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## Quantum Decoherence in a Four-Dimensional Black Hole Background

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### Abstract

We display a logarithmic divergence in the density matrix of a scalar field in the presence of an Einstein-Yang-Mills black hole in four dimensions. This divergence is related to a previously-found logarithmic divergence in the entropy of the scalar field, which cannot be absorbed into a renormalization of the Hawking-Bekenstein entropy of the black hole. As the latter decays, the logarithmic divergence induces a non-commutator term  $\delta H \rho$  in the quantum Liouville equation for the density matrix  $\rho$  of the scalar field, leading to quantum decoherence. The order of magnitude of  $\delta H$  is  $\mu^2/M_P$ , where  $\mu$  is the mass of the scalar particle.

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# 1 Introduction and Summary

There is currently much debate whether microscopic black holes induce quantum decoherence at a microscopic level. In particular, it has been suggested [1] that Planck-scale black holes and other topological fluctuations in the space-time background cause a breakdown of the conventional  $S$ -matrix description of asymptotic particle scattering in local quantum field theory, which should be replaced by a non-factorizable superscattering operator  $\mathcal{S}$  relating initial- and final-state density matrices:

$$\rho_{out} = \mathcal{S}\rho_{in} \quad (1)$$

It has further been pointed out that, if this suggestion is correct, there must be a modification of the usual quantum-mechanical time evolution of the wave function, taking the form of a modified Liouville equation for the density matrix [2]:

$$\partial_t \rho = i[\rho, H] + \mathcal{J}H\rho \quad (2)$$

The extra term in (2) is of the form generally encountered in the description of an open quantum-mechanical system, in which observable degrees of freedom are coupled to unobservable components which are effectively integrated over, which may evolve from a pure state to a mixed state with a corresponding increase in entropy. The necessity of a mixed-state description is generally accepted in the presence of a macroscopic black hole, but is far from being universally accepted in the case of microscopic virtual black-hole fluctuations.

Three of us have been analyzing the possibility of quantum decoherence in a non-critical formulation of string theory [3], in which the two-dimensional string black hole solution [4] is used as an example of a quantum fluctuation in space time. We exhibited in this model extra logarithmic singularities, associated with transitions between different conformal field theories on the world sheet, which induced an extra term in the quantum Liouville equation of the form conjectured in (2). This derivation of (2) was, however, incomplete, in that two dimensions are not the same as four, and even in two dimensions one does not have a complete non-perturbative formulation of string theory that includes transitions between different conformal field theories<sup>1</sup> In parallel, two of us have been studying a scalar field in the presence of a four-dimensional Einstein-Yang-Mills black-hole background [7], and have demonstrated that its entropy exhibits a logarithmic divergence that cannot be absorbed into a simple renormalization of the Bekenstein entropy of the black hole. This can be understood as a quantum reflection of the entanglement of the external scalar field with internal black hole properties that alter with its radius.

The purpose of this paper is to show that this logarithmic divergence, which also appears in the partition function  $Z$  but not at the same order in the Hamiltonian,

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<sup>1</sup>Progress in this direction is being made via studies of duality and conifolds [5], with considerable similarity to the approach taken in [3]: see [6].

induces an extra non-commutator term into the quantum Liouville equation for the density matrix of the scalar field. This follows from the identification of the renormalization scale with time, as in the two-dimensional non-critical string [3], and constitutes the first example of a derivation of equation (2) in four dimensions. We argue that, once one allows for the quantum decay of the black hole, the logarithmic divergence found in the previous static case becomes a new source of time dependence for the scalar field that leads to a monotonic increase in entropy, and hence cannot be absorbed within the conventional Liouville equation. The order of magnitude that we find for the extra term is

$$\delta H \simeq \frac{\mu^2}{M_P}, \quad (3)$$

where  $\mu$  is the mass of a quantum of the scalar field. The estimate (3) is the maximum that had been indicated by previous string black hole studies [3], and is not very far from the sensitivity of present experimental searches [8].

## 2 Entropy, Hair and Logarithmic Divergences

As a first step in this analysis, we now review relevant previous work and recall some of the ingredients we use. The observation that quantum black holes must be described by mixed quantum states goes back to the work of Bekenstein [9] and Hawking [10], who showed at the tree level that a generic black hole has non-zero entropy related to the area of its event horizon:

$$S = \frac{1}{4G_N} A \quad (4)$$

This entropy represents the number of quantum black hole states that are not distinguishable by traditional measurements using the varieties of hair known in conventional local four-dimensional quantum field theories. It is known that string theories possess an infinite number of local symmetries, and we have suggested that a corresponding infinite set of conserved quantum numbers, baptized  $W$ -hair [11], characterize string black hole states, at least in two dimensions. We have also pointed out [12] that the number of string black hole states grows in the same way as the logarithm of the Hawking-Bekenstein entropy (4), and suggested ways in which the  $W$  quantum numbers could in principle be measured and used to distinguish these string black hole states [11].

However, we have also pointed out that a complete set of these measurements is not possible in practice, and have gone on to suggest that the incompleteness of the  $W$ -hair measurements provides a measure of the loss of information associated with a string black hole [3]. The observable low-energy degrees of freedom are entangled by  $W$  symmetry with unobservable internal states of the black hole. Integration over these unobserved degrees of freedom provides a truncated subtheory which possesses

logarithmic divergences absent in a conformal field theory. The latter would represent string theory in a fixed classical background, in which scattering is described by a conventional  $S$  matrix. The extra logarithmic divergences are associated with fluctuations in the black-hole background, which require transitions between different conformal field theories. Two-dimensional examples of these divergences are provided by string world-sheet monopoles and instantons, which represent the creation and mass shifts of black holes, respectively [3]. After identification of the renormalization scale appearing in the logarithms with a time-like Liouville field, and thence with the target time variable, we have been able to derive (2) with an explicit representation for the non-commutator term.

We now point out the analogies with the study [7] of a scalar field coupled to an Einstein-Yang-Mills (EYM) black hole in four dimensions. This study was purely in the context of quantum gravity, with no string degrees of freedom. On the other hand, the presence of the gauge field endows the EYM black hole with additional gauge hair (c.f., the  $W$  hair of the two-dimensional string black hole) that is entangled with the scalar field via gauge interactions (c.f., non-diagonal  $W$  generators). The renormalizability of the EYM field theory enables the partition function and entropy of the four-dimensional black hole to be calculated with the inclusion of quantum corrections, with the results [7]:

$$-lnZ \equiv F = -\frac{2\pi^3 r_h^4 (1 - 2\hat{m}'_h)^{-2}}{45\hat{\epsilon} \beta^4} + \left\{ \frac{4}{45} r_h^3 \frac{\pi^3}{\beta^4} \frac{2 - 4\hat{m}'_h + \hat{m}''_h}{(1 - 2\hat{m}'_h)^3} - \frac{1}{6} r_h^3 \frac{\pi^2}{\beta^2} \mu^2 \frac{1}{1 - 2\hat{m}'_h} \right\} \log \hat{\epsilon} : \quad \beta \equiv \frac{4\pi r_h e^{\delta_h}}{1 - 2\hat{m}'_h} \quad (5)$$

and

$$S = S_{BH} + S_q = \frac{1}{4G_{N,0}} A + \left[ \frac{(1 - 2m'_h) e^{-3\delta_h}}{360\pi r_h \epsilon} \frac{1}{4} A - \left\{ \frac{1}{180} \left( 2 - 4m'_h + \frac{m''_h}{r_h} \right) e^{-3\delta_h} - \frac{1}{12} r_h^2 \mu^2 e^{-\delta_h} \right\} \log \left( \frac{\epsilon}{r_h} \right) \right] \equiv \frac{1}{G_{N,ren}} \pi r_q^2 \quad (6)$$

where  $S_{BH}$  is the tree-level Bekenstein-Hawking entropy (4), and the  $S_q$  are the quantum corrections, indicated by square brackets. The subscript  $h$  denotes quantities at the horizon of the black hole,  $r_h$  is the horizon radius, the symbols  $\hat{X}$  denote ratios  $X/r_h$ , primes denote differentiation with respect to the radial coordinate  $r$ ,  $m$  is the mass function,  $\delta$  is defined in ref. [7] and will be given explicitly below, and  $\epsilon$  is a small, positive fixed distance which will play the rôle of an ultra-violet cut-off. Its presence is associated with the ‘brick wall’ boundary condition [13] for the wavefunction of the (scalar) matter fields in the black hole background.

The first term in the quantum corrections to the entropy (6) can be absorbed into the bare lowest-order Hawking-Bekenstein entropy (4) via a renormalization of Newton's constant  $G_N : G_{N_0} \rightarrow G_{N,ren}$ . However, the second quantum correction cannot simply be absorbed into a bare parameter in this way, and therefore corresponds to a new effect beyond the reach of the conventional renormalization programme, as we discuss in the next section. Formally, it may be absorbed into a “quantum” horizon area, corresponding to a “quantum” radius  $r_q$ , as we shall discuss in section 4.

### 3 Hamiltonian of the Scalar Field in an EYM Black-Hole Background

In order to gather information needed for the interpretation of the logarithmic divergence in the partition function (5) and in the entropy (6) [7], we now compute the Hamiltonian of a scalar field of mass  $\mu$  described by the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\sqrt{-g}(g^{\mu\lambda}\partial_\mu\phi\partial_\lambda\phi + \mu^2\phi^2) \quad (7)$$

which satisfies the Klein-Gordon equation in an EYM black-hole background described by the metric

$$ds^2 = -e^\Gamma dt^2 + e^\Lambda dr^2 + r^2(d\theta^2 + \sin\theta d\varphi^2) \quad (8)$$

where the forms of the metric functions  $\Gamma, \Lambda$  are discussed in [7] and below. It is easy to verify that the canonical momentum  $\pi$  which is conjugate to  $\phi$  is given in this case by

$$\pi = \frac{\partial\mathcal{L}}{\partial(\partial_t\phi)} = -\sqrt{-g}g^{t\nu}\partial_\nu\phi = e^{\frac{1}{2}(-\Gamma+\Lambda)}r^2\sin\theta\partial_t\phi \quad (9)$$

Defining the Hamiltonian density  $\mathcal{H} = \pi\partial_\perp\phi - \mathcal{L}$  in the normal way, we arrive at the following expression for the Hamiltonian  $H$ :

$$\begin{aligned} H &= \int drd\theta d\varphi \mathcal{H} : \\ \mathcal{H} &= \frac{1}{2}e^{1/2(\Gamma+\Lambda)}r^2\sin\theta \left( e^{-\Gamma}(\partial_t\phi)^2 + e^{-\Lambda}(\partial_r\phi)^2 + \right. \\ &\quad \left. \frac{1}{r^2}(\partial_\theta\phi)^2 + \frac{1}{r^2}\text{cosec}^2\theta(\partial_\varphi\phi)^2 + \mu^2\phi^2 \right) \end{aligned} \quad (10)$$

which we now evaluate.

To evaluate (10), we first expand  $\phi$  in normal modes

$$\phi(t, r, \theta, \varphi) = e^{-iEt} f_{El}(r) Z_{El}(\theta, \varphi) \quad (11)$$

where  $Z_{lm}$  is a real spherical harmonic [14], and  $f_{El}$  satisfies the radial equation

$$e^\Lambda E^2 f_{El} + \frac{1}{r^2} e^{-\frac{1}{2}(\Gamma+\Lambda)} \frac{d}{dr} \left[ e^{\frac{1}{2}(\Gamma-\Lambda)} r^2 \frac{d}{dr} f_{El} \right] - \left( \frac{l(l+1)}{r^2} + \mu^2 \right) f_{El} = 0 \quad (12)$$

which we solve with the ‘brick-wall’ boundary conditions [13]  $\phi = 0$  at  $r = r_h + \epsilon$  and  $r = L : L \gg r_h$ . Here  $r_h$  is the horizon radius of the black hole,  $\epsilon$  is a small, positive fixed distance which will play the rôle of an ultra-violet cut-off, and  $L$  is an infra-red cut-off. Introducing annihilation and creation operators  $a$  and  $a^\dagger$  for each normal mode (11) in the usual way, we arrive at the following expression for the Hamiltonian:

$$H = -\frac{1}{2} \sum_{E,l,m} \left( a_{El} a_{El}^\dagger + a_{El}^\dagger a_{El} \right) \left[ \frac{1}{2} E + E^2 J_{El} \right]. \quad (13)$$

where all the dependence on  $\epsilon$  is contained in the last factor in (13) and

$$J_{El} = \int_{r_h+\epsilon}^L r^2 e^{\frac{1}{2}(\Gamma+3\Lambda)} f_{El}^2 dr. \quad (14)$$

Before we estimate this integral, a few comments are in order concerning (13).

First, the form of the Hamiltonian ensures that the Fock basis states are eigenstates of energy, so that the Hamiltonian can be written in the alternative form

$$H = \sum_{\text{states}} \langle {}^1n_{k_1}, {}^2n_{k_2}, \dots, {}^jn_{k_j} | H | {}^1n_{k_1}, {}^2n_{k_2}, \dots, {}^jn_{k_j} \rangle \times | {}^1n_{k_1}, {}^2n_{k_2}, \dots, {}^jn_{k_j} \rangle \langle {}^1n_{k_1}, {}^2n_{k_2}, \dots, {}^jn_{k_j} | \quad (15)$$

where the expectation values are, from (13),

$$\begin{aligned} \langle {}^1n_{k_1}, {}^2n_{k_2}, \dots, {}^jn_{k_j} | H | {}^1n_{k_1}, {}^2n_{k_2}, \dots, {}^jn_{k_j} \rangle &= -\frac{1}{2} \sum_{\text{modes}} \left[ \frac{1}{2} E + E^2 J_{El} \right] \\ &\quad - \frac{1}{2} \sum_i^i n_i \left[ \frac{1}{2} E_i + E_i^2 J_{E_i l_i} \right]. \end{aligned} \quad (16)$$

The first term in (16) is state-independent and contributes an infinite constant to the partition function  $Z = \text{Tre}^{-\beta H}$ . We shall ignore this factor from here on, since it does not contribute to correlation functions. The second term in (16) will be compared later in this section to that used in [7] to calculate the partition function.

In order to estimate the integral  $J_{El}$ , we shall use [7] the WKB approximation for the  $f_{El}$ :

$$f_{El}(r) = \frac{1}{r} e^{-\frac{1}{2}(\Gamma-\Lambda)} \frac{A_{El}}{\sqrt{K_{El}(r)}} \sin \left( \int_{r_h+\epsilon}^r K_{El}(r') dr' \right) \quad (17)$$

where the radial wave number  $K$  is given by

$$K_{El}(r) = e^\Lambda \left[ E^2 - \frac{l(l+1)}{r^2} e^{-\Lambda} - \mu^2 e^{-\Lambda} \right]^{\frac{1}{2}} \quad (18)$$

The constant  $A_{El}$  in each case is determined from the normalization conditions for the  $f$ 's, namely

$$\int_{r_h+\epsilon}^L r^2 e^{-\frac{1}{2}(\Gamma+\Lambda)} f_{El}^2 dr = \frac{1}{2E} \quad (19)$$

and the boundary condition

$$f_{El}(L) = 0 \quad (20)$$

implies that

$$\int_{r_h+\epsilon}^L K_{El}(r') dr' = n\pi \quad \text{for some integer } n. \quad (21)$$

Next introduce a dimensionless co-ordinate  $x$  given by

$$x = \frac{r}{r_h} \quad (22)$$

and the rescaled function  $\hat{K}$  as follows:

$$\hat{K}_{El} = r_h K_{El} \quad (23)$$

and

$$\hat{\epsilon} = \frac{\epsilon}{r_h}. \quad (24)$$

We decompose the integral in (19) as  $A_{El}^2(I_1 + I_2)$ , where  $I_1$  is the contribution to the integral for  $x$  very close to one, and  $I_2$  is the remainder of the integral, so that

$$A_{El}^2 = \frac{1}{2E(I_1 + I_2)}. \quad (25)$$

Similarly, we may write  $J_{El} = A_{El}^2(J_{1El} + J_{2El})$ .

Writing the metric functions in the form:

$$\begin{aligned} e^\Gamma &= \left( 1 - \frac{2m(r)}{r} \right) e^{-2\delta(r)} \\ e^\Lambda &= \left( 1 - \frac{2m(r)}{r} \right)^{-1}, \end{aligned} \quad (26)$$

it can easily be seen that, for spacetimes in which

$$e^{-2\delta(r)} \equiv 1, \quad (27)$$

the integrals (14) and (19) are identical. This is the case for a black hole without hair, i.e., the Schwarzschild solution. In this case,  $J_{El} = 1/2E$ , the Hamiltonian  $H$  (13) is independent of  $\log \epsilon$ , and the form of the partition function  $Z$  is identical to the expression (5) found in [7].



For geometries with hair, the contribution to the integral in (14) for  $x \simeq 1$  is dependent on  $\log \hat{\epsilon}$ , and is given by

$$\begin{aligned} J_{1El} &= e^{-2\delta_h} I_1. \\ &= \frac{A_{El}^2}{2E} \left[ -\log \hat{\epsilon} + \frac{1}{2E} \sin \left( -\frac{2Er_h}{1-2m'_h} \log \hat{\epsilon} \right) \right]. \end{aligned} \quad (28)$$

For other values of  $x$ , the integrand contains a factor of the generic form

$$\sin^2 \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right), \cos^2 \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right), \quad (29)$$

$$2 \sin \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right) \cos \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right), \quad (30)$$

which is the same for both  $I_2$  and  $J_2$ . Note that the logarithmic terms in the expressions (28) and (30) have as their argument  $\hat{\epsilon} = \epsilon/r_h$  rather than  $L/\epsilon$  as has been discussed by [15], the contribution to the entropy for large values of  $r$  being proportional to  $L^3$  [7]. The reason for this is that the expansion we are using for the integrand in (14) is valid only for values of  $x$  very close to 1. An alternative expansion in inverse powers of  $x$  has to be used for large values of  $x$ .

We therefore write

$$\begin{aligned} I_2 &= ({}^1\mathcal{F}) \sin^2 \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right) + ({}^2\mathcal{F}) \cos^2 \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right) \\ &\quad + ({}^3\mathcal{F}) 2 \sin \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right) \cos \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right) \\ J_{2El} &= ({}^1\mathcal{G}_{El}) \sin^2 \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right) + ({}^2\mathcal{G}_{El}) \cos^2 \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right) \\ &\quad + ({}^3\mathcal{G}_{El}) 2 \sin \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right) \cos \left( -\frac{2r_h E}{1-2m'_h} \log \hat{\epsilon} \right) \end{aligned} \quad (31)$$

where the  ${}^i\mathcal{F}$ 's and  ${}^j\mathcal{G}$ 's are constants which are independent of  $\hat{\epsilon}$  but do depend on  $L$ . In total, therefore, we have

$$\begin{aligned} J_{El} &= \frac{1}{2E} \frac{J_{1El} + J_{2El}}{e^{2\delta_h} J_{1El} + I_{2El}} \\ &= \frac{1}{2E} \left[ e^{-2\delta_h} + \frac{J_{2El} - I_{2El} e^{-2\delta_h}}{e^{2\delta_h} J_{1El} + I_{2El}} \right] \end{aligned} \quad (32)$$

The second term in this expression is of the order of  $(\log \hat{\epsilon})^{-1}$ , and hence is very small.

We now proceed to evaluate the rate of change of  $J_{El}$  with respect to  $\log \hat{\epsilon}$ . The result is:

$$\frac{\partial J_{El}}{\partial \log \hat{\epsilon}} = \frac{1}{2E} \frac{1}{e^{2\delta_h} J_{1El} + I_{2El}} \left[ \frac{\partial J_{2El}}{\partial \log \hat{\epsilon}} - e^{-2\delta_h} \frac{\partial I_{2El}}{\partial \log \hat{\epsilon}} \right] + \text{smaller terms.} \quad (33)$$

We therefore arrive at the following expression for the  $\epsilon$ -dependence of the scalar-field Hamiltonian  $H$ :

$$\frac{\partial H}{\partial \log \hat{\epsilon}} = \frac{1}{2} \sum_{E,l,m} (a_{El} a_{El}^\dagger + a_{El}^\dagger a_{El}) E^2 \frac{\partial J_{El}}{\partial \log \hat{\epsilon}}. \quad (34)$$

To estimate the order of magnitude of (34), we need to estimate the coefficients  $\mathcal{F}$  and  $\mathcal{G}$ . It is not possible to do this analytically, but it is straightforward to see that

$$I_2, J_{2El} \sim \mathcal{F}, \mathcal{G} \sim \frac{r_h}{\mu} \quad (35)$$

multiplied by a numerical factor of order unity. In addition we have

$$\frac{\partial J_2}{\partial \log \hat{\epsilon}} \sim r_h \mu J_2 \sim r_h^2, \quad \frac{\partial I_2}{\partial \log \hat{\epsilon}} \sim r_h \mu I_2 \sim r_h^2. \quad (36)$$

The leading order term in the denominator of (33) is  $J_1 \sim \mu^{-1} \log \hat{\epsilon}$  whence

$$\frac{\partial J_{El}}{\partial \log \hat{\epsilon}} \sim \frac{1}{\mu} \frac{1}{J_1} r_h^2 \sim r_h^2 (\log \hat{\epsilon})^{-1} \quad (37)$$

and our final estimate is

$$\frac{\partial H}{\partial \log \hat{\epsilon}} \sim \mu^2 r_h^2 (\log \hat{\epsilon})^{-1} \quad (38)$$

up to a numerical factor of order unity. This dependence on  $\log \hat{\epsilon}$  will be negligible in the regime where  $\epsilon \rightarrow 0$ . As we discuss in more detail in the next section, this provides an important justification for the adiabatic approximation used in this paper, which is the context in which we interpret the logarithmic divergences in the partition function (5) and the entropy (6).

We complete this section by commenting on the relationship of this analysis to that of [7]. We are fortunate in that the terms in  $H$  which are dependent on  $\log \hat{\epsilon}$  are very small, leaving in effect

$$H = -\frac{1}{4} \sum_{E,l,m} (a_{El} a_{El}^\dagger + a_{El}^\dagger a_{El}) E (1 + e^{-2\delta_h}) \quad (39)$$

which is to be compared with the expression

$$H = -\frac{1}{2} \sum_{E,l,m} (a_{El} a_{El}^\dagger + a_{El}^\dagger a_{El}) E \quad (40)$$

which was employed in [7]. The only difference between these two expressions is the  $e^{-2\delta_h}$  term. As discussed in [7], the WKB approximation used is only valid for black-hole solutions with large radii  $r_h$ , for which  $\delta_h$  is very small. The two expressions (39) and (40) are approximately the same in this regime.

## 4 Modified Quantum Liouville Equation

We now turn to the interpretation of the logarithmic divergences in the partition function (5), in the entropy (6), and in the effective Hamiltonian (38). As usual, we assume that the density matrix for the scalar field is given by the Gibbs formula:

$$\rho = \frac{e^{-\beta H}}{Z} \quad (41)$$

where  $\beta$  is the inverse of the effective temperature of the EYM black hole given in (5). The denominator in (41) ensures probability conservation:  $\text{Tr}\rho = 1$  at all times. One would expect (41) to be valid (at least approximately), if the black hole is (at least approximately) static, as we assume in the adiabatic approximation used in this paper. However, we emphasize that we do not have a proof of (41) in this particular case. If it is subject to corrections, there may be additional corrections to the quantum Liouville equation for  $\rho$ , beyond the one we exhibit shortly.

If the Gibbs formula (41) were exact, we would normally conclude that  $\partial\rho/\partial t = 0$ , since  $[\rho, H] = 0$ . However, we know from the analysis of Hawking [10] that quantum effects cause the black hole to decay, changing its effective radius, c.f., (6):

$$r_q^2 = \left( \frac{1}{G_{N,0}} + \frac{(1 - 2m'_h)e^{-3\delta_h}}{360\pi r_h \epsilon} \right) G_{N,ren} r_h^2 - \frac{G_{N,ren}}{\pi} \left[ \frac{1}{180} \left( 2 - 4m'_h + \frac{m''_h}{r_h} \right) e^{-3\delta_h} - \frac{1}{12} r_h^2 \mu^2 e^{-\delta_h} \right] \log\left(\frac{\epsilon}{r_h}\right) \quad (42)$$

The fact that the radius changes as the black hole decays, i.e., changes with time, motivates the identification of time  $t$  with  $\log\hat{\epsilon}$ .

This identification was supported in the two-dimensional string case by explicit calculations [3], for example of the black-hole metric [4]. In that case, we were able to incorporate the effect of back reaction using world-sheet instantons and the Liouville field, but here we do not have the corresponding formal tools at our disposal. Nevertheless, amplitudes and correlation functions in the four-dimensional case exhibit logarithmic divergences in complete formal analogy with the two-dimensional case, supporting the identification of  $\log\hat{\epsilon}$  with time.

Since both the Hamiltonian  $H$  (34) and the partition function  $Z$  (5) depend on  $\log\hat{\epsilon}$ , this identification implies that, even assuming the Gibbs formula (41), there are possible sources of time variation in  $\rho$  which are not given by the usual commutator term  $[\rho, H]$ :

$$\frac{\partial\rho}{\partial t} = \left[ -\beta \frac{\partial H}{\partial t} + \beta \frac{\partial F}{\partial t} \right] \rho \quad (43)$$

where the free energy  $F$  is calculated in [7]. Since the normal commutator term  $[\rho, H]$  vanishes, the result (43) implies a modification of the quantum Liouville term by the addition of a non-commutator term as in (2), with

$$\begin{aligned} \delta H = \beta \left( -\frac{\partial H}{\partial t} + \frac{\partial F}{\partial t} \right) = O[(\log \hat{\epsilon})^{-1}] + \\ \frac{1}{1440} (1 - 2m'_h) e^{-3\delta_h} \frac{1}{\hat{\epsilon}} + \frac{1}{720} \left( 2 - 4m'_h + \frac{m''_h}{r_h} \right) e^{-3\delta_h} - \frac{1}{24} r_h^2 \mu^2 e^{-\delta_h} \end{aligned} \quad (44)$$

which is our main result. We note that the first term, obtained from the Hamiltonian (13), is suppressed relative to the terms obtained from the free energy. Such a suppression is necessary for the consistency of our adiabatic approximation. The term in (44) that depends on the energy content of the scalar matter field is the last one, which is of  $O[\mu^2]$ , where  $\mu$  is the mass of the scalar particle.

The rest of the terms depend on details of the black-hole background, and in particular on its hair [7]. For extreme macroscopic black holes with  $m_h \gg 1$  (in units of the Planck mass  $M_P$ ), for instance, the dominant term is the  $\mu^2$  term. This appears not to be the case for non-extreme black holes, where the usual Bekenstein-Hawking term seems to be the dominant one, for  $\epsilon \rightarrow 0$ . However, such terms may always be absorbed in a renormalization of the gravitational coupling constants of the model [15, 7], as explained at the end of section 2. Therefore, even in this case, it is the matter  $\mu^2$  term in (44) that determines the order of magnitude of the modifications of quantum mechanics [2].

In general, the quadratic dependence of  $\delta H$  on the scalar mass  $\mu$ , divided in order of magnitude by just one power of  $M_P$ , is the largest that could be expected for any such modification of the quantum Liouville equation: *a priori*, it could have been suppressed by one or more additional powers of  $\mu/M_P$ , or an exponential, or even absent all together. We are not in a position to estimate the coefficient of this  $\mu^2/M_P$  term. Nor, indeed, can we be sure that such a parametric dependence would persist in a complete quantum theory of gravity. However, we would like to remind the reader that just such a quadratic dependence also appeared as a possibility in a previous string analysis [3] of the quantum Liouville equation. We cannot resist pointing out also that such a dependence may not be many orders of magnitude from the experimental sensitivity to such a modification of the quantum Liouville equation for the  $K^0$  system [2, 3, 8].

Notice also that the main time-dependence in (44) comes from the divergences occurring when one traces over an infinite number of states in  $Z$ . Any explicit time-dependence of the Hamiltonian operator  $H$  is subleading at large times (38). This is not in contradiction with the adiabatic approximation made above, in which the effect of quantum fluctuations on the Hawking temperature  $\beta$  was neglected. This

approximation is valid for macroscopic black holes, for which the *WKB* approximation method we have used to derive expressions for the entropy and the free energy is valid [7].

It is easy to check that the  $\epsilon$ -dependence in (34), combined with the Gibbs formula (41), reproduces the  $\epsilon$ -dependence (6) of the entropy found previously in [7]. The interpretation developed earlier of the  $\epsilon$ -dependence as a time-dependence implies that the entropy of the scalar field  $\phi$  increases with time:

$$\frac{\partial S}{\partial t} = -\frac{1}{360}(1 - 2m'_h)e^{-3\delta_h}\frac{1}{\hat{\epsilon}} - \frac{1}{180}\left(2 - 4m'_h + \frac{m''_h}{r_h}\right)e^{-3\delta_h} + \frac{1}{12}r_h^2\mu^2e^{-\delta_h} \quad (45)$$

Thus the  $\phi$  state evolves to become more mixed, as a result of the non-commutator term (44) in the quantum Liouville equation (2). This can be thought of as being due to a change in the entanglement of the external  $\phi$  field with the unmeasurable modes interior to the black hole. The amount of this entanglement depends on the quantum numbers of the EYM background, as can be seen in (6). The more hair it has, the smaller the  $\epsilon$ -dependence in (34), and, correspondingly, the slower the rate of information loss in (45).

Although our analysis breaks down in the limit of an extremal black hole, the results (44) and (45) suggest that there may be information loss even in this case. This is because it is no longer true in general at the quantum level that the entropy is proportional to the area of the horizon (4). There will be information loss (entropy increase) even in the absence of any finite-temperature effects, if there is entanglement with modes beyond the horizon at the quantum level, as we have illustrated. This observation is related to the phenomenon of entropy generation during inflation in cosmology, which may also be regarded as a non-equilibrium process associated with information loss beyond the Hubble horizon. In the context of non-critical strings [3], we have discussed in ref. [16] how such an information loss can lead to a stochastic framework for time evolution, during such non-equilibrium processes. The stochasticity of the time evolution in the Liouville string [16], where the time variable is identified with a RG scale, can be derived from some specific properties of the RG evolution in two-dimensional spaces (world sheets) [17, 3]. One can hope that a similar framework may be developed here, identifying a renormalization group (UV) scale  $\log \hat{\epsilon}$  with time. However, we are not yet in a position to prove that a similar stochasticity characterizes the RG evolution in the four-dimensional case. However, the presence of logarithmic infinities in the (entanglement) entropy of black holes does seem to be a generic phenomenon, independent of the dimensionality of space-time, in view of the fact that they are present even in two-dimensional models [18].

The above analysis is not complete yet, as a result of the absence of a satisfactory treatment of quantum gravity. In the absence of such a treatment, the results of this

paper can only be regarded as indicative. However, we think that they constitute interesting circumstantial evidence in favour of the picture advanced previously [1, 2, 3], namely that microscopic quantum fluctuations in the space-time background may induce a loss of quantum coherence in apparently isolated systems. Moreover, the magnitude of  $\delta H$  that we find is consistent with previous string estimates, and may not lie many orders of magnitude beyond the reach of particle physics experiments [8].

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