1. Introduction

Making decisions is a cognitive process that aims to achieve the most desired objectives with the least expected penalties. Multi-criteria Decision-Making (MCDM) refers to making decisions in the presence of several, and often conflicting, criteria and objectives. The criteria in MCDM are not always supposed to be tangible, and hence may not be measurable in well-defined units. Saaty (2008) highlighted this need to measure the relative importance of given options for intangible criteria and declared the use of pairwise comparison (PC) as being "central" for this purpose. When n elements are to be ranked, the PC judgments can be used to construct a matrix of order $n \times n$.

A complete set of judgments in the PC matrix (PCM) creates an opportunity to have inconsistent information, primarily due to the redundancy inherent in its structure. The issue of inconsistency in PCs has been discussed by many authors e.g. (Ramanathan and Ramanathan, 2009; Saaty, 2008; Laininen and Hämäläinen, 2003). There exist situations in practice where the acquired judgments cannot be revised. For example, it may not be possible to revise judgments collected through anonymous surveys. Another possible reason could be to avoid cost of the revision process. In such situations, a prioritization method has to be applied in order to elicit preferences from an inconsistent set of provided judgments. Several prioritization methods have been proposed in the literature (Lin, 2007; Choo and Wedley, 2004).

The Analytic Hierarchy Process (AHP), proposed in (Saaty, 1977), is an MCDM technique based on PC to assess relative importance of criteria and alternatives. The main benefit of using this approach is to convert both objective and subjective judgments into relative weights of importance. The applications of AHP have been numerous (Vaidya and Agarwal, 2006; Ngai and Chan, 2005), and it was recently considered to be the most active area of research in MCDM (Wallenius et al., 2008).

This paper presents a priority estimation tool, PriEsT, that has been developed to support AHP decision making. In contrast to existing software tools based on AHP, PriEsT better assists decision makers (DMs) to interactively identify and revise their inconsistent judgments based on newly proposed consistency measures. Further, PriEsT offers multiple equally-good solutions using multi-objective optimization - hence the DM has the flexibility to select any of these non-dominated solutions according to his/her requirements. PriEsT is an open-source software and is freely available on the world wide web.

The paper is structured as follows: the PC method and advances in the area of PC-based decision making are discussed in Section 2; Section 3 provides an overview of the PriEsT tool; the rationale and design approach of the PriEsT tool is discussed in Section 4; Section 5 demonstrates and evaluates the features of PriEsT through an application which highlights the presence of intransitive judgments in the acquired data, and how their correction leads to a different ranking of the available alternatives; finally, conclusion are presented in Section 6.

2. Background

The decision problems are usually decomposed into four steps i.e. defining a problem, structuring the problem, acquiring judgments and finally eliciting preferences from the acquired judgments. AHP (Saaty, 1980) is a decision making technique that enables DMs to evaluate the relative importance of alternatives with the help of both, objective and subjective types of judgments. In AHP, the criteria are usually structured in a hierarchical fashion where ultimate goal is represented as root node and alternatives are placed at the bottom of this hierarchy.

Suppose, a city dweller wants to choose a mode of transporation for him to commute to the office which is about 3 miles away from his home. He has four alternatives available in that city i.e. by bus, by car, by walk, or using a bi-cycle. The dweller - DM in this case - has to consider two main criteria of cost and convenience. Cost can be further sub-divided into one-time payments, daily charges, and maintenance cost. Similarly, the convenience criterion can also be divided into travel time, health, fatigue, and safety hazards.

After structuring the problem, the next step is to explore alternatives and acquire judgments from DM. In AHP, the judgments are acquired using the PC method where only two criteria or alternatives are compared at one time.

2.1. Pairwise Comparison Judgments

Consider a prioritization of n elements. In the PC method, a DM assesses the relative importance of any two elements, E_i and E_j, by providing a ratio judgment a_{ij}, specifying by how much E_i is preferred to E_j. The judgment is provided with respect to some predetermined preference scale. In the case of *tangible* criteria, this can be derived from the directly measured information as, for example, weights (in kgs) or price (in euros). In the case of *intangibles*, a set of verbal judgments may be provided that correspond to the ratio-scale of 1 to 9 (Saaty, 1977). These judgments can be used to construct a matrix $A = [a_{ij}]$ of the order $n \times n$. The PC matrix (PCM) includes all the self-comparison and reciprocal judgments.

2.2. Inconsistency in PC Judgments

A complete set of judgments in the PC method creates an opportunity to have inconsistent information, primarily due to the redundancy inherent in its structure. There are several causes of inconsistency including psychological reasons, clerical errors and an insufficient model structure (Sugden, 1985). Consistency in PCs is generally of two types i.e. *cardinal consistency* (CC) and *ordinal consistency* (OC). The judgments of DMs are cardinally consistent, if $a_{ij} = \frac{1}{a_{ji}}$ and $a_{ij} = a_{ik}a_{kj}$ for all i, j and k. OC states that if E_i is preferred to E_j and E_j is preferred to E_k, then E_i should be preferred to E_k i.e. If $E_i \to E_j \to E_k$ then $E_i \to E_k$.

2.3. Measuring Inconsistency in Judgments

There exist several measures proposed to assist a DM in accepting and/or updating the acquired judgments. Widely used measures are *Consistency Ratio* (Saaty, 1977), *Logarithmic residual mean square* (Crawford and Williams, 1985) and *Consistency Measure* (Koczkodaj, 1993). Siraj (2011) investigated these consistency measures with the help of Monte-Carlo simulations and the results suggested a need to propose new measures for consistency. Considering the consistency test between E_i and E_j i.e. $a_{ij} = a_{ik}a_{kj}$ (for all i, j, k), he proposed a cardinal consistency measure, called *congruence*, as:

$$\theta_{ij} = \frac{1}{n-2} \sum_{k=1}^{n} \left|\log\left(a_{ij}\right) - \log\left(a_{ik}a_{kj}\right)\right| \tag{1}$$

and an ordinal consistency measure, called *dissonance*, as:

$$\psi_{ij} = \frac{1}{(n-2)} \sum_{k} step\left(-\log a_{ij} \log a_{ik}a_{kj}\right) \tag{2}$$

where $i \neq k \neq j \neq i$ and the *step* function returns 1 for positive values and 0 otherwise.

The two measures can be used together to detect and highlight *outlying judgments*. The congruence measure can also detect the presence of *consistency deadlock* where all the provided judgments are equally inconsistent. It is recommended to use these measures as a useful addition to PC-based decision support tools.

2.4. Prioritization from Inconsistent PC Judgments

Suppose that there exists a preference vector $r = (r_1, r_2, ..., r_n)^T$ such that r_i represents the preference intensity of E_i where $i = 1, 2, ..., n$. However, the preference vector r is unknown to a DM and should be estimated. The prioritization problem is to determine a priority vector $w = (w_1, w_2, ..., w_n)^T$ which estimates the unknown preference vector r. The priority weights in ratio-comparisons are considered to have non-zero positive values ($w_i > 0$) and usually calculated with the additional constraint of normalization i.e. $\sum w_i = 1$.

There are many prioritization methods that can be applied to derive a priority vector from a set of PC judgments (Choo and Wedley, 2004). The most widely-used are the Eigenvector (EV) method (Saaty, 1977) and the Geometric Mean (GM) method (Crawford, 1987). It was shown that all prioritization methods give equal results in the case of error-free (consistent) judgments, however, the results are different when the PCM is inconsistent (Choo and Wedley, 2004).

Except for the EV Method, all the widely-used methods are based on optimization. In optimization-based methods, an objective function is formulated that needs to be minimized. For example, Chu et al. (1979) proposed to minimize the total deviation (TD) between the given judgments, a_{ij} and the estimated weights, w. The distance function, $TD(w)$ (or *minimand*), for the *direct least squares*

(DLS) can be formulated as:

$$TD(w) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(a_{ij} - \frac{w_i}{w_j}\right)^2 \tag{3}$$

where $\sum w_i = 1$.

When E_i is preferred to E_j, or $a_{ij} > 1$, it is assumed that the estimated priority vector should preserve the preference direction i.e. $w_i > w_j$. However, while eliciting preferences, if E_j receives a larger priority weight i.e. $w_i < w_j$, then a *priority violation* occurs. Considering the ratio judgments, a violation can be formulated as a logarithmic test: $v_{ij} = step\left(\log a_{ij} \log \frac{w_j}{w_i}\right)$, where the *step* function returns 1 for positive values and 0 otherwise.

2.4.1. Prioritization using Indirect Judgments

Mikhailov (2006) highlighted that minimizing TD produces a solution with a greater number of priority violations (NV) and therefore introduced a Two-Objective Prioritization (TOP) method to optimize both TD and NV. The use of an evolutionary multi-objective optimization technique was proposed for this purpose (Mikhailov and Knowles, 2010).

The concept of using TOP has further been developed in (Siraj et al., 2012c) which has proposed minimization of second-order deviations, TD2, along with the two objectives of TD and NV. This method of prioritization using indirect judgments (PrInT) can be formulated as:

$$\begin{aligned} &minimize \quad [TD(w),\, TD2(w),\, NV(w)]^T \\ &s.t. \quad \sum_i w_i = 1, w_i > 0, i \in \{1, 2, ..., n\} \end{aligned}$$

where

$$TD2(w) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\left(a_{ik}a_{kj} - \frac{w_i}{w_j}\right)^2 \tag{4}$$

The generated priority vectors using this method are then provided to the DM to select one according to his/her requirements. Similar to TOP, the PrInT solutions can also be generated using an evolutionary multi-objective optimization approach (Mikhailov and Knowles, 2010).

These consistency measures and prioritization methods have been implemented as part of a decision aid tool called PriEsT, which is overviewed in the next section.

3. An Overview of PriEsT

PriEsT, a priority estimation tool, has been developed as a decision support tool based on AHP. There exist software tools based on AHP, for example, ExpertChoice (Forman et al., 1983) and HIPRE (Hämäläinen and Lauri, 1995), however, they lack the capability to visualize inconsistency among the given PC judgments. This has been the primary consideration whilst developing the new tool. PriEsT can assist DMs to interactively explore and revise their judgments based on the *congruence* and *dissonance* measures (Siraj, 2011). PriEsT also offers multiple equally-good solutions using multi-objective optimization; unlike other tools which offer only a single solution (see the companion work in (Siraj et al., 2012a)). PriEsT implements the proposed technique offering a wide range of Pareto-optimal solutions where the DM has the flexibility to select any of these non-dominated solutions according to his/her requirements.

3.1. Decision Aid

PriEsT offers different ways to help users identify inconsistency in their judgments. The proposed measure of *congruence* and *dissonance* are useful in finding the contribution of individual judgments towards overall inconsistency of a PCM and, therefore, can be used to detect and correct inconsistent judgments.

Consider the example in Fig. 1 where four alternatives are compared on the ratio scale of 1.0 to 99.0. PriEsT clearly shows the level of inconsistency for each judgment provided by a DM. The congruence and dissonance measures are plotted as bar graphs against their respective judgments. The most inconsistent triple (set of three judgments) is also shown with the help of small dots on the blamed judgments.

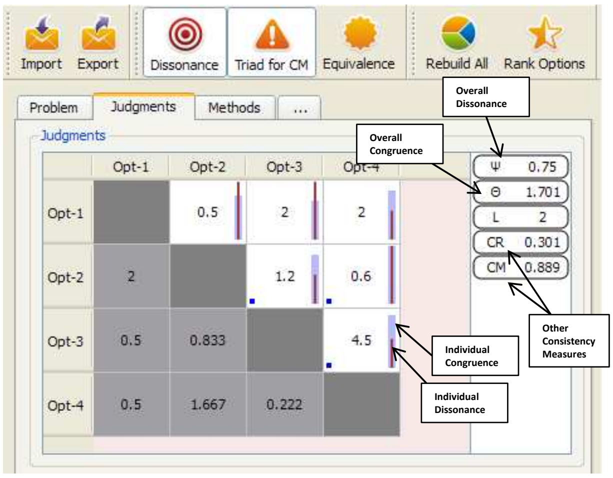


Figure 1: Visualizing Inconsistency in Table View

The use of a graph view has much potential in helping experts to analyze acquired judgments. The graph view proves particularly helpful in visualizing inconsistency in provided judgments. A car selection example is shown in Fig. 2 where the judgments provided by the user violates the transitive property of ratio judgments. The set of judgments is provided in a table (on bottom-left of the figure), and is also plotted graphically on the right side. Each judgment is shown as a connector between two elements (nodes) wherein the bolder side of each line shows the dominating element. The preference of BMW over Toyota and Toyota over Audi suggests that BMW should be preferred over Audi. However, the judgment provided by the user has violated this rule of transitivity. In the case where there are multiple intransitive judgments, PriEsT provides the user a set of buttons below the graph view to highlight them one at a time.

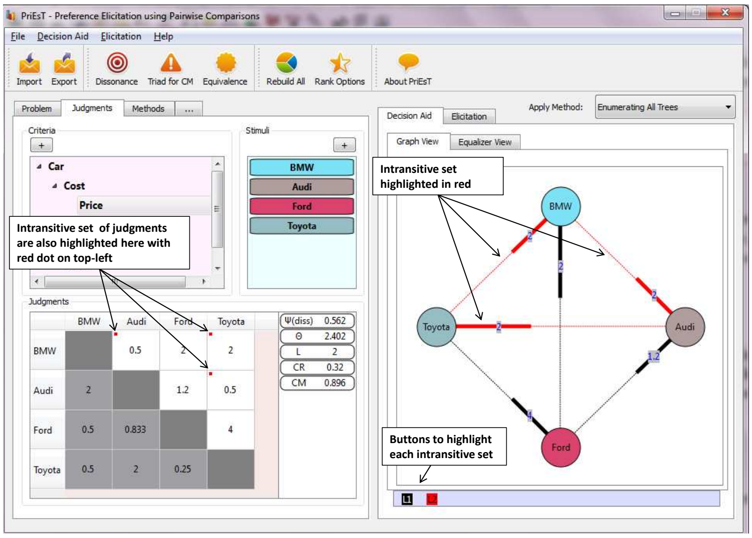


Figure 2: Graph View for Intransitive Set of Judgments

Plotting all judgments on a measurement scale has also been found useful to analyze inconsistency between direct and indirect judgments. This helps in visualizing cognitive dissonance present in the set of provided judgments (Siraj, 2011). Fig. 3 shows how this aids the DM in finding the potential cause of priority violations. For example, BMW has been preferred by DM over Audi, and all the other (indirect) judgments also support this order of preference. Therefore, there is no priority violation amongst these judgments. In contrast, Audi has been preferred over Ford which is in contradiction to what other judgments have suggested. This indirect judgment is highlighted as a small dot pointed by an arrow emerging from the label "Latent Violation" on the right side of Fig. 3.

3.2. Elicitation

Users of PriEsT are allowed to select different prioritization methods to estimate preferences from the same set of judgments. PriEsT therefore qualifies as an appropriate research and experimentation tool to evaluate such methods.

3.2.1. List of Solutions and Gantt View

The solutions generated by different methods are displayed as a list containing all numerical values of the generated weights. An alternative option is also provided for users to view the generated weights in the form of a Gantt chart. A method producing a different set of rankings can easily be spotted when viewed as Gantt chart.

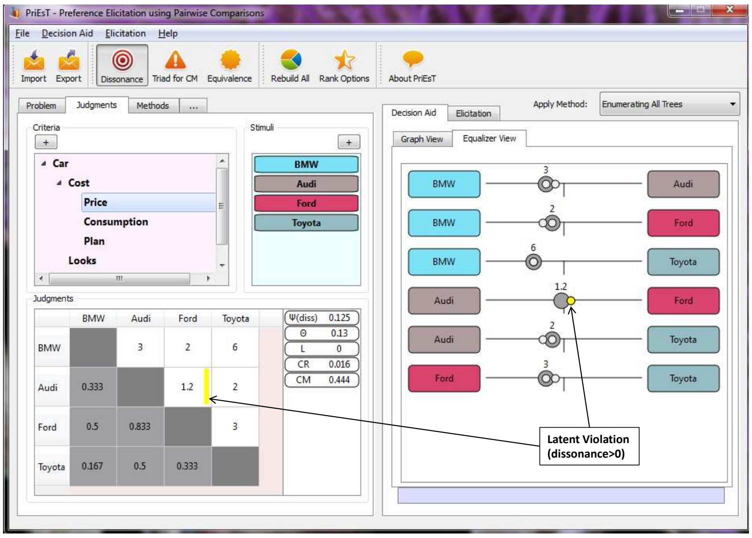


Figure 3: Dissonance Visible in Equalizer View

3.2.2. Objective Space

Along with TD, the need to minimize NV and TD2 has been highlighted in Section 2. PriEsT offers the DMs an interactive selection of any non-dominated solution by plotting them on two different objective spaces. The first is TD-NV space as shown in Fig. 4a, while the second is TD-TD2 space, proposed in (Siraj et al., 2012a) (shown in Fig. 4b). The objective space of TD2 versus NV needs to be investigated, and will be considered for implementation in future.

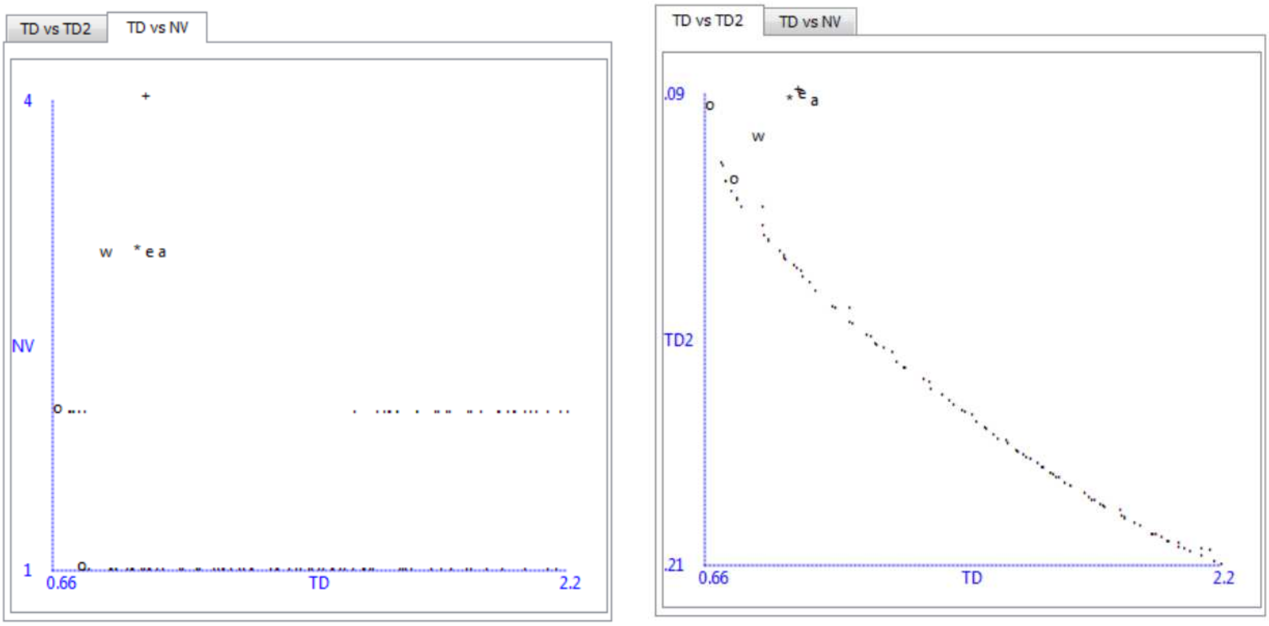
PriEsT offers the DMs an interactive selection of a non-dominated solution by plotting them on a two-objective space (as proposed in (Mikhailov and Knowles, 2010)). The solutions generated by different methods are displayed as a list containing all numerical values of the generated weights. An alternative option is also provided for users to view the generated weights in the form of a Gantt chart.

3.3. Other features

The use of the XML format enables easy integration of PriEsT with other tools and web technologies without necessitating major changes in its architecture. The use of XML also allows integration with spreadsheet applications (e.g. Microsoft Excel) and the importing of data from other software tools.

4. Rationale and Design Approach

PriEsT has been developed in Java using the NetBeans IDE. The application is based on the Model-View-Controller (MVC) architecture. Each "View" class has an associated "Delegate" class to communicate with its respective Model. All the data is ultimately preserved in a relational database.



(a) TD vs. NV (b) TD vs. TD2

Figure 4: Visualizing Solutions in Objective Space

4.1. PriEsT Engine

The core of the PriEsT software is its engine, which has been designed to be independent of user-interface libraries. The front-end has then been built on top of this engine. The engine consists of several building blocks: the *base* to implement basic operations and data-structures, *pre-processors* to remove outliers and/or estimate missing judgments, *prioritization methods* implementation, and *analysts* to calculate various properties for given problems. The building blocks are briefly discussed below.

4.1.1. Base

This block consists of the basic classes required to support pairwise comparisons i.e. *PC* (for Pairwise Judgments), *JudgmentScale* (for Measurement Scales) and *W* (for Priority Vectors).

4.1.2. Factories

A set of factory classes generate PCMs possessing different properties e.g. consistent, intransitive, acceptable etc. In addition to this, *PersistentFactory* allows save and/or load of PCMs from text files (serialization).

4.1.3. Analysts

This work has involved the analysis of several different properties of PCMs. This block contains the code for analysis of PCMs: *ConsistencyAnalyzer* calculates Eigenvalues, CR and CM for a given PCM; *IndirectAnalyzer* is useful to calculate θ and ψ based on indirect judgments; the *TournamentAnalyzer* class calculates the number of intransitive judgments (three-way cycles).

4.1.4. Pre-processors

The pre-processors cover possible pre-processing of PCMs before prioritization. For example, the *CyclesRemover* class suggests the removal of intransitive judgments by implementing the heuristic algorithm proposed in (Siraj et al., 2012b).

4.1.5. Methods

All prioritization methods are implemented in this block of code, including both the matrix-based and the optimization-based algorithms. Each optimization algorithm provides an objective function from the set of available objectives.

4.1.6. Objectives

This block implements the major objective functions proposed for prioritization in the PC literature. This includes TD, NV, TD2, logarithmic deviations and absolute errors. Satisfaction index is also implemented for the *Fuzzy Preference Programming* (FPP) method (Mikhailov, 2000).

4.2. Front-end Application

The user-interface application is based on the Model-View-Controller (MVC) architecture. Each "View" class has an associated "Delegate" class to communicate with its respective "Model". All the data is ultimately preserved in a relational database. The user-interface of the PriEsT Engine was developed using the Qt framework - an open-source cross-platform software development kit (SDK) for writing applications in C++ or Java.

5. Case Study: Telecom Backbone Selection

In order to demonstrate the utility of the features of PriEsT, we consider the practical data acquired in a recent study: the selection of a backbone infrastructure for telecommunication in rural areas (Gasiea, 2010). This application is primarily focused on the rural areas of developing countries, where the lack of adequate telecommunications infrastructure remains a major obstacle for providing affordable services.

The four alternatives are Fiber-optic cable (G1), Power-line communication (G2), Microwave link (G3) and Satellite communication (G4). The problem was solved using AHP and the criteria used to compare these alternatives were grouped into six major categories including technical, infrastructural, economic, social, regulatory and environmental factors. These categories and their constituent criteria are presented in Fig. 5. The PCM, A_{top}, acquired for prioritizing these six categories (top-level criteria) is shown in Fig. 6a. Although A_{top} is a transitive PCM, the estimated vectors produce a priority violation (NV=1) with the widely used EV and GM methods.

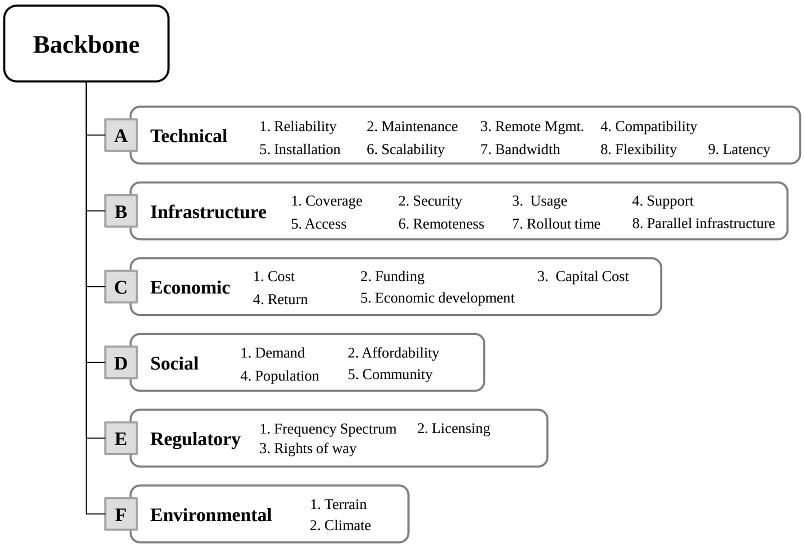


Figure 5: Criteria to compare the available backbone infrastructures

The final weights calculated using the EV and GM methods are found to be almost identical, as given in Table 1 in normalized form. Satellite communication (G4) is considered the most preferred alternative with a weight of 29.95% (using EV), followed by Microwave (G3) with a weight around 28.34% (using EV).

	Optic Fiber w_{G1}	Power-line w_{G2}	Microwave w_{G3}	Satellite w_{G4}
EV:	21.7%	20.1%	28.3%	29.9%
GM:	21.7%	20.1%	28.4%	29.8%

Table 1: Estimated weights for the available backbone infrastructure options

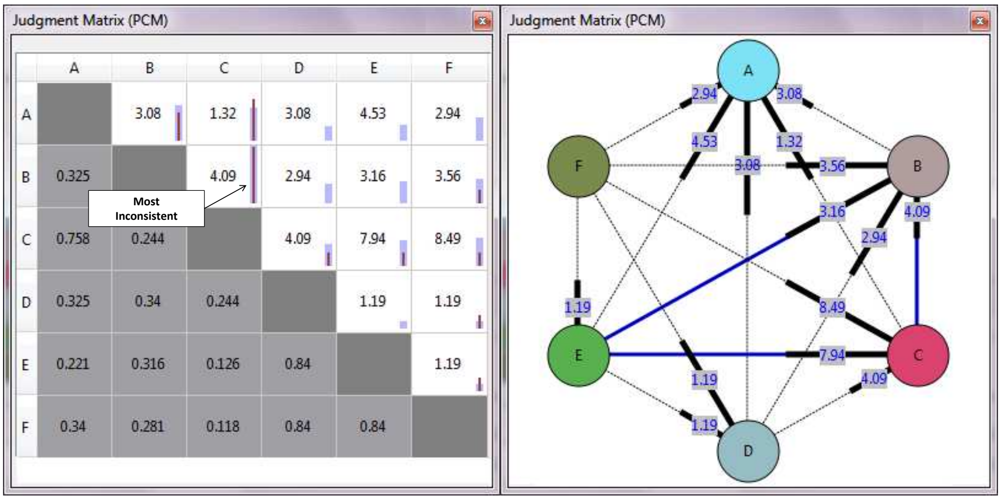
Most criteria lie under the *Technical* and *Infrastructure* categories. The *Technical* category includes nine criteria whilst the *Infrastructure* category has eight criteria used to compare the alternatives. The two PCMs for the *Technical* and *Infrastructure* categories, A_{tech} and A_{infra}, have been found to be intransitive and should be investigated along with A_{top} for their impact on the final result.

5.1. Investigation using PriEsT

The three matrices, A_{top}, A_{tech} and A_{infra} have been analyzed using PriEsT, using both the table-view and the graph-view. Next, we discuss these PCMs individually.

5.1.1. A_{top}

The two views for A_{top} are shown in Fig. 6. Fig. 6a is a snapshot of the PCM when viewed as a table and the graph view is shown in Fig. 6b. The labels A to F in these figures correspond to the labels listed in Fig. 5.



(a) Table-view for A_{top} (b) Graph-view for A_{top}

Figure 6: Visualizing PC Judgments in A_{top}

The CR for this PCM is equal to 0.136 and therefore unacceptable in AHP terms. The contribution of each judgment towards overall inconsistency is visible in the table view. The most inconsistent judgment according to the *congruence* and *dissonance* measures is determined to be $a_{23} = 4.09$. The graph view helps to highlight the most inconsistent set of judgments. i.e. a_{23}, a_{25} and a_{35} (see Fig. 6b). This also suggests that the judgment a_{23} is amongst the most inconsistent.

PCM	Method	w (Estimated weights)								
A_{top}										
	EV:	[.3046	.2811	.2444	.0649	.0521	.0530]^T			
	GM:	[.3074	.2461	.2524	.0760	.0595	.0585]^T			
A_{tech}										
	EV:	[.2091	.2021	.1225	.0648	.1199	.0391	.1841	.0357	.0228]^T
	GM:	[.2202	.1931	.1234	.0635	.1161	.0406	.1883	.0342	.0205]^T
A_{infra}										
	EV:	[.3934	.0888	.0244	.0606	.0834	.1500	.0526	.1468]^T	
	GM:	[.4102	.0821	.0246	.0530	.0928	.1398	.0555	.1420]^T	

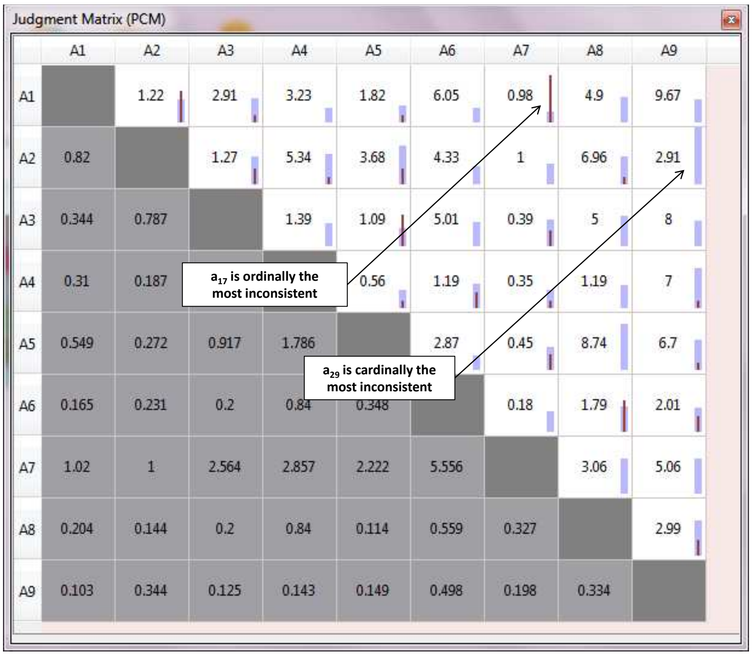
Table 2: Estimated values for the criteria weights

The priority vectors obtained using EV and GM are given in Table 2. The *ideal* ranking possible for this PCM is $A \to B \to C \to D \to E \to F$, however, the ranking order suggested by EV is $A \to B \to C \to D \to F \to E$. Although the judgments were found to be *transitive*, the EV method has violated order of preference for one judgment i.e. the judgment $a_{56} = 1.19$ suggests $E_5 \to E_6$ but the estimated value w_5 is less than w_6. GM produces a different ranking order:

$A \to C \to B \to D \to E \to F$. This method has also generated a priority violation but at a different location i.e. $w_2 < w_3$ when $a_{23} > 1$.

5.1.2. A_{tech}

The table-view for A_{tech} is shown in Fig 7. The most inconsistent judgment according to the *congruence* measure is found to be a_{29}. However, the ordinal consistency measure, *dissonance*, suggests a_{17} as the most inconsistent. There exists a three-way cycle in this PCM i.e. $E_1 \to E_2 \sim E_7 \to E_1$. The judgment a_{29} does not contribute to this three-way cycle present in the PCM.



	A1	A2	A3	A4	A5	A6	A7	A8	A9
A1		1.22	2.91	3.23	1.82	6.05	0.98	4.9	9.67
A2	0.82		1.27	5.34	3.68	4.33	1	6.96	2.91
A3	0.344	0.787		1.39	1.09	5.01	0.39	5	8
A4	0.31	0.187	[illegible]		0.56	1.19	0.35	1.19	7
A5	0.549	0.272	0.917	1.786		2.87	0.45	8.74	6.7
A6	0.165	0.231	0.2	0.84	0.348		0.18	1.79	2.01
A7	1.02	1	2.564	2.857	2.222	5.556		3.06	5.06
A8	0.204	0.144	0.2	0.84	0.114	0.559	0.327		2.99
A9	0.103	0.344	0.125	0.143	0.149	0.498	0.198	0.334	

Figure 7: Table-view for A_{tech}

EV and GM solutions are given in Table 2. Both solutions give $NV = 1.5$ i.e. the judgment a_{17} suggests $E_7 \to E_1$ whilst w_1 is higher than w_7 and the half-violation is added due to the presence of *preference equivalence* - the judgment a_{27} suggests $E_2 \sim E_7$ but w_2 is greater than w_7.

An intransitive PCM cannot produce a solution with $NV = 0$ therefore, the three-way cycle has to be removed. The *dissonance* measure suggests a_{17} should be revised. Therefore, inverting the judgment of a_{17} will make the PCM transitive.

5.1.3. A_{infra}

The table-view of A_{infra} is given in Fig 8. The most inconsistent judgment according to the *congruence* measure is found to be a_{46}. The ordinal consistency measure, *dissonance*, also suggests a_{46} as the most inconsistent. There exists four three-way cycles in this PCM i.e.

$$L_1: \quad E_2 \to E_4 \to E_6 \to E_2$$

$$L_2: \quad E_6 \rightarrow E_5 \rightarrow E_4 \rightarrow E_6$$
$$L_3: \quad E_4 \rightarrow E_6 \rightarrow E_8 \rightarrow E_4$$
$$L_4: \quad E_4 \rightarrow E_6 \rightarrow E_7 \sim E_4$$

The judgment $a_{46} = 1.666$ has contributed the most to the three-way cycles present in the PCM. By inverting only the judgment a_{46}, all the three-way cycles can be rectified.

The EV and GM solutions for A_{infra} are provided in Table 2 here both vectors generate two and a half violations i.e. $NV = 2.5$. The EV solution has violated a_{25} while the GM solution violated a_{68} instead. The judgments a_{46} and a_{47} have been violated by both the EV and GM solutions.

Judgment Matrix (PCM)

	B1	B2	B3	B4	B5	B6	B7	B8
B1		5.18	9	6.96	5.18	2.23	6.65	7.17
B2	0.193		6.88	3.31	0.89	0.2	3.08	0.19
B3	0.111	0.145		0.64	0.24	0.32	0.19	0.2
B4	0.144	0.302	1.562		0.38	1.67	1	0.32
B5	0.193	1.124	4.167			0.46	1.97	0.57
B6	0.448	5	3.125	0.599	2.174		3	1.19
B7	0.15	0.325	5.263	1	0.508	0.333		0.46
B8	0.139	5.263	5	3.125	1.754	0.84	2.174	

a_{46} is ordinally and cardinally the most inconsistent

Figure 8: Table-view for A_{infra}

5.2. Improving Consistency

Exploration of the judgment space has highlighted the main sources of inconsistency i.e.,

1. A_{top} is found to be unacceptable in AHP terms (CR=0.136); the major source of inconsistency is found to be $a_{23} = 4.09$, which is both ordinally and cardinally most inconsistent.

2. A_{tech} is found to be ordinally inconsistent (intransitive); inverting the judgment of a_{17} can make the PCM transitive.

3. A_{infra} is also an intransitive PCM with $L = 4$, i.e. with four three-way cycles; each three-way cycle can be removed by inverting a single judgment i.e. $a_{46} = 1.67$.

These judgments should be revised in order to improve the overall consistency of these matrices. As the judgments here cannot be revised manually, the suggested new values for the blamed judgments are:

1. A_{top}: Change a_{23} from 4.09 to 0.99
2. A_{tech}: Change a_{17} from 0.98 to 1.01
3. A_{infra}: Change a_{46} from 1.67 to 0.99

The suggested values are calculated using methods discussed in Siraj (2011). The final weights calculated after these improvements are given in Table 3 in normalized form.

	Optic Fiber w_{G1}	Power-line w_{G2}	Microwave w_{G3}	Satellite w_{G4}
EV:	20.7%	21.2%	29.4%	28.7%
GM:	20.8%	21.4%	29.2%	28.6%

Table 3: Weights suggested by PriEsT for the backbone infrastructure options

Satellite communication (G4) is no longer the most preferred alternative, its weight has been reduced to 28.74% from 29.95%. The new results indicate that Microwave (G3) is the best alternative with a weight of 29.35% (using EV). The results for both EV and GM are almost in-differentiable.

5.3. Prioritization using PrInT

As mentioned earlier, there exist situations when revision of judgments is not allowed and prioritization is required without attempting to remove inconsistency. PriEsT has the ability to solve this problem using different prioritization methods. The solutions for the three matrices, A_{top}, A_{tech} and A_{infra} have been obtained in PriEsT using EV, GM and PrInT. The results are discussed below.

Table 4 lists the solutions for A_{top} generated by EV, GM and PrInT. When seen in the TD-TD2 plane, shown in Fig. 9, the EV and GM solutions are clearly dominated by the PrInT solutions.

PrInT has produced several solutions with $NV = 0$ and $NV = 1$. Fig. 9 shows all these solutions; the solutions having $NV > 0$ are not listed in Table 4 being less relevant.

Method	TD	TD2	NV	w_1	w_2	w_3	w_4	w_5	w_6
EV	1.4208	5.9541	1	0.3046	0.2811	0.2444	0.0649	0.0521	0.053
GM	1.329	6.1757	1	0.3074	0.2461	0.2524	0.076	0.0595	0.0585
PrInT-1	1.3065	5.9825	0	0.2342	0.1937	0.3887	0.0797	0.051	0.0528
PrInT-2	1.3073	5.9577	0	0.2138	0.2017	0.4027	0.0806	0.0521	0.0491
PrInT-3	1.3522	5.8889	0	0.2923	0.2746	0.2704	0.0676	0.0494	0.0457
PrInT-4	1.3562	5.8643	0	0.2859	0.2787	0.2744	0.0685	0.0479	0.0446
PrInT-5	1.383	5.8498	0	0.2832	0.282	0.2713	0.0737	0.046	0.0437
PrInT-6	1.4477	5.813	0	0.2933	0.2787	0.2744	0.0659	0.0456	0.0421
PrInT-7	1.5404	5.8032	0	0.306	0.2893	0.2457	0.0683	0.0473	0.0435

Table 4: Solutions for A_{top}

Similarly, the solutions for A_{tech} and A_{infra} are listed in Tables 5 and 6. In both cases, the EV and GM solutions generate more violations than the PrInT solutions. Moreover, the EV and GM solutions are again dominated by the PrInT solutions, as shown in Fig. 10 and Fig. 11.

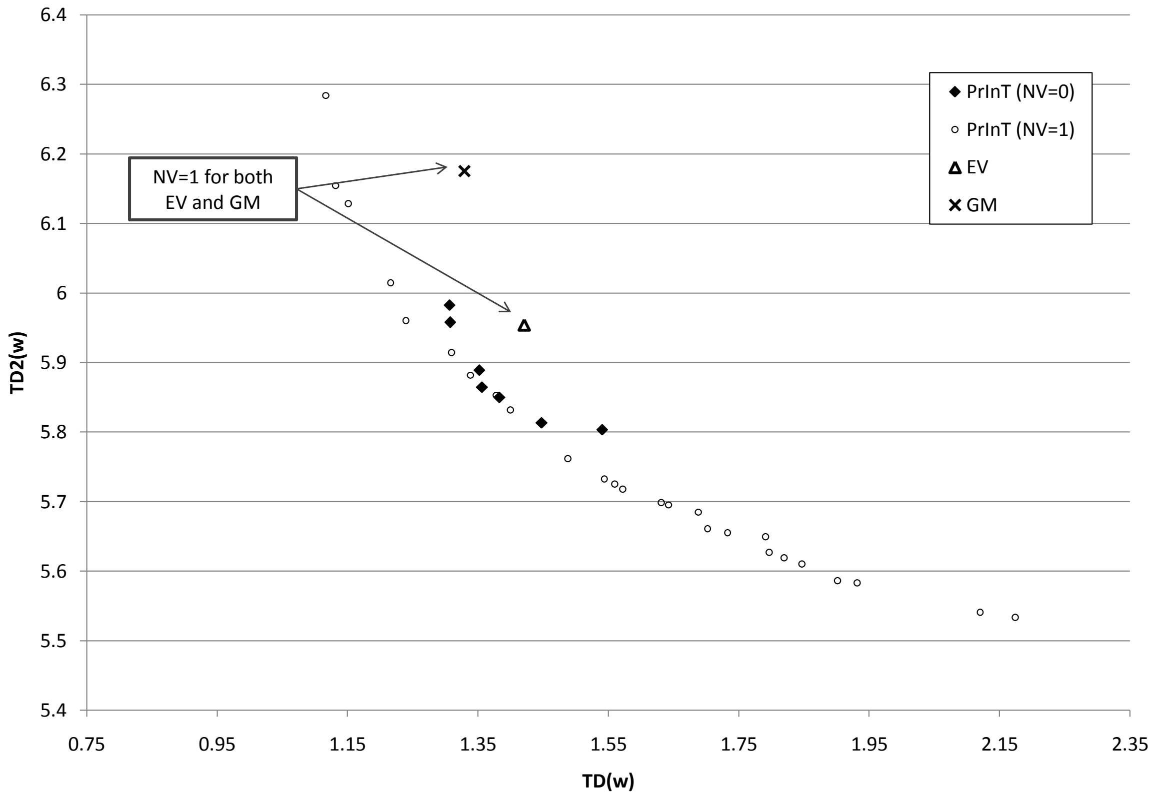


Figure 9: Solutions for A_{top} in TD-TD2 plane

The solutions generated by PrInT are equally good and therefore any of them could be selected by the DM. Consider a situation where the solutions selected are PrInT-7 for A_{top}, PrInT-2 for A_{tech} and PrInT-32 for A_{infra} (see tables 4, 5 and 6). The overall weights generated with the help of these solutions will be $w_{G1} = 21.3\%$, $w_{G2} = 19.7\%$, $w_{G3} = 29.4\%$ and $w_{G4} = 30.6\%$.

Choosing a different solution from the set of non-dominated ones will obviously result in different weights. Although different, no solution can be declared to be inferior.

It can be argued that PriEsT should produce a single solution to support situations where user interaction is not possible. We consider this to be a future area of research: 'the selection of the most appropriate solution from within a set of Pareto-optimal solutions' in the context of pairwise comparisons.

6. Conclusion

This article has discussed the rationale, design and evaluation of a priority estimation tool (PriEsT) that offers several new features. PriEsT offers innovative ways to help users explore and identify inconsistency in their judgments. The graph view proves helpful in visualizing such inconsistency in provided judgments. PriEsT also assists the DM in revising his/her judgments

Method	TD	TD2	NV	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9
EV	1.3461	4.2035	1.5	0.209	0.2021	0.1225	0.0648	0.1199	0.0391	0.1841	0.0357	0.0228
GM	1.4026	4.0775	1.5	0.2202	0.1931	0.1234	0.0635	0.1161	0.0406	0.1883	0.0342	0.0205
PrInT-1	1.3383	4.1109	0.5	0.1973	0.1652	0.147	0.0613	0.1453	0.0366	0.1994	0.0274	0.0204
PrInT-2	1.3803	4.0947	0.5	0.1956	0.1691	0.1461	0.0636	0.1316	0.0357	0.209	0.0291	0.0202
PrInT-3	1.4025	4.0309	0.5	0.1962	0.1841	0.1333	0.0673	0.1331	0.0353	0.2016	0.0293	0.0198
PrInT-4	1.4289	4.0119	0.5	0.1945	0.1791	0.1392	0.0621	0.1353	0.0401	0.2014	0.0294	0.0191
PrInT-5	1.4448	4.0111	0.5	0.1973	0.1775	0.138	0.0615	0.1341	0.0439	0.1996	0.0291	0.0189

Table 5: Solutions for A_{tech}

and highlights intransitive set of judgments present in a given PCM. In the case of inconsistent judgments, PriEsT offers a wide range of Pareto-optimal solutions based on multi-objective optimization. The DM has the flexibility to select any of these non-dominated solutions according to his/her requirements. So far, PriEsT has been developed as a prototype; the future aim is to develop PriEsT in accordance with international standard ISO/IEC 9126.

The features of PriEsT have been demonstrated and evaluated through its application to a real-world case study: the selection of the most appropriate Telecom infrastructure for rural areas. This use of PriEsT has highlighted the presence of intransitive judgments in the acquired data and the correction of these judgments has led to a different ranking of the available alternatives.

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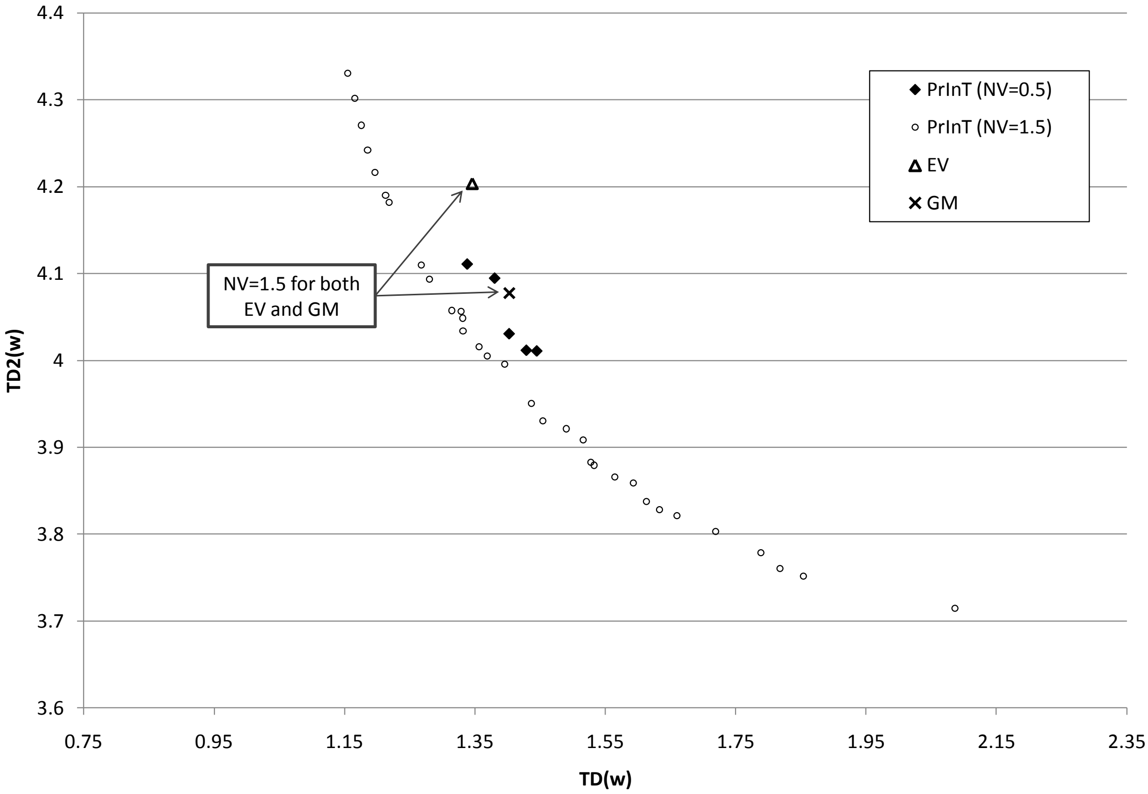

Figure 10: Solutions for A_{tech} in TD-TD2 plane

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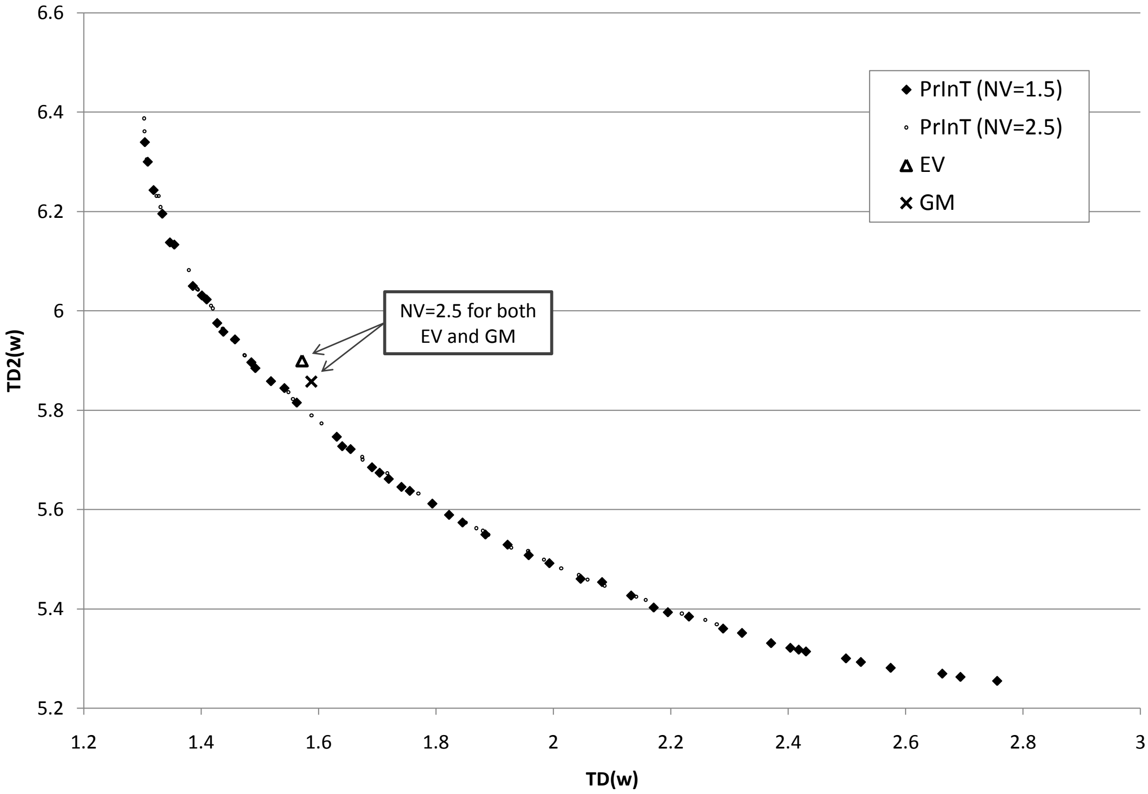

Figure 11: Solutions for A_{infra} in TD-TD2 plane

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Method	TD	TD2	NV	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
EV	1.5716	5.8996	2.5	0.3934	0.0888	0.0244	0.0606	0.0834	0.15	0.0526	0.1468
GM	1.5875	5.8581	2.5	0.4102	0.0821	0.0246	0.053	0.0928	0.1398	0.0555	0.142
PrInT-1	1.3041	6.3398	1.5	0.3768	0.0919	0.0339	0.051	0.0923	0.1639	0.0544	0.1358
PrInT-2	1.3089	6.2998	1.5	0.3884	0.0761	0.0339	0.0511	0.0876	0.1637	0.0613	0.1379
PrInT-3	1.3188	6.2426	1.5	0.3851	0.0822	0.0317	0.0509	0.0918	0.163	0.0612	0.134
PrInT-4	1.334	6.1952	1.5	0.3657	0.0855	0.0309	0.0474	0.0996	0.1712	0.0513	0.1484
PrInT-5	1.3468	6.1379	1.5	0.3712	0.0712	0.0306	0.0465	0.093	0.1663	0.0553	0.1659
PrInT-6	1.354	6.1332	1.5	0.3886	0.0766	0.0313	0.0482	0.0855	0.1609	0.052	0.157
PrInT-7	1.386	6.05	1.5	0.3739	0.0809	0.0285	0.0462	0.0893	0.1641	0.0542	0.163
PrInT-8	1.4015	6.0309	1.5	0.3627	0.0738	0.0279	0.0434	0.0946	0.1707	0.0568	0.1701
PrInT-9	1.4092	6.0227	1.5	0.3622	0.0732	0.0277	0.0421	0.1002	0.1706	0.0567	0.1673
PrInT-10	1.427	5.9753	1.5	0.3739	0.0717	0.0274	0.0449	0.0926	0.1672	0.0556	0.1668
PrInT-11	1.4376	5.958	1.5	0.3768	0.0714	0.0272	0.0447	0.0922	0.1665	0.0552	0.1659
PrInT-12	1.4574	5.942	1.5	0.394	0.076	0.027	0.0479	0.0847	0.1589	0.0557	0.1558
PrInT-13	1.4853	5.8961	1.5	0.3852	0.061	0.027	0.0484	0.0856	0.1716	0.0523	0.1689
PrInT-14	1.4925	5.8848	1.5	0.3814	0.0607	0.0268	0.044	0.0993	0.1713	0.052	0.1646
PrInT-15	1.5187	5.8578	1.5	0.3933	0.0602	0.0272	0.0429	0.0848	0.1728	0.0524	0.1665
PrInT-16	1.5421	5.8442	1.5	0.3651	0.0587	0.0262	0.0427	0.0824	0.1917	0.0507	0.1826
PrInT-17	1.563	5.8152	1.5	0.3884	0.0605	0.0267	0.0408	0.0881	0.1822	0.0478	0.1654
PrInT-18	1.6307	5.7461	1.5	0.3785	0.0595	0.024	0.0427	0.0874	0.183	0.0574	0.1676
PrInT-19	1.6404	5.7275	1.5	0.3803	0.0594	0.0242	0.0429	0.0881	0.1839	0.0526	0.1685
PrInT-20	1.6546	5.7216	1.5	0.3793	0.053	0.0244	0.0429	0.0881	0.1837	0.0523	0.1763
PrInT-21	1.6912	5.6845	1.5	0.3946	0.0615	0.0239	0.0431	0.0854	0.1776	0.0507	0.1632
PrInT-22	1.7043	5.6739	1.5	0.3894	0.0567	0.0239	0.042	0.0862	0.1782	0.0512	0.1725
PrInT-23	1.7194	5.6612	1.5	0.3842	0.064	0.0233	0.0427	0.0877	0.1824	0.0477	0.168
PrInT-24	1.7415	5.645	1.5	0.3825	0.0631	0.0229	0.0426	0.0864	0.1801	0.0497	0.1729
PrInT-25	1.7554	5.6372	1.5	0.3788	0.0632	0.0229	0.0421	0.0865	0.1804	0.0465	0.1794
PrInT-26	1.7938	5.6113	1.5	0.396	0.0575	0.0227	0.0444	0.0856	0.1793	0.0509	0.1636
PrInT-27	1.8223	5.5886	1.5	0.3919	0.0625	0.0225	0.0416	0.0855	0.1783	0.0462	0.1714
PrInT-28	1.8455	5.5736	1.5	0.3897	0.0547	0.0223	0.0437	0.0842	0.1826	0.0485	0.1745
PrInT-29	1.8842	5.5492	1.5	0.3868	0.0529	0.022	0.0428	0.0884	0.1843	0.0472	0.1757
PrInT-30	1.9217	5.5288	1.5	0.383	0.057	0.0212	0.0427	0.0868	0.1834	0.0484	0.1775
PrInT-31	1.9579	5.5079	1.5	0.3994	0.0579	0.0215	0.0416	0.0879	0.1783	0.0458	0.1675
PrInT-32	1.9931	5.4913	1.5	0.4028	0.0518	0.0215	0.042	0.0862	0.1807	0.0462	0.1689
PrInT-33	2.0467	5.4605	1.5	0.3867	0.0519	0.0209	0.0405	0.0833	0.2007	0.0449	0.1712
PrInT-34	2.0827	5.4536	1.5	0.376	0.0504	0.0202	0.0394	0.081	0.1951	0.049	0.1888
PrInT-35	2.1712	5.4026	1.5	0.3862	0.0528	0.0198	0.0392	0.0882	0.1935	0.043	0.1773
PrInT-36	2.2311	5.3844	1.5	0.3961	0.0525	0.0195	0.0423	0.0778	0.1922	0.0442	0.1754
PrInT-37	2.2893	5.3603	1.5	0.3822	0.0555	0.0189	0.0377	0.0847	0.2029	0.0434	0.1747
PrInT-38	2.3217	5.3511	1.5	0.3798	0.0545	0.0187	0.0374	0.0842	0.2086	0.0431	0.1736
PrInT-39	2.3708	5.331	1.5	0.3897	0.0506	0.0188	0.037	0.0842	0.2015	0.0422	0.176
PrInT-40	2.4036	5.3214	1.5	0.3919	0.0502	0.0187	0.0368	0.0839	0.2002	0.0419	0.1763
PrInT-41	2.418	5.3173	1.5	0.3922	0.0504	0.0186	0.0373	0.0839	0.2007	0.0419	0.175
PrInT-42	2.4985	5.2996	1.5	0.3869	0.0507	0.0179	0.0371	0.0847	0.1951	0.0422	0.1855
PrInT-43	2.524	5.2925	1.5	0.3958	0.0507	0.0179	0.0371	0.0847	0.1958	0.0423	0.1755
PrInT-44	2.5748	5.2811	1.5	0.3894	0.0499	0.0176	0.0365	0.0834	0.1989	0.0416	0.1827
PrInT-45	2.6626	5.2694	1.5	0.3842	0.0505	0.0171	0.0344	0.0845	0.2018	0.0421	0.1855
PrInT-46	2.6936	5.2629	1.5	0.3976	0.0503	0.0173	0.0349	0.0851	0.2033	0.0408	0.1706
PrInT-47	2.756	5.2547	1.5	0.392	0.0506	0.0169	0.034	0.0835	0.1996	0.0398	0.1835
PrInT-48	2.8598	5.246	1.5	0.3797	0.0481	0.0165	0.0335	0.0813	0.2235	0.039	0.1785

Table 6: Solutions for A_{infra}