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# Critical Analysis of the Eigenfilter Method for the Design of FIR Filters and Wideband Beamformers

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***Abstract*—The least squares based eigenfilter method has been applied to the design of both finite impulse response (FIR) filters and wideband beamformers successfully. It involves calculating the resultant filter coefficients as the eigenvector of an appropriate Hermitian matrix, and offers lower complexity and less computation time with better numerical stability as compared to the standard least squares method. In this paper, we revisit the method and critically analyze the eigenfilter approach by revealing a serious performance issue in the passband of the designed FIR filter and the mainlobe of the wideband beamformer, which occurs due to a formulation problem. A solution is then proposed to mitigate this issue, and design examples for both FIR filters and wideband beamformers are provided to demonstrate the effectiveness of the proposed method.**

***Index Terms*—Least squares, eigenfilter, filter design, wideband beamformer design.**

## I. INTRODUCTION

FIR filters and wideband beamformers have numerous application ranging from SONAR, RADAR, audio processing, ultrasound imaging, radio astronomy, earthquake prediction, medical diagnosis, to communications, etc [1, 2]. Many optimization methods have been employed in the past to design FIR filters and wideband beamformers with required specifications. General convex optimization is one of the techniques that has been extensively explored from this perspective [3–6] with the inherent drawback of long computation time required to reach the feasible solution. Although it can be considered as a special case of the convex optimization approach, least squares based design has been adopted as a simple but effective solution to both design problems, which minimizes the mean squared error between the desired and designed responses [2, 7, 8]. The solution of the standard least squares cost function involves matrix inversion to obtain the required weight vector. Since matrix inversion poses numerical instability with long filters [9], another method was proposed based on the least squares approach by performing eigenvector decomposition of a cost function to extract the required weight vector in the form of an eigenvector. This method is called eigenfilter design and has been explored for designing both filters and beamformers [10–15]. Moreover, the design of linear-phase FIR Hilbert transformers and arbitrary order digital differentiators were considered by Pei and Shyu [16, 17], who also investigated the design of arbitrary complex coefficient nonlinear-phase filters [18, 19]. Two-dimensional (2-D) extension to the eigenfilter method was proposed by Nashashibi and Charalambous [20], and later considered by Pei [21, 22]. Eigenfilters have also been used to design Infinite Impulse Response (IIR) and all-pass filters [23, 24].

In this work, we revisit the eigenfilter method for designing FIR filters and wideband beamformers and reveal a serious performance issue in the passband of the designed FIR filters and the mainlobe of the designed wideband beamformers in the light of an inherent design formulation flaw. An overall critical analysis of the performance of this approach is presented with the suggested modification for tackling this issue.

This paper is organized as follows. The eigenfilter based design formulation for FIR filters and wideband beamformers along with the critical analysis is presented in Section II. The proposed solution to the highlighted problem is given in Section III. Design examples for different types of FIR filters and wideband beamformers affected by the problem are provided in Section IV followed by results using the proposed solution. Conclusions are drawn in Section V.

## II. LEAST SQUARES BASED DESIGN AND CRITICAL ANALYSIS

### A. *FIR filter design*

Consider an $N-$tap FIR filter. Its frequency response $W(e^{j\omega})$ is given by

$$W(e^{j\omega}) = \sum_{n=0}^{N-1} w_n e^{-jn\omega} \ , \tag{1}$$

where $w_n$ is the $n-$th tap/coefficient of the filter. In vector form, we have

$$W(e^{j\omega}) = \mathbf{w}^H \mathbf{c}(\omega) \tag{2}$$

where $\mathbf{w}$ is the $N \times 1$ weight vector holding the coefficients $w_n$, $n = 0, 1, \ldots, N-1$, and

$$\mathbf{c}(\omega) = [1, e^{-j\omega}, \cdots, e^{-j(N-1)\omega}]^T \ . \tag{3}$$

Now consider designing a lowpass filter as an example and the desired response $D(\omega)$ is expressed as

$$D(\omega) = \begin{cases} e^{-j\omega\frac{N-1}{2}}, & 0 \leq \omega \leq \omega_p \\ 0, & \omega_s \leq \omega \leq \pi \end{cases} \tag{4}$$

where $e^{-j\omega\frac{N-1}{2}}$ represents the desired linear phase at the passband along with the desired stopband response equal to zero.

The design process involves formulating the cost function in the standard eigenfilter form, which is based on Rayleigh-Ritz principle which states that for any Hermitian matrix **R**, its Rayleigh-Ritz ratio is given by

$$\frac{\mathbf{w}^H\mathbf{R}\mathbf{w}}{\mathbf{w}^H\mathbf{w}} \tag{5}$$

This ratio reaches its maximum/minimum when **w** is the eigenvector corresponding to the maximum/minimum eigenvalue of **R**. The maximum and minimum values of this ratio are respectively the maximum and minimum eigenvalues. For FIR filter design, a reference frequency point was introduced by Nguyen in the passband region of the cost function to help represent it into the quadratic form as desired by (5) [11]. The cost function with reference frequency point incorporated is given as

$$E = \frac{1}{\pi}\int_{\omega} v(\omega)\left|\frac{D(\omega)}{D(\omega_r)}W(e^{j\omega_r}) - W(e^{j\omega})\right|^2 d\omega \tag{6}$$

where $v(\omega)$ is the weighting function and $D(\omega_r)$ and $W(e^{j\omega_r})$ represent the desired and designed responses at reference frequency, respectively. Then, we have

$$\begin{aligned} E = \frac{1}{\pi}\int_{\omega} v(\omega)&\left(\frac{D(\omega)}{D(\omega_r)}W(e^{j\omega_r}) - W(e^{j\omega})\right) \\ &\left(\frac{D(\omega)}{D(\omega_r)}W(e^{j\omega_r}) - W(e^{j\omega})\right)^H d\omega \end{aligned} \tag{7}$$

For stopband, the desired response $D(\omega) = 0$. We have

$$E_s = \frac{1}{\pi}\int_{\omega_s}^{\pi} v(\omega)W(e^{j\omega})W(e^{j\omega})^H d\omega \tag{8}$$

Substituting the expression in (2) into (8), we have

$$E_s = \frac{1}{\pi}\int_{\omega_s}^{\pi} v(\omega)\mathbf{w}^H\mathbf{c}(\omega)\mathbf{c}(\omega)^H\mathbf{w}d\omega \tag{9}$$

Then we can express (9) as

$$E_s = \mathbf{w}^H\mathbf{P}_s\mathbf{w} \tag{10}$$

where $\mathbf{P}_s$ is a symmetric, positive definite matrix of order $N$ x $N$ given by

$$\mathbf{P}_s = \frac{1}{\pi}\int_{\omega_s}^{\pi} v(\omega)\mathbf{c}(\omega)\mathbf{c}(\omega)^H d\omega \tag{11}$$

The passband cost function is derived by incorporating the desired passband response $D(\omega) = e^{-j\omega\frac{N-1}{2}}$ into (7)

$$\begin{aligned} E_p = \frac{1}{\pi}\int_{0}^{\omega_p} v(\omega)&\left(\frac{e^{-j\omega\frac{N-1}{2}}}{e^{-j\omega_r\frac{N-1}{2}}}W(e^{j\omega_r}) - W(e^{j\omega})\right) \\ &\left(\frac{e^{-j\omega\frac{N-1}{2}}}{e^{-j\omega_r\frac{N-1}{2}}}W(e^{j\omega_r}) - W(e^{j\omega})\right)^H d\omega \end{aligned} \tag{12}$$

After simplification, we have

$$\begin{aligned} E_p = \frac{1}{\pi}\int_{0}^{\omega_p} v(\omega)\mathbf{w}^H&\left(e^{-j\frac{N-1}{2}(\omega-\omega_r)}\mathbf{c}(\omega_r) - \mathbf{c}(\omega)\right) \\ &\left(e^{-j\frac{N-1}{2}(\omega-\omega_r)}\mathbf{c}(\omega_r) - \mathbf{c}(\omega)\right)^H \mathbf{w}d\omega \end{aligned} \tag{13}$$

This expression can also be written as

$$E_p = \mathbf{w}^H\mathbf{P}_p\mathbf{w} \ , \tag{14}$$

where $\mathbf{P}_p$ is a symmetric, positive definite matrix of order $N$ x $N$ given by

$$\begin{aligned} \mathbf{P}_p = \frac{1}{\pi}\int_{0}^{\omega_p} v(\omega)&\left(e^{-j\frac{N-1}{2}(\omega-\omega_r)}\mathbf{c}(\omega_r) - \mathbf{c}(\omega)\right) \\ &\left(e^{-j\frac{N-1}{2}(\omega-\omega_r)}\mathbf{c}(\omega_r) - \mathbf{c}(\omega)\right)^H d\omega \end{aligned} \tag{15}$$

The total cost function is a combination of the passband and stopband cost functions with a trade-off factor $\alpha$

$$E = \alpha E_p + (1-\alpha)E_s \ , \quad 0 \le \alpha \le 1 \tag{16}$$

which can be transformed into

$$E = \mathbf{w}^H\mathbf{P}\mathbf{w} \tag{17}$$

where

$$\mathbf{P} = \alpha\mathbf{P}_p + (1-\alpha)\mathbf{P}_s, 0 \le \alpha \le 1 \tag{18}$$

Combining (11) and (15) in (18) and taking the real part, we have

$$\begin{aligned} \mathbf{P} = \alpha\int_{0}^{\omega_p}\mathbf{Re}[&\left(e^{-j\frac{N-1}{2}(\omega-\omega_r)}\mathbf{c}(\omega_r) - \mathbf{c}(\omega)\right) \\ &\left(e^{-j\frac{N-1}{2}(\omega-\omega_r)}\mathbf{c}(\omega_r) - \mathbf{c}(\omega)\right)^H]d\omega \\ &+(1-\alpha)\int_{\omega_s}^{\pi}\mathbf{Re}[\mathbf{c}(\omega)\mathbf{c}(\omega)^H]d\omega \end{aligned} \tag{19}$$

The solution rests in finding the eigenvector **w** corresponding to the minimum eigenvalue of **P** which minimizes $E$. The norm constraint $\mathbf{w}^H\mathbf{w} = 1$ is also incorporated to avoid trivial solution. The final expression of solution for the eigenfilter based FIR filter design problem is given by

$$\underset{\mathbf{w}}{\text{Min}} \quad \frac{\mathbf{w}^H\mathbf{P}\mathbf{w}}{\mathbf{w}^H\mathbf{w}} \tag{20}$$

After investigating the designed filter's performance, it is found that although the design performs well for most of the cases with varying specifications for short filters, it produces ever increasingly inconsistent results as the number of filter taps increases for the same set of specifications. With those longer filters, the passband performance starts varying and switches from one case with flatness around near unity gain to another case with flatness achieved at almost zero magnitude.

This unstable performance can be attributed to the formulation in (19) where the first part of the cost function measures the difference between the filter's response at the reference frequency $\omega_r$ and those at the other frequencies $\omega$ in the passband. The term $e^{-j\frac{N-1}{2}(\omega-\omega_r)}$ compensates for different phase shifts of the response at different frequencies.

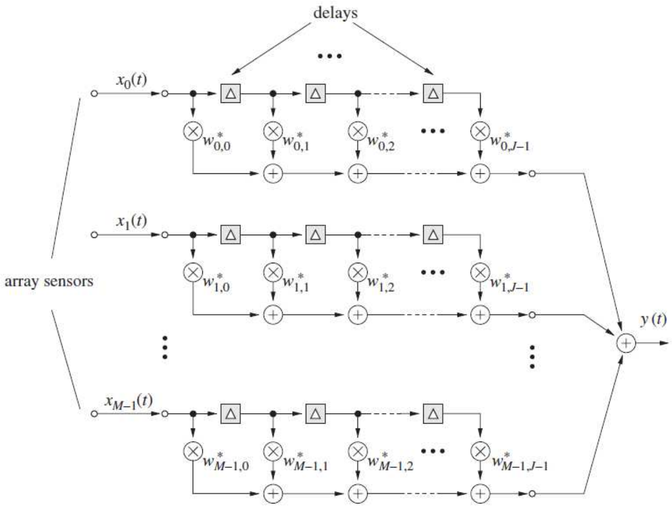



Fig. 1: A general structure for wideband beamforming.

This expression minimizes the relative variation of the filter's response at different passband frequencies and ensures a flat passband response. However, there is no control over the absolute value of the filter's response in passband, which can lead to inconsistent design performance.

### B. Wideband beamformer design

Consider a wideband beamformer with tapped delay lines (TDLs) or FIR filters shown in Figure 1, where $J$ is the number of delay elements associated with each of the $M$ sensors. The wideband beamformer samples the propagating wave field in both space and time. Its response as a function of signal angular frequency $\omega$ and direction of arrival $\theta$ is given by [2]

$$P(\omega,\theta) = \sum_{m=0}^{M-1}\sum_{k=0}^{J-1} w_{m,k} e^{-j\omega(\tau_m + kT_s)} \tag{21}$$

where $T_s$ is the delay between adjacent taps of the TDL and $\tau_m$ is the spatial propagation delay between the $m-th$ sensor and the reference sensor. We can also express (21) as

$$P(\omega,\theta) = \mathbf{w}^T \mathbf{d}(\omega,\theta) \tag{22}$$

where $\mathbf{w}$ is the coefficient vector

$$\mathbf{w} = [w_{0,0}, \cdots w_{M-1,0}, \cdots w_{0,J-1}, \cdots, w_{M-1,J-1}]^T \tag{23}$$

and $\mathbf{d}(\omega,\theta)$ is the $M$ x $J$ steering vector

$$\mathbf{d}(\omega,\theta) = \mathbf{d}_{Ts}(\omega) \otimes \mathbf{d}_{\tau_m}(\omega,\theta) \tag{24}$$

where $\otimes$ denotes the Kronecker product. The terms $\mathbf{d}_{Ts}(\omega)$ and $\mathbf{d}_{\tau_m}(\omega,\theta)$ are defined as

$$\mathbf{d}_{Ts}(\omega) = [1, e^{-j\omega T_s}, \cdots, e^{-j(J-1)\omega T_s}]^T \tag{25}$$

$$\mathbf{d}_{\tau_m}(\omega,\theta) = [e^{-j\omega\tau_0}, e^{-j\omega\tau_1}, \cdots, e^{-j\omega\tau_{M-1}}]^T \tag{26}$$

For a uniform linear array (ULA) with an inter-element spacing $d$, and angle $\theta$ measured from the broadside, the spatial propagation delay $\tau_m$ is given by $\tau_m = m\tau_1 = \frac{md\sin\theta}{c}$. With normalized angular frequency, $\Omega = \omega T_s$, and $\mu = \frac{d}{cT_s}$, the steering vector is given by

$$\mathbf{d}(\Omega,\theta) = \mathbf{d}_{T_s}(\Omega) \otimes \mathbf{d}_{\tau_m}(\Omega,\theta) \tag{27}$$

$$\mathbf{d}_{T_s}(\Omega) = [1, e^{-j\Omega}, \cdots, e^{-j(J-1)\Omega}]^T \tag{28}$$

$$\mathbf{d}_{\tau_m}(\Omega,\theta) = [1, e^{-j\mu\Omega sin\theta}, \cdots, e^{-j(M-1)\mu\Omega sin\theta}]^T \tag{29}$$

Now we have (22) as a function of $\Omega$ and $\theta$, given by

$$P(\Omega,\theta) = \mathbf{w}^T \mathbf{d}(\Omega,\theta) \tag{30}$$

The desired response for the wideband beamformer is represented by $P_d(\Omega,\theta)$. Then, the eigenfilter based cost function can be expressed as

$$J_{ef}(\mathbf{w}) = \int_{\Omega_{pb}}\int_{\Theta} v(\Omega,\theta)\left|P(\Omega,\theta) - P(\Omega_r,\theta_r)\frac{P_d(\Omega,\theta)}{P_d(\Omega_r,\theta_r)}\right|^2 d\Omega d\theta \tag{31}$$

where $(\Omega_r,\theta_r)$ is the reference point. We can change this expression into

$$J_{ef}(\mathbf{w}) = \mathbf{w}^H \mathbf{G}_{ef}\mathbf{w} \tag{32}$$

where

$$\mathbf{G}_{ef} = \int_{\Omega_{pb}}\int_{\Theta} v(\Omega,\theta)\left(\mathbf{d}(\Omega,\theta) - \mathbf{d}(\Omega_r,\theta_r)\frac{P_d(\Omega,\theta)}{P_d(\Omega_r,\theta_r)}\right) \left(\mathbf{d}(\Omega,\theta) - \mathbf{d}(\Omega_r,\theta_r)\frac{P_d(\Omega,\theta)}{P_d(\Omega_r,\theta_r)}\right)^H d\Omega d\theta \tag{33}$$

Consider a typical design case with desired sidelobe response equal to zero and response at look direction $\theta_0$ given by $e^{-j\frac{J}{2}\Omega}$ equal to a pure delay; $\Omega_r$ and $\Omega_{pb}$ represent the reference frequency and passband frequency range, respectively, and $\alpha$ is the weighting factor for the mainlobe. The expression in (33) is modified accordingly for real-valued beamformer coefficients and given by

$$\begin{aligned}\mathbf{G}_{ef} = \alpha\int_{\Omega_{pb}} &\mathbf{Re}[\Big(\mathbf{d}(\Omega,\theta_0) - e^{-j\frac{J}{2}(\Omega-\Omega_r)}\mathbf{d}(\Omega_r,\theta_r)\Big) \\ &\Big(\mathbf{d}(\Omega,\theta_0) - e^{-j\frac{J}{2}(\Omega-\Omega_r)}\mathbf{d}(\Omega_r,\theta_r)\Big)^H]d\Omega \\ &+(1-\alpha)\int_{\Omega_{pb}}\int_{\Theta_{ls}}\mathbf{Re}[\mathbf{d}(\Omega,\theta)\mathbf{d}(\Omega,\theta)^H]d\Omega d\theta\end{aligned} \tag{34}$$

Then, the solution to the wideband beamformer design problem is given by

$$\underset{\mathbf{w}}{\text{Min}} \quad \frac{\mathbf{w}^H\mathbf{G}_{ef}(\Omega,\theta)\mathbf{w}}{\mathbf{w}^H\mathbf{w}} \tag{35}$$

Similar to the FIR filter design case, testing of the designed wideband beamformer through the eigenfilter method showed an inconsistent performance. The design performed well for some look directions, while attained a very poor response for other look directions.

This variable nature of look direction response for the same set of specifications can again be traced back to the design formulation in (34), where the first part of the expression calculates the difference between the beamformer response at reference point $(\Omega_r, \theta_r)$ and those at other frequencies in the look direction $\theta_0$ . The term $e^{-j\frac{J}{2}(\Omega-\Omega_r)}$ compensates for the different phase shifts experienced by the wideband signal at different frequencies. The formulation ensures minimzation of the relative error at the look direction for different frequencies, thus providing flat response at $\theta_0$. However, just like the FIR filter case, there is a lack of control for absolute response in the look direction which can lead to design failure.

## III. PROPOSED SOLUTION

As shown in our analysis of the eigenfilter design for both FIR filters and wideband beamformers in Section II, the key issue is its lack of control of the achieved response at the passband/look direction compared to the desired one in the formulation. To solve this problem, we add an additional constraint to the formulation to specify the required response explicitly at the reference point. Since the original formulation will minimize the variation of the achieved response in the passband/look direction, the explicit control of the response of the designed filter/beamformer at one reference point of the passband/look direction will guarantee the design reaches the desired response for the whole considered passband/look direction region with a minimum overall error.

Now, constraining the reference frequency response to unity by adding a linear constraint to (20) gives us the following modified design formulation

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{P} \mathbf{w} \text{ Subject to } \mathbf{C}^H \mathbf{w} = \mathbf{f} \tag{36}$$

where the constraint matrix $\mathbf{C}$ and the response vector $\mathbf{f}$ provide the required constraint on the weight vector $\mathbf{w}$ so that the resultant design can have the required exact response at the reference frequency. Note that we can add other constraints to the formulation of $\mathbf{C}$ and $\mathbf{f}$ so that more flexible constraint can be imposed on the design. For example, we can add a constraint to make sure the resultant design has an exact zero response at some stopband frequencies.

The solution to (36) can be obtained by the Lagrange multipliers method and it is given by

$$\mathbf{w}_{opt} = \mathbf{P}^{-1}\mathbf{C}(\mathbf{C}^H \mathbf{P}^{-1}\mathbf{C})^{-1}\mathbf{f} \tag{37}$$

For the wideband beamformer design, the modified problem is given by

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{G}_{ef} \mathbf{w} \text{ Subject to } \mathbf{C}^H \mathbf{w} = \mathbf{f} \tag{38}$$

where $\mathbf{C}$ and $\mathbf{f}$ can be formulated in a similar way as before and the solution to (38) is given by

$$\mathbf{w}_{opt} = \mathbf{G}_{ef}^{-1}\mathbf{C}(\mathbf{C}^H \mathbf{G}_{ef}^{-1}\mathbf{C})^{-1}\mathbf{f} \tag{39}$$

Note that there are matrix inversion operations in (37) and (39), which can be computationally intensive for larger filters and beamformers. However, there are other approaches available in literature e.g. null space based methods to solve (36) and (38) without the need to compute matrix inversion [2].

## IV. DESIGN EXAMPLES

In this section, design examples are provided to show the inconsistent performance produced by the original unconstrained eigenfilter design method. The examples are then re-designed through the proposed constrained eigenfilter method to show the improvement.

### A. *Unconstrained eigenfilter design*

First consider the lowpass filter design scenario. The whole frequency range from $[0, \pi]$ was discretized into 400 points. The design specifications include passband from [0, $0.5\pi$] and stopband from [$0.8\pi$, $\pi$]. A 70-tap filter with trade-off parameter $\alpha$ = 0.97 and reference frequency at $0.35\pi$ is then designed. The result is shown in Figure 2 in blue colour (solid curve) with a clearly satisfactory design performance. In the second case, we change the number of taps to 76 and all the other specifications are the same as the first case. The result is shown in Figure 2, highlighted in dashed curve with red colour. We can see that the passband response is out of control, with a flat response of around -118 dB, and the resulting ratio between passband and stopband is just around 19 dB (if ignoring the unacceptable response at the transition band).

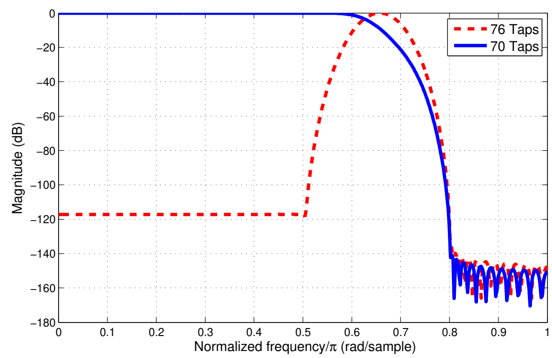



Fig. 2: The designed lowpass FIR filters using the original formulation.

For highpass filters, again two cases are presented. For the first case, we have 81 taps, stopband is from [0, $0.4\pi$] and passband from [$0.7\pi$, $\pi$]. The tradeoff factor $\alpha$ = 0.71 and the reference frequency is $0.74\pi$. The satisfactory design result is shown in Figure 3 with solid curve and blue colour.

For the second case, we just change the reference frequency to $0.94\pi$ and the result is shown in Figure 3 with dashed red colour, which is without any doubt unacceptable, with a passband response only at around -130 dB.

For the wideband beamformer design, we consider an array with 10 sensors and 10 taps. The look direction is $\theta_0 = 10°$ with desired response $e^{-j5\Omega}$. The frequency band consists of $\Omega_{pb} = [0.4\pi, \pi]$ with $\Omega_r = 0.7\pi$ and $\theta_r = 10°$ as the reference point. The weighting function is set as $\alpha = 0.6$ at look direction and 0.4 at sidelobe region from $-90^0$ to $-10^0$ and $30^0$ to $90^0$. The frequency range is discretized into 20 points, while the angle range divided into 360 points. The

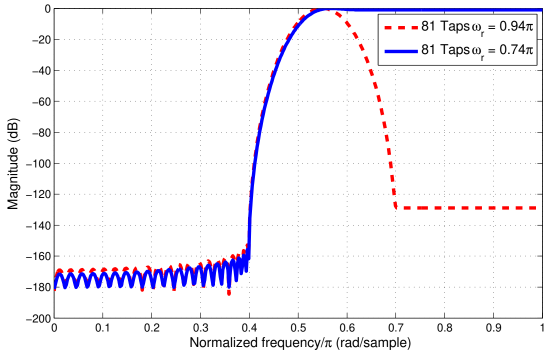
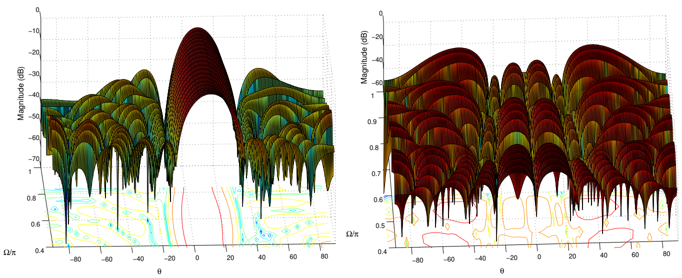



Fig. 3: The designed highpass FIR filters using the original formulation.




(a) $\theta_0 = 10^0$ (b) $\theta_0 = 0^0$

Fig. 4: The designed wideband beamformer using the original formulation.

result is shown in Figure 4(a), where it can be seen that the mainlobe and the sidelobe have a reasonable ratio of 20 dB.

Now we change the look direction to $\theta_0 = 0^0$ with sidelobe region from $-90^0$ to $-20^0$ and $20^0$ to $90^0$ with the remaining specifications unchanged. The result is shown in Figure 4(b), it can be observed that the look direction has a flat response at around -40 dB, even lower than the sidelobe.

### *B. Constrained eigenfilter design*

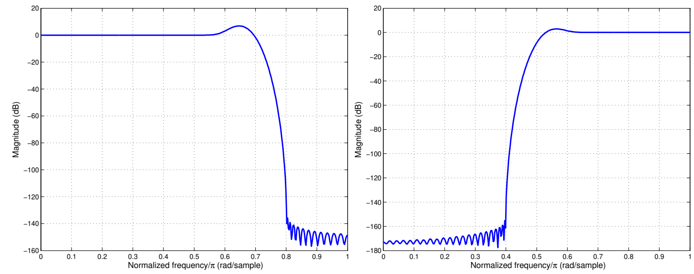
We now apply the constrained eigenfilter formulation in (36) to design the lowpass and highpass filters specified in Section IV-A. The new results are presented in Figure 5(a) and 5(b). Although there is still a noticeable bump at the transition band, the overall response has been improved significantly compared to the results in Figures 2 and 3.

For the beamformer presented in Figure 4(b), we re-design it using the constrained formulation in (38) and the result is provided in Figure 6, where the look direction and the sidelobe now have a reasonable ratio of 26 dB.

We have tried various designs and the proposed method performs consistently well in different scenarios.

## V. Conclusion

The classic eigenfilter approach has been revisited and critically analyzed, where a formulation problem is highlighted in the passband/look direction part of the cost function, leading to an inconsistent design performance. A solution was then




(a) 76 taps low pass filter (b) 81 taps high pass filter

Fig. 5: The designed lowpass and highpass filters using the constrained design.

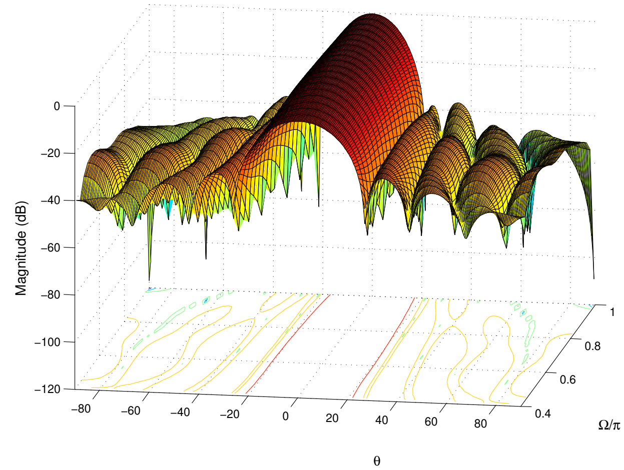



Fig. 6: The designed wideband beamformer with $\theta_0 = 0^0$

proposed by adding a linear constraint, explicitly setting the designed passband response at a reference point to the desired one. Results have been provided for different design scenarios to demonstrate the crucial issue of the original formulation and the satisfactory performance by the proposed one.

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