interpret the columns of \( H \) as the basis of a lattice, then \( HX \) belongs to a lattice spanned by the columns of \( H \) \([3]\).

As we know, when the lattice basis \( H \) is orthogonal, linear equalizer has the same performance as ML decoder. However, in general \( H \) is not orthogonal which degrades the performance of linear equalizer. Note the orthogonality deficiency (od) of \( N_r \times N_t \) matrix is defined \([33]\) to quantify the orthogonality. In a word, the closer \( H \) is to an orthogonal matrix, the smaller performance gap will be between the linear equalizer and the ML decoder. Therefore, if we can find another basis \( \tilde{H} \) with better orthogonality than \( H \) to describe the same lattice and use linear equalizer based on \( \tilde{H} \), the performance should be closer to that of ML compared with linear equalizer based on \( H \). Lattice reduction (LR) is such a technique used to find a more orthogonal matrix \( \tilde{H} \) given a matrix \( H \).

In MIMO systems, a new channel matrix \( \tilde{H} = HT \) can generate the same lattice as that of \( H \), if and only if the square matrix of \( N_t \) order \( T \) is unimodular \([5]\), i.e. all elements of \( T \) are Gaussian integers and \( \det(T) = \pm 1 \).

\[
\mathcal{L}(\tilde{H}) = \mathcal{L}(H) \Leftrightarrow \tilde{H} = HT \text{ if } T \text{ is unimodular}.
\]

Thereby, in MIMO systems, using lattice reduction to find a more orthogonal matrix given \( H \) means to find a unimodular matrix \( T \) to transform basis \( H \) into a new basis \( \tilde{H} \) with roughly orthogonal basic vectors. And in MIMO system, it is beneficial to have the basis vectors as short as possible. A famous and efficient reduction criterion named LLL algorithm is first proposed in \([55]\) which finds a vector not much longer than the shortest nonzero vector. Since LLL was originally introduced in real-valued lattice, while lattices in digital communications are complex-valued. A standard approach to deal with this problem is to convert complex lattices into real lattices, but this nearly doubles the computational complexity. Therefore, the complex LLL (CLLL) algorithm was proposed by \([16,3]\) to reduce the complexity by directly using complex basis rather than converting it into real basis. Later, a modified CLLL algorithm with less complexity and negligible BER performance loss was proposed in \([7]\). Note that, in this work, we utilize the CLLL algorithm described in \([7]\).

IV. LR-AIDED DETECTION FOR GSM

In this section, we introduce the conventional LR-aided linear decoders for GSM, namely GSM-LR-ZF and GSM-LR-MMSE, and the proposed precoding aided GSM-LR-ZF and GSM-LR-
MMSE which are termed as PGSM-LR-ZF and PGSM-LR-MMSE. Note that all the above LR aided linear detection schemes need more receive antennas than transmit antennas, unless the underdetermined equation would cause error floor of BER performance.

A. Proposed PGSM-LR-ZF and PGSM-LR-MMSE

As aforementioned, a linear equalizer is optimal with an orthogonal channel matrix. With the newly generated channel matrix $\hat{H} = HT$ using CLLL algorithm, the received signal (2) can be rewritten as:

$$y = HX + n = HT T^{-1}X + n = \hat{H}Z + n.$$  \hfill (7)

where $Z = T^{-1}X$. The idea behind LR aided linear detection is to firstly perform linear detection based on $Z$ instead of $X$, then calculate $X$ using $X = TZ$. The estimated $\hat{Z}$ are obtained as:

$$\hat{Z}_{ZF} = ((H^H \hat{H})^{-1}H^H)y,$$  \hfill (8)

$$\hat{Z}_{MMSE} = ((H^H \hat{H} + \sigma_n^2 T^H T)^{-1}H^H)y.$$  \hfill (9)

Thus the estimated $\hat{X}$ can be calculated as:

$$\hat{X}_{ZF} = T\hat{Z}_{ZF} = X + T(H^H \hat{H} + \sigma_n^2 T^H T)^{-1}H^H n,$$  \hfill (10)

$$\hat{X}_{MMSE} = T\hat{Z}_{MMSE} = X + T(H^H \hat{H} + \sigma_n^2 T^H T)^{-1}H^H n.$$  \hfill (11)

Note that the estimated vectors $\hat{X}_{ZF}$ and $\hat{X}_{MMSE}$ are not necessarily the legal constellation points in the three-dimensional constellation diagram of GSM, so they need to be rounded off to the closest point in the constellation diagram by quantization operation. And the quantized symbol vector $\tilde{X}_{ZF}$ and $\tilde{X}_{MMSE}$ are expressed as $Q(\hat{X}_{ZF})$ and $Q(\hat{X}_{MMSE})$, where $Q(\cdot)$ means the quantization operation.

B. Proposed PGSM-LR-ZF and PGSM-LR-MMSE

Precoding can be viewed as some kind of decoding at the transmitter. In this section, two novel LR-aided decoders with LR-aided precoding at the transmitter are proposed for GSM. In this research, we consider the MIMO system in Time Division Duplex (TDD) mode as suggested by many massive MIMO investigations. In TDD mode MIMO systems, due to the channel reciprocity, the channel state information at transmitter (CSIT) can be acquired directly at the transmitter.

The proposed GSM system is depicted in Fig 1. In PGSM-LR-MMSE/PGSM-LR-ZF system, the input bits $Q$ are modulated to $X$ after GSM mapping, then the modulated vector $X$ is multiplied by the precoding matrix $P$. After that, the transmitted signal is emitted through the flat fading channel. At the receiver, a simplified LR-aided linear decoder is employed.

The transmitted signal can be formulated as:

$$X' = PX = TX,$$  \hfill (12)

where $P = T$ and $T$ is generated by CLLL algorithm given the channel matrix $H$. $X$ is the same as that defined in (1). The received signal in (2) can be rewritten as:

$$y = HX' + n = HTX + n = \hat{H}X + n.$$  \hfill (13)

Then, the estimated modulated vectors, i.e. $\hat{X}_{PZF}$ and $\hat{X}_{PMMSE}$ can be formulated as:

$$\hat{X}_{PZF} = ((\hat{H}^H \hat{H})^{-1}\hat{H}^H)y = X + (\hat{H}^H \hat{H})^{-1}H^H n,$$  \hfill (14)

$$\hat{X}_{PMMSE} = ((\hat{H}^H \hat{H} + \sigma_n^2 T^H T)^{-1}\hat{H}^H)y = X + (\hat{H}^H \hat{H} + \sigma_n^2 T^H T)^{-1}H^H n.$$  \hfill (15)

Then the estimated transmitted vector $\tilde{X}_{PZF}$ and $\tilde{X}_{PMMSE}$ are quantized to the closest point in the constellation diagram of GSM.

V. COMPUTATIONAL COMPLEXITY ANALYSIS OF PGSM-LR-ZF AND PGSM-LR-MMSE

In this section, the receiver computational complexities of GSM-ML, PGSM-LR-ZF and PGSM-LR-MMSE are analyzed. The complexity is computed as the number of required complex addition and multiplication operations at the receiver.

A. GSM-ML Decoder

The complexity of GSM-ML receiver is mainly introduced in computing (4), $(N_t N_t + N_r)$ multiplication operations and
and GSM-LR-ZF/GSM-LR-MMSE is due to the noise enhancement at the receiver when $X$ is estimated from $Z$. In PGSM-LR-ZF and PGSM-LR-MMSE systems, this kind of noise enhancement can be avoided with the help of LR-aided precoding at the transmitter. Among the proposed schemes, the performance of PGSM-LR-MMSE is slightly better than that of PGSM-LR-ZF. Compared to that of GSM-ML, PGSM-LR-ZF/PGSM-LR-MMSE provides fairly lower complexity with small BER performance degradation. More specifically, according to the computational complexities shown in Table 1, PGSM-LR-ZF offers 50% and 60% complexity reductions compared to that of GSM-ML under conditions with 4 and 5 transmit antennas respectively. More impressive complexity reduction can be observed in Fig.3. The ML detection provides 4 dB SNR gain over PGSM-LR-ZF and PGSM-LR-MMSE schemes at the BER of $10^{-5}$ with $N_t = 4, n_t = 2$, and this SNR gain reduces to 2.5 dB when $N_t = 5, N_r = 8, n_t = 2$.

**VI. SIMULATION RESULTS**

In this section, Monte Carlo simulation results for at least $10^6$ flat fading channel realisations are presented to compare the BER performances and computational complexities of GSM-ML, PGSM-LR-ZF/PGSM-LR-MMSE and GSM-LR-ZF/GSM-MMSE. 4-QAM is considered for all the simulations. The performances of GSM-ZF/GSM-MMSE are also provided for reference.

1. BER comparisons between GSM-ML, PGSM-LR-ZF/PGSM-LR-MMSE and GSM-LR-ZF/GSM-MMSE.

Fig. 2 and Fig.3 show the BER comparisons between GSM-ML, GSM-MMSE/GSM-ZF/GSM-LR-MMSE/GSM-LR-ZF; and PGSM-LR-MMSE/PGSM-LR-ZF for $N_r = N_t = 4, n_t = 2$ and $N_r = 5, n_t = 2, N_r = 8$ respectively. It can be observed that the performances of the GSM-LR-ZF and GSM-LR-MMSE are unsatisfactory. PGSM-LR-MMSE and PGSM-LR-ZF achieve significant performance improvements compared to that of GSM-MMSE/GSM-ZF and GSM-LR-MMSE/GSM-LR-ZF. For example, with $N_t = 5, n_t = 2, N_r = 8$ PGSM-LR-MMSE provides 5 dB and 4.5 dB SNR gains over GSM-MMSE and GSM-LR-MMSE respectively. And from Table I, the computation complexity of PGSM-LR-MMSE is slightly lower than that of GSM-LR-MMSE. The BER performance gap between the proposed PGSM-LR-ZF/PGSM-LR-MMSE
2. Complexity comparisons between PGSM-LR-ZF/PGSM-LR-MMSE and GSM-ML.

Fig.4 compares the computational complexity of ML, PGSM-LR-ZF and PGSM-LR-MMSE with different numbers of transmit antennas and different modulation orders. It can be obviously observed that the complexity of ML decoder is much higher than that of the other two decoders. For example, in a system with $N_t = N_r = 16, M = 8$, PGSM-LR-MMSE offers more than 90% complexity reduction compared to that of GSM-ML. Furthermore, the complexity of PGSM-LR-ZF/PGSM-LR-MMSE is mainly determined by $N_t$ and $N_r$. However, the complexity of GSM-ML is greatly affected by $N_t$, $N_r$ and $M$.

<table>
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<tr>
<th>Detection schemes</th>
<th>$N_t$</th>
<th>$N_r$</th>
<th>$N_i$</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSM-ML</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GSM-LR-ZF</td>
<td>4</td>
<td>8</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>GSM-LR-MMSE</td>
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<tr>
<td>PGSM-LR-MMSE</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

### Table 1: Computational Complexity of Each Detection Scheme with Specific System Parameters

In this paper, we introduced the conventional Lattice reduction (LR) aided linear decoders in GSM systems for the first time and proposed two LR-aided detection schemes with LR-aided precoding at the transmitter for GSM. Their BER and complexity performances for different system parameters are investigated. BER and complexity performances of GSM with ZF, MMSE, and ML decoders are also introduced for comparison. Simulation results show that the conventional LR-aided linear decoders are not suitable to be directly applied to GSM. The proposed PGSM-LR-ZF/PGSM-LR-MMSE achieves significant BER improvements compared to that of GSM-LR-ZF/GSM-LR-MMSE with even lower complexity. Compared to that of GSM-ZF/GSM-MMSE, PGSM-LR-ZF/PGSM-LR-MMSE offers much better BER performance with the same complexity. Moreover, PGSM-LR-ZF/PGSM-LR-MMSE provides fairly lower complexity with a small BER performance degradation compared to that of GSM-ML.

### References