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Effect of a ring electrode on the cone-jet characteristics of ethanol in small-scale electro-spraying combustors

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Abstract

Two new micro-scale electro-spraying combustors of nozzle-grid and nozzle-ring-grid electrode configurations were designed. The experimental studies on the electro-spraying cone-jet characteristics in the micro-scale combustors were carried out using liquid ethanol as fuel. A digital camera was applied to visualize the electro-spraying at the cone-jet mode. The size and velocity distributions of the electro-spraying droplets were measured by a Phase Doppler Anemometer for both electrode configurations. Numerical calculation was performed to investigate the effects of the ring electrode on the electric field. A non-dimensional analysis was proposed to explain the liquid electro-spraying phenomena at the cone-jet mode. The results show that the electric field strength near the nozzle tip decreased when using the ring electrode with an appropriate voltage based on the superposition principle of the electric field, and the cone-jet mode could form at lower nozzle voltage without “satellite trapping”. Compared with the nozzle system without the ring electrode, the nozzle-ring system could produce smaller and more uniform droplets because of the more stable cone-jet mode. The lower velocity corresponded to the smaller droplet size, the velocity of the droplets was lower and the droplet size was smaller accordingly in the nozzle-ring system. It was found that the dimensionless droplet sizes in both the nozzle system and nozzle-ring system has a 1/2 power dependence on dimensionless liquid flow rate.

Keywords: ring electrode; cone-jet mode; electro-spraying; droplet size; electric field strength distribution; non-dimensional analysis
Nomenclature

\( d \)  \quad \text{droplet diameter (mm)}

\( d_o \)  \quad \text{the corresponding jet diameter of the reference flow (mm)}

\( d_{pred} \)  \quad \text{the droplet sizes predicted by the available correlations (mm)}

\( d_{exp} \)  \quad \text{the droplet sizes measured in the experiments (mm)}

\( d_n \)  \quad \text{inner diameter of nozzle (mm)}

\( d_i \)  \quad \text{inner diameter of ring electrode (mm)}

\( D_n \)  \quad \text{outer diameter of nozzle (mm)}

\( D \)  \quad \text{outer diameter of grid (mm)}

\( D_i \)  \quad \text{outer diameter of ring electrode (mm)}

\( D_{mean} \)  \quad \text{arithmetic mean diameter (\( \mu \)m)}

\( D_i \)  \quad \text{diameter of the individual particles (\( \mu \)m)}

\( E_t \)  \quad \text{tangential electric field on the surface (V/m)}

\( E_{n^o} \)  \quad \text{outer normal electric field on the surface (V/m)}

\( E_{n^i} \)  \quad \text{inner normal electric field on the surface (V/m)}

\( K \)  \quad \text{electrical conductivity (S/m)}

\( L \)  \quad \text{the distance between the nozzle tip and grid (mm)}

\( L_1 \)  \quad \text{the distance between nozzle tip and ring electrode (mm)}

\( L_2 \)  \quad \text{the distance between ring electrode and grid (mm)}

\( M \)  \quad \text{the number of data points}

\( N \)  \quad \text{total number of particles}

\( p \)  \quad \text{pressure (Pa)}
\( \Delta p \) pressure jump over the interface from air to liquid (Pa)

\( Q \) flow rate (ml/h)

\( Q_\sigma \) the reference flow rate (ml/h)

\( r \) radial coordinate (mm)

\( r_\sigma \) the corresponding jet radius of the reference flow (mm)

\( r_s \) cone-jet radius (mm)

\( \delta \) thickness of ring electrode (mm)

\( u \) velocity component in the \( z \) direction (m/s)

\( u_{\text{mean}} \) arithmetic mean axial velocity (m/s)

\( u_i \) axial velocity of the individual particles (m/s)

\( v \) velocity component in the \( r \) direction (m/s)

\( V_n \) the voltage applied to nozzle (kV)

\( V_r \) the voltage applied to ring electrode (kV)

\( V_{f1} \) the volume fractions of liquid

\( V_{f2} \) the volume fractions of air

\( z \) axial coordinate (mm)

*Greek symbols*

\( \alpha \) reinitialization parameter

\( \varepsilon \) dielectric constant of liquid

\( \varepsilon_r \) relative permittivity

\( \varepsilon_0 \) dielectric constant of vacuum

\( \varepsilon_{ls} \) parameter controlling the interface thickness (mm)
\[ \mu \quad \text{dynamic viscosity (Pa \cdot s)} \]
\[ \rho \quad \text{fluid density (kg/m}^3\text{)} \]
\[ \rho_s \quad \text{charge density along the surface (C/m}^2\text{)} \]
\[ \rho_v \quad \text{the space charge density (C/m}^3\text{)} \]
\[ \sigma \quad \text{gas-liquid surface tension (N/m)} \]
\[ \tau \quad \text{tangential shear stress (N)} \]

1. **Introduction**

With the development of new technologies, more and more attention has been focused on the micro-power systems using micro-scale combustors (Cao & Xu, 2007; Gan et al., 2014; Mikami et al., 2013). The specific energy of liquid hydrocarbons is much higher than that of batteries. So, the liquid hydrocarbons-based micro-combustor can become competitive with the batteries (Chen et al., 2009; Xu et al., 2013). The liquid hydrocarbon must be evaporated and mixed with the air in premixed systems or injected directly in non-premixed systems, due to the large surface area to volume ratio in microchannels. The phase change from liquid to vapor in the microchannels occurs in an unstable way (Gan et al., 2015a). Recently, because the electro-spraying can produce monodisperse liquid droplets easily, motions of the charged droplets can be controlled by electric field (Gañán-Calvo, 1994; Gañán-Calvo, 2004; Gañán-Calvo and Montanero, 2009; Ghazian et al., 2014; Taylor, 1964), it has been adapted for liquid fuel injection (Agathou & Kyritsis, 2012). The injected liquid is supplied to a nozzle at a low flow rate, under an externally applied electric field,
and the electro-spraying is operated at typical modes (spindle mode, dripping mode, pulsating mode and cone-jet mode) (Li & Tok, 2008; Verdoold et al., 2014). At cone-jet mode, the interface between air and the liquid is charged to a high potential, the liquid meniscus becomes a stable cone, whose apex issues a narrow jet (Rezvanpour et al., 2012; Yan et al., 2003).

According to the scaling laws, the dimensionless electrospay droplet size has a monotonic dependence on the 1/2 power of the dimensionless liquid flow rate (Gañán-Calvo, 1994). Smaller droplets can be achieved at very low flow rate (Gañán-Calvo et al., 2013), the low throughput becomes the main drawback of the electrospay system. In order to improve the throughput, many approaches have been considered (Holbrook et al., 2015; Li et al., 2014; Liu & Chen, 2014), which can be divided into three categories: multiplex, multi-jet and dual electrodes.

The electrospay nozzles with very small diameters were used in the multiplex system, and the nozzle structures needed extra fabrication steps. The electrospay sources were arranged in a hexagonal pattern which offers the highest packing density with a fixed distance between two neighboring nozzles (Oh et al., 2008). It was very important to obtain a uniform electric field. The nozzle diameter was very small to increase the throughput, or some dummy nozzles were introduced at the outmost region. So the device was complex and the nozzles might be plugged by the liquid easily.

The multi-jet mode operated with several jets issued around the tip of the nozzle, and some shape grooves were made at the tip of the nozzle to increase the electric
field around the nozzle tip (Kang et al., 2013). The multi-jet mode had much instability, and the liquid meniscus was in an unstable state, leading to a spread in droplet size and formation locations. The system required the sprays forming at the same time, and the geometry must be accurately reproduced from groove to groove.

A separation of the cone-jet region from the spraying region was generally desirable for stability and application considerations (Gan et al., 2015b). The dual electrodes spraying system consisted of a nozzle, a ring electrode, and a grounded grid was applied. An appropriate potential of the ring electrode should be selected to prevent the reversing of droplets paths. The spray passed through the ring electrode, and a stable cone-jet was established.

In present study, two different electro-spraying systems were compared to investigate the effect of the ring electrode on the cone-jet characteristics. The electro-spraying characteristics at cone-jet mode were visualized by a digital camera. The droplets size and velocity distributions were measured by a Phase Doppler Anemometer for both the electro-spraying systems. A numerical calculation was performed to investigate the effects of the ring electrode on the electric field. A non-dimensional analysis was carried out to explain the electro-spraying phenomena at cone-jet mode.

2. Experimental setup

2.1 The test rig

Fig.1 shows the test rig, which consisted of a liquid fuel feeding system, a test
section, a high voltage supply system and a droplet size and velocity measuring system. A capillary was used as a nozzle, and the fuel was supplied though a plastic tube to the nozzle by a syringe pump (KDS100, KD SCIENTIFIC) with an uncertainty of ±1%. The nozzle was supported by a copper sleeve and a ceramic package. The different electro-spraying modes were visualized by a digital single-lens reflex camera (Cannon EOS 5D Mark III) with a green laser light as an illuminating light source. The effective pixels are 22.3 million pixels, the highest resolution of the photos is 5760×3840, the highest shutter speed is 1/8000 s, ISO range is 100 to 25600. In present study, a setting of aperture at F 2.8, shutter speed at 1/2560 s, and ISO at 800 was chosen. The liquid fuel was pure ethanol (CH₃CH₂OH, molecular weight of 46.07, purity > 99.5%).

2.2 Test section

Two different of test sections with or without a ring electrode are illustrated in Fig.2. There were two electro-spraying systems: (1) nozzle system: the nozzle was maintained at high potential by connecting it to a direct-current (DC) power source, and the stainless steel grid was grounded; (2) nozzle-ring system: the nozzle and the ring electrode were connected to two DC power source separately, and the stainless steel grid was also grounded.

The inner and outer diameters of the stainless steel nozzle were 0.9 mm and 1.2 mm respectively (dₙ=0.9 mm, Dₙ=1.2 mm). The length of protruding section of the nozzle was 1.5 mm (h= 1.5 mm). The inner and outer diameters of the ring electrode were 12.4 mm and 16 mm respectively (dₑ=12.4 mm, Dₑ=16 mm), its thickness was
5 mm (δ=5 mm). The stainless steel grid had a diameter of \( D = 16 \) mm, and each hole in it had a diameter of 1 mm. A high DC power source (Model DW-P103-1AC) was used to supply high voltage on the nozzle \((V_n)\). For the nozzle system, the stainless steel grid was arranged above the tip of the nozzle with a vertical distance of \( L = 26.1 \) mm. For the nozzle-ring system, the ring electrode was arranged above the tip of the nozzle with a distance of \( L_1 = 1.1 \) mm. Another DC power source (Model 71030P) was used to supply high voltage on the ring electrode \((V_r)\). The stainless steel grid was arranged above the ring electrode with a distance of \( L_2 = 20 \) mm.

2.3 Droplet size and velocity measurement

A Phase Doppler Anemometer (PDA) (Dantec Dynamics classic PDA) was used to measure the sizes of the droplets (Li et al., 2013). The PDA system included a fiber optic probe, a signal processor, a receiver probe and an argon-ion laser. Droplet distributions at different spatial locations were measured by adjusting a three dimensional auto control coordinate system with a dimension of 560x560x580 mm. An argon-ion laser generator and optical splitter were used to generate two laser beams with a spacing of 40 mm. The wavelength was 514.5 nm for beam U1 (green) and 488 nm for beam U2 (blue). The beam diameters were 1.35 mm and beam expander ratios were set to 1. The shift frequency of one of the two beams is 40 MHz. The fringe spacing was 6.87 \( \mu m \) and 6.51 \( \mu m \) for U1 and U2, respectively, fringes moved in a negative direction. A dedicated electronic processor sampled and analyzed the signal using BSA Flow Software. The spherical validation band was set to 10% to ensure a high sphericity of recorded droplets and a high accuracy of droplet size. The
sample number at every measuring position was set to 2000 and sampling time was 10 s which enable estimation of droplet size and velocity statistics with confidence. Droplets size was measured from the phase difference between the quasi-periodic signals collected by two appropriately positioned detectors. An automatic check rejects signals emitted from non-spherical drops. Other signals from droplets crossing the measuring volume far from its center were rejected because the linear relationship between droplet diameter and phase shift ceases to be valid for those cases. The intersection of the two beams was located as near to the axis (Gañán-Calvo, 1997). After comparing the droplet size distribution at the different cross sections, measurements were performed at the section \((z=10 \text{ mm})\) up the tip of the nozzle, and size distribution was determined based on 17 points. Fig.3 shows the distribution of measuring points, which distribute on the concentric circles of different radiiuses.

2.4 Error Analysis

In present study, ethanol was used as the liquid fuel and its physical properties at 25 °C are shown in Table 1. The experiment was conducted in the following variation ranges of operating parameters: liquid ethanol flow rate \(Q\) of 0.2-4 ml/h, applied voltage on the nozzle electrode \(V_n\) of 0-6.8 kV, the applied voltage on the ring electrode \(V_r\) was 1 kV, the distance \(L_1\) was 1.1 mm, the distance \(L_2\) was 20 mm, the distance \(L\) was 26.1 mm. The measurement errors of a set of parameters are listed in Table 2.

3. Numerical simulation
3.1 Modeling Methodology

The liquid was supplied into a nozzle at a very low flow rate. A stable cone-jet mode can be formed under an externally applied electric field. Then the liquid ethanol was broken into small droplets, which passed through the ring electrode and then gathered on the stainless steel grid. The geometric model of numerical simulation is shown in Fig.4. The region between the nozzle and the ring electrode was named as the cone-jet region, and the space between the ring electrode and the grid was named as the spraying region.

The liquid flow would change the electric field distribution around the nozzle. Thus, the electric field distribution needed to be solved for the whole region (Lim et al, 2011). The governing equations for both fluid and electric field were solved for the cone-jet region. To track the liquid-gas interface, the level set method had been incorporated into the simulations using a software package Fem-Lab (Comsol). The coupling simulation area was \(0 \text{ mm} \leq r \leq 8 \text{ mm}, -5 \text{ mm} \leq z \leq 26.1 \text{ mm}\), the electric field simulation area was \(0 \text{ mm} \leq r \leq 50 \text{ mm}, -30 \text{ mm} \leq z \leq 70 \text{ mm}\). Two-dimensional axisymmetric calculation model and free triangular mesh were used. A steady-state solver was adopted in the calculation process. A normal element size was used in present simulation. In order to check the effect of mesh size on the numerical results, three mesh sizes were considered. The first one was 1256 domain units and 463 boundary elements, the second was 1500 domain units and 500 boundary elements, the third was 900 domain units and 400 boundary elements. The numerical results did not vary significantly with the mesh size, thus the first element size mentioned above
was used in present simulation.

3.2 Governing equations

The governing equations were solved in the cone-jet region to obtain the details of fluid flow and its interaction with the electric field. Additional body forces were added to the Navier-Stokes equations for considering the surface tension ($F_{st}$) and electric stress ($F_{es}$) (Ghazian et al., 2014).

\[ \nabla \cdot \mathbf{u} = 0 \]  
\[ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} - \nabla \cdot [-p \mathbf{I} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] = \rho \mathbf{g} + F_{st} + F_{es} \]  

where $\mathbf{u}$ denotes fluid velocity, $\rho$ is the fluid density, $\mathbf{g}$ is the gravitational acceleration, $\mu$ is the dynamic viscosity, $I$ is the 3×3 identity matrix, and $p$ is the pressure.

To obtain the electric field distribution, the Poisson’s equation for the electrical potential was expressed as follows:

\[ \nabla \cdot \mathbf{D} = \rho_v \]  
\[ \mathbf{E} = -\nabla V \]  
\[ \mathbf{D} = \varepsilon \mathbf{E} \]

Where $\mathbf{D}$ is the electric displacement, $\mathbf{E}$ is the electric field intensity, $V$ is the voltage, $\varepsilon$ is the dielectric constant of the liquid, $\rho_v$ is the space charge density.

The electric stress could be calculated by taking the divergence of the Maxwell stress tensor, neglecting the effect of magnetic field, the Maxwell stress tensor could be defined as follows:

\[ TM_{ij} = \varepsilon_r \varepsilon_0 E_i E_j - \frac{1}{2} (\varepsilon_r \varepsilon_0 E^2) \delta_{ij} \]

\[ F_{es} = \nabla \cdot TM \]
\[ \varepsilon_r = \varepsilon_{r1}V_{f1} + \varepsilon_{r2}V_{f2} \]  

where \( \varepsilon_r \) is the relative permittivity, \( \varepsilon_0 \) is the dielectric constant of vacuum, \( E_i, E_j \) is the electric field component in the i, j direction, \( i=j, \delta_{ij}=1, \) or \( \delta_{ij}=0. \) \( \varepsilon_{r1} \) and \( \varepsilon_{r2} \) denote the relative permittivity of liquid and air, \( V_{f1} \) and \( V_{f2} \) are the volume fractions of each fluid, respectively. Although the relative permittivity of liquid and air are constant, the volume fractions of each fluid changed with the interface between the liquid and air, the relative permittivity \( \varepsilon_r \) changed during the numerical simulation.

The level set method described the evolution of the interface between the liquid and air tracing an iso-potential curve of the level set function \( (\varphi) \). The function \( \varphi \) is governed by:

\[
\frac{\partial \varphi}{\partial t} + \nabla \cdot (\varphi \mathbf{u}) = \alpha \nabla \cdot \left[ \varepsilon_{ls} \nabla \varphi - \varphi (1-\varphi) \frac{\nabla \varphi}{|\nabla \varphi|} \right]
\]

Where \( \varepsilon_{ls} \) is the parameter controlling the interface thickness, \( \alpha \) is the reinitialization parameter.

### 3.3 Boundary conditions

The fluid-air interface was affected by the electrostatic field force and the surface tension. The effect of fluid viscous force was ignored. The force balance was reached under the proper voltage, and a stable cone jet atomization model was obtained. The boundary conditions for the hydrodynamic equations were listed as follows (Yan et al., 2003).

At the inlet of the nozzle:

\[ z = -h \]

\[ u = \frac{4Q}{\pi d_h^2}, v = 0 \]
At the tip of the nozzle:

\[ z=0, \quad 0 \leq r \leq r_s \]

\[ u = \frac{Q}{\pi r^2}, \quad v = 0 \] \hspace{1cm} (11)

Along the axis:

\[ r=0, \quad -h \leq z \leq L_1 + L_2 \]

\[ \frac{\partial u}{\partial r} = 0, \frac{\partial v}{\partial r} = 0, \quad v = 0 \] \hspace{1cm} (12)

Along the liquid-air interface:

\[ r=r_s, \quad -h \leq z \leq L_1 + L_2 \]

\[ \tau = \rho_s \cdot E_t \] \hspace{1cm} (13)

\[ \Delta p = \sigma \left( \frac{1}{r_1} - \frac{1}{r_2} \right) - \frac{\varepsilon_0}{2} \left[ E_n^o \frac{2}{r_1} - \varepsilon_r E_n^i \frac{2}{r_2} + (\varepsilon_r - 1)E_t^2 \right] \] \hspace{1cm} (14)

where \( u, v \) is the velocity component in the \( z, r \) directions, \( r_s \) is the cone-jet radius, \( Q \) is the liquid flow rate, \( d_n \) is the inner diameter of nozzle, \( \rho_s \) is the charge density along the surface, \( \varepsilon_r \) is the relative permittivity, \( \varepsilon_0 \) is the dielectric constant of vacuum, \( \tau \) and \( E_t \) are the tangential shear stress and tangential electric field on the surface, respectively. \( E_n^o, E_n^i \) is the outer and inner normal electric field on the surface, \( \sigma \) is the surface tension, \( r_1, r_2 \) is the inner and outer radii of curvature of the free surface, \( \Delta p \) is the pressure jump over the interface from air to liquid.

### 3.4 Numerical implementation and model verification

Two levels of the numerical procedures were adopted in this simulation, an outer loop of electric potential field in the whole region and an inner loop for cone-jet simulation in cone-jet region. The procedures are listed below.

1. Both the cone-jet region and spraying region were used for the initial electric
field simulation using Eqs. (3) - (5), no liquid presented in it.

(2) The data from the electric field simulation for the whole region was extracted and applied to cone-jet region as the boundary conditions of the electrical field.

(3) The cone-jet simulation was carried out using Eqs. (1), (2) and (9).

(4) Once a cone-jet was obtained, a new electric field simulation would be carried out again, because the relative permittivity $\varepsilon_r$ changed.

(5) The electrical field simulation in the whole region was checked for convergence by comparing the electrical field applied to the cone-jet region with the new electric field distribution in step (4). If convergence was reached, the simulation finished, or steps (2)-(5) were repeated.

When the flow rate was 1 ml/h, and the applied voltage on the ring electrode $V_r$ was 1 kV, the nozzle voltage $V_n$ was 4.19 kV, the semi-cone angle simulated by Comsol was 37.3°, that measured in the experiments was 38.1°, a good agreement with them was obtained, and the accuracy of the simulation was well verified.

4 Results and discussions

4.1 The electric field distributions

The electrical potential distribution in the absence of the liquid was calculated based on the Poisson’s equation. The electrical potential distribution with the liquid was simulated by using the relative permittivity $\varepsilon_r$ as described in the Eqs. (8). The electric field distribution could be obtained through calculating the electrical potential
distribution. The results of the numerical simulation using software (Comsol) are shown in Fig.5 and Fig.6.

Fig.5 (a) shows the electrical field distribution near the nozzle tip without liquid at the cone-jet region \((z=0 \text{ mm})\) for the two electro-spraying systems. According to the superposition principle, the electric field strength in the nozzle-ring system was smaller than that in the nozzle system. Both of the electric field of the two systems was higher than the breakdown limit of air \((3\times10^6 \text{ V/m})\). When the voltage was applied to the electrode, ambient discharge formed near the nozzle tip, generating an audible noise.

Fig. 5(b) shows the electrical field distribution without the liquid at the spraying region \((z=5\text{ mm}, 10\text{ mm}, 20\text{ mm})\) for the two electro-spraying systems. For the nozzle system, the electric field strength \((z=5\text{ mm})\) decreased rapidly with the increasing of the radius \(r\). For the nozzle-ring system, the section of \(z=5\text{ mm}\) was in the ring electrode, the electric field strength was zero \((6.2\text{ mm}<r<8\text{ mm})\) because of the ring electrode. For both nozzle system and nozzle-ring system, the electric field distribution \((z=10\text{ mm}, 20\text{ mm})\) became more uniform with the increasing of the radius \(r\). The electric field strength decreased with the increasing of the axial coordinate \(z\), and the electric field strength near the grid \((z=20\text{ mm})\) increased because of the reverse electric field. As mentioned above in section 2.3, the experimental measurements of droplet sizes performed at \(z=10\text{ mm}\) were acceptable to identify the difference between the two systems.

Fig.6 (a) shows the electrical field distribution near the nozzle tip with liquid at
the cone-jet region \((z=0 \text{ mm})\) for the two electro-spraying systems. Both of the electric field of the two systems was lower than the breakdown limit of air \((3\times10^6 \text{ V/m})\), the experiments were performed without ambient discharge. For the nozzle-ring system, the electric field near the nozzle tip decreased due to the effect of the ring electrode. The rapid increment was reduced in the electric field near the cone-tip, and the sudden changes were prevented. Thus, a more stable cone-jet formed.

Fig.6 (b) shows the electrical field distribution with the liquid at the spraying region \((z=5 \text{ mm, 10 mm, 20 mm})\) for the two electro-spraying systems. Comparing with the nozzle system, the electric field strength was much lower in the nozzle-ring system. The electric field between the ring electrode and the ground electrode supplied the driving force for the movement of the droplets. If the ring electrode voltage was too low, the satellite droplets were driven out from the shield of the spraying core. They would reverse their paths and flowed back to the ring electrode, causing flooding and finally interruption of the flow. This phenomenon was called “satellite trapping” (Deng & Gomez, 2007), which should be prevented. For the nozzle-ring system, the electric field strength was suitable to drive the droplets to the grounded electrode without “satellite trapping”.

### 4.2 The cone-jet mode

Fig.7 shows the photos of electro-spraying at cone-jet mode obtained by a digital camera. The experiments were carried out at the fixed flow rate of 1.0ml/h. Some different electro-spraying modes were observed along with the increasing of the nozzle voltage \(V_n\). At the cone-jet mode, the interface between air and the liquid was
charged to a high potential, the liquid meniscus became a stable cone, whose apex issued a narrow jet. For the nozzle system, the distance between the stainless steel grid and the nozzle tip $L$ was 26.1 mm, the stable cone-jet mode was observed when the nozzle voltage $V_n$ ranged from 5 to 5.5 kV. Fig.7 (a) is the photo of electro-spraying at cone-jet mode when $V_n=5.2$ kV. For the nozzle-ring system, the distance between the nozzle tip and the ring electrode $L_1$ was 1.1 mm, the distance between the ring electrode and the stainless steel grid $L_2$ was 20 mm. The applied voltage on the ring electrode $V_r$ was 1 kV. The stable cone-jet mode was observed when the nozzle voltage $V_n$ ranged from 4.1 to 4.6 kV. Fig.7 (b) is the photo of electro-spraying at cone-jet mode when $V_n=4.2$ kV.

For the nozzle system, a high voltage was applied to the nozzle to form a strong electric field. The cone-jet was obtained when the electric field strength exceeded the critical value. And if the voltage was too high, the strong electric field may destabilize the cone and the spraying.

For the nozzle-ring system, the spraying passed through the ring electrode. The electric field near the nozzle tip decreased due to the effect of the ring electrode. The rapid increment was reduced in the electric field near the cone-tip and the sudden changes were prevented, so the ring electrode shielded the cone-jet region, the stable cone-jet mode can be achieved at the lower nozzle voltage. If the ring electrode $V_r$ was too low, the “satellite trapping” phenomenon described in section 4.2 appeared. The satellite droplets were trapped by the ring electrode and thus they couldn’t reach the ground electrode completely. On the other hand, if the ring electrode $V_r$ was too
high, the stable cone-jet would be obtained at higher nozzle voltage, which was unnecessary.

4.3 Droplet size and velocity at four different modes

Fig. 8 shows the influence of the nozzle voltage $V_n$ on the droplet size and velocity for the nozzle system at the flow rate of 1.0 ml/h. The distance between the stainless steel grid and the nozzle tip $L$ was 26.1 mm. The measurements were performed at the position of $z=10$ mm. The arithmetic mean diameter can be defined as follows:

$$D_{\text{mean}} = \frac{1}{N} \sum_{i=1}^{N} D_i$$  \hspace{1cm} (15)

Where $N$ is the total number of particles, $D_i$ is the diameter of the individual particles.

The arithmetic mean axial velocity can be defined as follows:

$$u_{\text{mean}} = \frac{1}{N} \sum_{i=1}^{N} u_i$$  \hspace{1cm} (16)

Where $N$ is the total number of particles, $u_i$ is the axial velocity of the individual particles.

Fig. 8 shows the average diameters and velocity of the droplets at different positions which were described in Fig. 3. Fig. 8 (a) shows the average diameters of the droplets for nozzle system at different modes and positions. As the nozzle voltage $V_n$ increased from 4.3 kV to 5 kV (at pulse-jet mode), the droplet sizes decreased from 25 $\mu$m to 13 $\mu$m, and the droplet sizes were nonuniform at different positions. When $V_n$ increased from 5 to 5.5 kV (at cone-jet mode), nearly monodisperse droplets appeared with the average sizes of around 12 $\mu$m. If $V_n$ exceeded 5.5 kV (at skewed cone-jet or
multi-jet modes), the droplet sizes became nonuniform. Fig. 8 (b) shows the average axial velocity of the droplets for nozzle system at different modes and positions. The droplets at positions 0-8 which were described in Fig. 3 were close to the axis of the core (central area). The lower velocity corresponded to the smaller droplet, and the more uniform velocity corresponded to the more uniform droplet size. But for the droplets which were far away from the center (satellite area), droplets size and velocity had no obvious regularity, due to the effect of other forces, such as the coulomb force.

Fig. 9 shows the influence of the nozzle voltage $V_n$ on the droplet sizes and velocity for the nozzle-ring system at the flow rate of 1.0 ml/h. The distance between the nozzle tip and the ring electrode $L_1$ was 1.1 mm, the distance between the ring electrode and the stainless steel grid $L_2$ was 20 mm, and the applied voltage on the ring electrode $V_r$ was 1.0 kV. The measurements were performed at the position of $z=10$ mm. Fig. 9 (a) shows the average diameters of the droplets for nozzle system at different modes and positions. When $V_n$ ranged from 4.1 to 4.6 kV (at cone-jet mode), nearly monodisperse droplets appeared with the average sizes of around 9 $\mu$m. Fig. 9(b) shows the average velocity of the droplets at different modes and positions. The change trend of the droplets size and velocity were the same as that described in the nozzle system. But the velocity was lower than that in the nozzle system, and the corresponding droplet size was smaller too.

Comparing with the nozzle system, smaller and more uniform droplets were found in the nozzle-ring system at the cone-jet mode and the same flow rate. The
droplets breakup process was dominated by the axisymmetric disturbance. For the nozzle-ring system, the axisymmetric disturbance was protected by the ring electrode, due to the decreasing of the electric field strength near the cone-tip. Thus, the cone was more stable and produced smaller and more uniform droplets.

4.4 The scaling law

4.4.1 Non-dimensional analysis

The scaling law for droplet sizes is a function of the parameters governing the problem. The droplet diameter $d$ depends on the liquid properties (density $\rho$ and viscosity $\mu$, electrical conductivity $K$, gas-liquid surface tension $\sigma$, and relative permittivity $\varepsilon_r$) as well as the flow rate $Q$, nozzle voltage $V_n$, ring voltage $V_r$, vacuum permittivity $\varepsilon_0$, and a given geometrical configuration. For the electro-spraying at the cone-jet mode, the droplet sizes keep constant as the voltage changes, so the influence of the voltage can be negligible. The relationship between droplet sizes and flow rate was developed by Gañán-Calvo (Gañán-Calvo et al., 2009).

$$\frac{r_d}{(\rho Q^2/\Delta p)^{1/4}} = k_d$$

(17)

$$\Delta p = k_p \left( \frac{\sigma^2 K^2 \rho}{\varepsilon_0^2} \right)^{1/3}$$

(18)

Where $r_d$ is the radius of the resulting droplets, $\rho$ is the density of the liquid, $Q$ is the volume flow rate of the liquid, $\Delta p$ is the pressure jump caused by the Maxwell stresses and surface tension, $k_d$ is a nearly universal constant, $k_p$ is a constant of order unity, $\varepsilon_0$ is the permittivity of a vacuum, $\sigma$ is the gas-liquid surface tension, $K$ is the electrical conductivity of the liquid.
The reference flow rate and the corresponding jet radius were defined as follows:

\[ Q_\sigma = \left( \frac{\sigma^4}{\rho \Delta p^3} \right)^{1/2} \]  \hspace{1cm} (19)

\[ r_\sigma = \frac{\sigma}{\Delta p} \]  \hspace{1cm} (20)

When Eqs. (19) and (20) were put into Eq. (17), it can be obtained as follows:

\[ \frac{d}{d_\sigma} = k_d \left( \frac{Q}{Q_\sigma} \right)^{1/2} \]  \hspace{1cm} (21)

Where \( d \) is the diameter of the resulting droplets, \( d_\sigma \) is the corresponding jet diameter of the reference flow rate, \( Q_\sigma \) is the reference flow rate.

The scaling law Eq. (21) shows the relationship between the dimensionless diameter \( d/d_\sigma \) and a function of the dimensionless flow rate \( Q/Q_\sigma \).

4.4.2 Droplet size scaling law

The droplet sizes were measured at cone-jet mode both for the nozzle system and the nozzle-ring system. The droplet size distributions of these two different electro-spraying systems are shown in the Fig.8 and Fig.9. The droplet size distributions at cone-jet mode were uniform at various positions, so the droplets measured could be considered as nearly monodisperse. In present study, the arithmetic mean diameter \( D_{\text{mean}} \) was chosen as the droplet diameter \( d \). Fig.10 shows the dimensionless diameter \( d/d_\sigma \) as a function of the dimensionless flow rate \( Q/Q_\sigma \). The dimensionless droplet sizes in both nozzle system and nozzle-ring system were found to obey the scaling law as dimensionless flow rate. Base on the present experimental results, the values of \( k_d \) were fitted. For the nozzle system, \( k_d = 0.75 \); and for the nozzle-ring system, \( k_d = 0.50 \).

The predictive capability of these correlations was overall evaluated by the mean
relative error (MRE), defined as

$$\text{MRE} = \frac{1}{M} \sum \frac{|d_{\text{pred}} - d_{\text{exp}}|}{d_{\text{exp}}} \times 100\%$$ (22)

where $M$ is the number of data points, $d_{\text{pred}}$ is the droplet sizes predicted by the available correlations, $d_{\text{exp}}$ is the droplet sizes measured in the experiments. Based on present experimental results, the fitted equation (23) had a MRE of 14.3%, and the fitted equation (24) had a MRE of 25.2%.

$$\frac{d}{d_{\sigma}} = 0.75 \left( \frac{Q}{Q_{\sigma}} \right)^{1/2}, \quad \text{for the nozzle system (23)}$$

$$\frac{d}{d_{\sigma}} = 0.50 \left( \frac{Q}{Q_{\sigma}} \right)^{1/2}, \quad \text{for the nozzle-ring system (24)}$$

5 Conclusions

In present study, two different micro-combustors were designed, and some experiments on electro-spraying in the combustors using liquid ethanol as fuel were carried out. The droplet size and velocity distributions were measured by PDA at different electro-spraying modes. The electric field distributions were numerically calculated at the cone-jet mode. And the non-dimensional analysis was performed based on the scaling law at the cone-jet mode. Some main conclusions can be achieved as following:

(1) The electric field strength near the nozzle tip in the nozzle-ring system at cone-jet mode and the same flow rate was smaller than that for the nozzle system according to the superposition principle of electric field.

(2) The rapid increment was reduced in the electric field near the cone-tip and the sudden changes were prevented, a more stable cone-jet mode was obtained in the nozzle-ring system.
Comparing with the nozzle system, the nozzle-ring electro-spraying system could produce smaller and more uniform droplets with lower velocity because of the more stable cone-jet mode.

It was found that the dimensionless droplet sizes in both the nozzle system and nozzle-ring system has a 1/2 power dependence on dimensionless liquid flow rate.

Acknowledgements

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References


ethanol diffusion flames from mini tube nozzles. *Combustion and Flame*, 156, 460-466.


Table captions

**Table 1** Physical properties of ethanol (25°C)

**Table 2** Error analysis
<table>
<thead>
<tr>
<th>Density</th>
<th>Viscosity</th>
<th>Surface tension</th>
<th>Conductivity</th>
<th>Relative permittivity</th>
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<td>N/m</td>
<td>S/m</td>
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<td>0.022</td>
<td>5.1×10⁻⁵</td>
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</table>
**Table 2** Error analysis

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Measure tool</th>
<th>Ranges</th>
<th>Error</th>
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<tr>
<td>Flow rate, $Q$</td>
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<td>Voltage, $V_n$</td>
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<tr>
<td>Voltage, $V_t$</td>
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<td>±1.0%</td>
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<tr>
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<td>Thickness of ring electrode, $\delta$</td>
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<td>Droplet size</td>
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<tr>
<td>Droplet velocity</td>
<td>PDA</td>
<td>0-50 m/s</td>
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Figure captions

Fig.1 Schematic diagram of the test rig

Fig.2 Electro-spraying systems (a) nozzle system; (b) nozzle-ring system

Fig.3 Distribution of measurement points using PDA

Fig.4 Geometric model of numerical simulation

Fig.5 Electric field distributions at cone-jet region and spraying region without fuel

(a) at the cone-jet region; (b) at the spraying region

Fig.6 Electric field distributions at cone-jet region and spraying region with fuel

(a) at the cone-jet region; (b) at the spraying region

Fig.7 Typical images of the electro-spraying in the two different systems

(a) nozzle system ($L=26.1$ mm, $V_n=5.2$ kV, $Q=1.0$ ml/h)

(b) nozzle-ring system ($L_1=1.1$ mm, $L_2=20$ mm, $V_n=4.2$ kV, $V_r=1$ kV, $Q=1.0$ ml/h)

Fig.8 Droplet size and velocity distributions for nozzle system at different modes and positions ($L=26.1$ mm, $z=10$ mm, $Q=1$ ml/h)

(a) Droplet size distribution for nozzle system at different modes and positions

(b) Droplet velocity distribution for nozzle system at different modes and positions

Fig.9 Droplet size and velocity distributions for nozzle-ring system at different modes and positions ($L_1=1.1$ mm, $L_2=20$ mm, $z=10$ mm, $V_2=1$ kV, $Q=1$ ml/h)

(a) Droplet size distribution for nozzle-ring system at different modes and positions

(b) Droplet velocity distribution for nozzle-ring system at different modes and positions

Fig.10 Dimensionless average droplet size versus dimensionless average flow rate at cone-jet mode
1- syringe pump; 2- syringe; 3- plastic pipe; 4- substrate; 5- nozzle; 6- laser lamp;
7- ring electrode; 8- high-voltage DC power supply; 9- laser; 10-focusing lens; 11-
steel grid; 12-computer;

**Fig.1** Schematic diagram of the test rig
Fig. 2 Electro-spraying systems with or without a ring electrode
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(a) nozzle system ($L=26.1$ mm, $V_n=5.2$ kV, $Q=1.0$ ml/h)

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(a) Droplet size distribution for nozzle system at different modes and positions

(b) Droplet velocity distribution for nozzle system at different modes and positions

**Fig. 8** Droplet size and velocity distributions for nozzle system at different modes and positions ($L=26.1$ mm, $z=10$ mm, $Q=1$ ml/h)
(a) Droplet size distribution for nozzle-ring system at different modes and positions

(b) Droplet velocity distribution for nozzle-ring system at different modes and positions

**Fig.9** Droplet size and velocity distributions for nozzle-ring system at different modes and positions ($L_1=1.1 \text{ mm}, L_2=20 \text{ mm}, z=10 \text{ mm}, V_2=1 \text{ kV}, Q=1 \text{ ml/h}$)
**Fig. 10** Dimensionless average droplet size versus dimensionless average flow rate at cone-jet mode