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Bit Error Rate of Underlay Decode-and-Forward Cognitive Networks with Best Relay Selection

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Abstract: **This paper provides an analytic performance evaluation of the bit error rate (BER) of underlay decode-and-forward cognitive networks with best relay selection over Rayleigh multipath fading channels. A generalized BER expression valid for arbitrary operational parameters is firstly presented in the form of a single integral, which is then employed for determining the diversity order and coding gain for different best relay selection scenarios. Furthermore, a novel and highly accurate closed-form approximate BER expression is derived for the specific case where relays are located relatively close to each other. The presented results are rather convenient to handle both analytically and numerically, while they are shown to be in good agreement with results from respective computer simulations. In addition, it is shown that as in the case of conventional relaying networks, the behaviour of underlay relaying cognitive networks with best relay selection depends significantly on the number of involved relays.**

Index Terms: **Bit error rate, cognitive radios, cooperative relaying, underlay communication, relay selection, Rayleigh fading.**

I. INTRODUCTION

An extensive survey on frequency spectrum utilization carried out by the Federal Communications Commission has reported a severe spectrum under-utilization [1]. However, this is in contrast with the currently witnessed spectrum scarcity due to the highly increasing spectrum demand for emerging wireless communication services. Fortunately, it has been shown that this issue can be effectively resolved with the aid of cognitive radio (CR) technology which allows secondary users (SUs) to co-exist with primary users (PUs) on the frequency bands inherently allocated to the latters [2]. As a result, the corresponding spectrum utilization efficiency can be substantially improved.

Ensuring the avoidance of undesired interference on PUs is the most critical task and challenge in CR technology. To this end, the involved SUs can typically operate in three different modes: interweave; overlay; and underlay [3]. Due to the advantageous feature of low implementation complexity, the underlay mode has recently attracted a notable deal of attention, e.g. [3–17] and the references therein. In this mode, SUs must adaptively control their transmit power in order for the induced interference to be strictly maintained within levels that can be tolerated by PUs. This ultimately leads to the drastically shortened transmission range of SUs, which can be compensated in turn with the aid of cooperative relaying techniques [18]. Indeed, by taking advantage of intermediate users −so called relays− located between the source and the destination to relay source information, underlay relaying cognitive networks can overcome the aforementioned drawback thanks to the resulting short range communication with low path-loss effects. The relays can operate according to various cooperative relaying schemes such as the decode-and-forward (DF) and amplify-and-forward (AF) [19]. In the former scheme, the relays decode the received signal and then re-encode the decoded information before relaying it to the destination. In the latter scheme, the relays just amplify the received signal and forward it to the destination. It is recalled here that cooperative relaying with selection of a single relay among a set of possible candidates requires less system resources, such as bandwidth and power, than multi-relay assisted transmission while maintaining the same diversity order [3, 20–23].

Outage probability (OP) of underlay DF cognitive networks with relay selection has been extensively studied in several research works, such as [3–12]. Specifically, the authors in [3], [5–12] assume single-carrier transmission, while [4] considers multi-carrier transmission. Furthermore, in order to guarantee certain quality of service for PUs, the authors in [3], [5], [6], [11], [12] investigate both interference power and maximum transmit power constraints, while [7], [9], [10] study only the interference power constraint. The OP constraint at PUs was considered in [8], while several relay selection methods have been proposed in [3, 6–8, 11, 24–26]. For instance, in the method of [3, 24], the selected relay is the one that maximizes the end-to-end signal-to-noise ratio (SNR). The authors in [6–8, 25] select the relay among all possible candidates (i.e., all relays are assumed to successfully decode source information) that results in the largest SNR at the destination while the authors in [26] opt for the relay among all possible candidates (i.e., relays are assumed to satisfy the interference power constraint) that results in either the largest or smallest SNR at the destination, or the minimum level of interference to PUs. In [11], the N-th best relay selection method is proposed. However, in spite of the potential of underlay DF cognitive networks, only few works have addressed the BER analysis of these systems [26–30]. Nevertheless, the

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works in [27–30] have not investigated the impact of relay selection, which will be shown to be a particularly cumbersome task, even in deriving an approximate BER expression. It is also noted that the work in [26] studies the effect of relay selection on the BER performance but with a simplified system model, where the relays are assumed geographically close, the source does not interfere with the PU and only interference power constraint is considered. It is recalled here that the OP analysis can provide an insight into the information-theoretic performance limit and motivate practical code designs to reach it. However, there is no systematic tool that determines when this limit is reached, but instead the BER analysis provides the realistic measure of system performance for a target spectral efficiency, i.e. signal's modulation level. This renders the theoretical and practical importance of the BER analysis more significant.

Motivated by the above, the aim of the present work is to evaluate analytically the BER performance of underlay DF cognitive networks with the best relay selection scheme proposed in [3], which is proven to be capacity optimal. The corresponding analysis takes into account both the interference power constraint and the maximum transmit power constraint. For the sake of computer simulation time and energy savings, it is imperative to possess the BER performance. However, since deriving an exact closed-form BER expression is extremely difficult, if not impossible, in this paper we resort to the derivation of a tractable closed-form approximation. It is extensively shown that the derived expression is highly accurate and this is verified through comparisons with results obtained from corresponding Monte Carlo simulations. As a result, the proposed closed-form approximate BER expression facilitates in assessing effectively the system behaviour and performance in key operational parameters, without necessarily resorting to energy exhaustive and time consuming simulations. It is additionally shown that, as in the case of conventional relaying networks, the BER performance of underlay relaying cognitive networks with best relay selection depends significantly on the number of employed relays.

The contributions of this paper are summarized as follows[1]:

- An exact BER analysis framework is proposed for underlay DF cognitive networks with best relay selection under general operational conditions, such as arbitrary number of relays, unequal fading powers among channels, both interference power and maximum transmit power constraints. The derived BER expression is in the form of single integral, which can be easily evaluated numerically.
- Under general operational conditions, we obtain the diversity order and coding gain for underlay DF cognitive networks with best relay selection. It is shown that this type of networks achieves the full diversity order.

[1]It should be emphasized that the analysis presented in this paper is completely different and more complicated than [26] for the following reasons: Firstly, the relay selection scheme considered in this paper is different from that in [26]; the former is a capacity-optimal selection scheme while the latter is not. Secondly, we consider both interference power and maximum transmit power constraints whereas, [26] only considers the interference power constraint, which definitely renders the analysis presented hereinafter more complex than [26]. Thirdly, our system model investigates both cases of arbitrarily and closely located relays, while [26] only demonstrates the case of closely located relays. Finally, our analysis is more thorough (including the analysis of the exact and approximate BER as well as the diversity order and coding gain) than [26], where only an approximate BER analysis is presented.

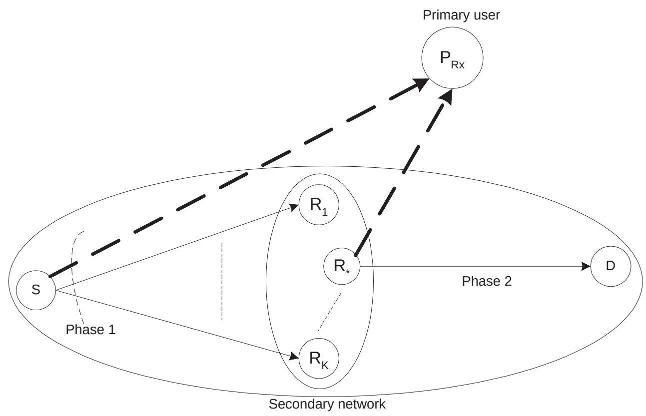

Fig. 1. The considered underlay relaying cognitive network.

- In the specific case where relays are located relatively close to each other, we propose a tight approximation for the corresponding BER. This expression is given in closed form and appears to be particularly useful in analytically evaluating the BER performance of underlay DF cognitive networks with best relay selection.

The remainder of this paper is organized as follows: The system model is described in Section II. The corresponding BER analysis for underlay DF cognitive networks with best relay selection is presented in Section III. Simulated and analytical results for the evaluation and validation of the presented BER expressions are provided in Section IV. Finally, useful remarks and conclusions are included in Section V.

II. SYSTEM MODEL

We investigate an underlay relaying cognitive network as depicted in Fig. 1. In the secondary network, the source S transmits its information to the destination D with the help of the best relay R_*, selected from the cluster of K relays $\mathcal{R} = \{R_1, R_2, ..., R_K\}$. It is also assumed that the operation of S and R_* interferes with that of the PU P_{Rx}. Wireless channels are considered independent and frequency flat with fading following the Rayleigh distribution. To this effect, the channel coefficient between a transmitter t and a receiver r can be modelled as[2] $h_{t,r} \sim \mathcal{CN}(0, \lambda_{t,r}^{-1})$ where $t \in \{S, R_1, R_2, ..., R_K\}$ and $r \in \{R_1, R_2, ..., R_K, D, P_{Rx}\}$.

As illustrated in Fig. 1, cooperative relaying operates in two phases; in the first phase, S broadcasts a sequence of q modulated symbols $\mathbf{x}_S = \{x_S(1), x_S(2), ..., x_S(q)\}$ with symbol energy $\mathcal{P}_S = \mathcal{E}\{|x_S(u)|^2\}$, $u = 1, 2, ..., q$, where $\mathcal{E}\{\cdot\}$ denotes statistical expectation. Subsequently, the best relay R_* demodulates this symbol sequence while the other relays remain idle, and the demodulated symbols are re-modulated as $\mathbf{x}_{R_*} = [x_{R_*}(1), x_{R_*}(2), ..., x_{R_*}(q)]$ with symbol energy $\mathcal{P}_{\mathcal{R}_*}$, before forwarded to D in the second phase. For notation simplicity and without loss of generality, the time index q is hereinafter ignored. To this end, the received signal at the relays and

[2]$h \sim \mathcal{CN}(a, p)$ denotes a circular symmetric complex Gaussian random variable with mean a and variance p.

the destination can be modelled as

$$y_{t,r} = h_{t,r}x_t + n_{t,r} \tag{1}$$

where $n_{t,r} \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN) at user r, while $t \in \{S, R_*\}$ and $r \in \{R_1, R_2, ..., R_K, D\}$.

It is recalled that operating in the underlay mode as in [3], the SU t (i.e. both S and R_*) is required to set its transmit power as $\mathcal{P}_t = \min(\bar{I}/|h_{t,P_{Rx}}|^2, \bar{P})$ for maximizing the transmission range while meeting both the interference power constraint, i.e. $\mathcal{P}_t \leq \bar{I}/|h_{t,P_{Rx}}|^2$, and the maximum transmit power constraint, i.e. $\mathcal{P}_t \leq \bar{P}$. The notation $\bar{I}$ represents the maximum interference power that PU can tolerate and $\bar{P}$ is the maximum transmit power designed for the corresponding SU. It is also noted that $\bar{I}$ implicitly stands for the interference limit from SU and excludes any interference from other PUs [3]. Likewise, the primary network is implicitly assumed to operate reliably for interference levels caused by SUs up to $\bar{I}$, regardless of the interference already existing in this network. In other words, PU-to-PU interference is not necessarily accounted when setting $\mathcal{P}_t$. With this transmit power setting, (1) renders the following instantaneous SNR expression:

$$\gamma_{t,r} = \frac{\mathcal{P}_t|h_{t,r}|^2}{N_0} = \min\left(\frac{\bar{I}}{|h_{t,P_{Rx}}|^2}, \bar{P}\right)\frac{|h_{t,r}|^2}{N_0}. \tag{2}$$

By letting $\eta_{t,r} = \min(\bar{I}/|h_{t,P_{Rx}}|^2, \bar{P})|h_{t,r}|^2$, the cumulative density function (cdf) of $\eta_{t,r}$, denoted as $F_{\eta_{t,r}}(x)$, is given by [3, eq. (8)]. To this effect and since $\gamma_{t,r} = \eta_{t,r}/N_0$, the cdf of $\gamma_{t,r}$ is $F_{\gamma_{t,r}}(x) = \Pr\{\gamma_{t,r} \leq x\}$ which can be expressed as

$$\begin{aligned} F_{\gamma_{t,r}}(x) &= \Pr\left\{\frac{\eta_{t,r}}{N_0} \leq x\right\} = F_{\eta_{t,r}}(N_0 x) \\ &= 1 + \left(\frac{e^{-\frac{\lambda_{t,r}\Lambda_{t,r}I}{P}}}{1+\frac{\Lambda_{t,r}I}{x}} - 1\right)e^{-\frac{\lambda_{t,r}x}{P}} \end{aligned} \tag{3}$$

where $\Lambda_{t,r} = \lambda_{t,P_{Rx}}/\lambda_{t,r}$, $I = \bar{I}/N_0$ and $P = \bar{P}/N_0$, while $\Pr\{X\}$ is the probability of the event X.

According to the proactive DF relaying principle in [3], the best relay R_* is the one having the largest end-to-end SNR. Thus, the end-to-end SNR can be mathematically expressed as

$$\gamma_{e2e} = \max_{R_k \in \mathcal{R}}\left(\min\left(\gamma_{S,R_k}, \gamma_{R_k,D}\right)\right). \tag{4}$$

Hence, since γ_{S,R_k} and $\gamma_{R_k,D}$ are statistically independent, it follows that the corresponding cdf of γ_{e2e} is given by $F_{\gamma_{e2e}}(x) = \Pr\{\gamma_{e2e} < x\}$, which yields

$$\begin{aligned} F_{\gamma_{e2e}}(x) &= \prod_{k=1}^{K}\Pr\left\{\min\left(\gamma_{S,R_k}, \gamma_{R_k,D}\right) < x\right\} \\ &= \prod_{k=1}^{K}\left(1 - \Pr\left\{\min\left(\gamma_{S,R_k}, \gamma_{R_k,D}\right) \geq x\right\}\right) \\ &= \prod_{k=1}^{K}\left(1 - \Pr\left\{\gamma_{S,R_k} \geq x\right\}\Pr\left\{\gamma_{R_k,D} \geq x\right\}\right) \\ &= \prod_{k=1}^{K}\left\{1 - \left[1 - F_{\gamma_{S,R_k}}(x)\right]\left[1 - F_{\gamma_{R_k,D}}(x)\right]\right\} \end{aligned} \tag{5}$$

Therefore, by substituting (3) in (5), one obtains (6) at the top of the next page. Importantly, the above expression is particularly useful in the subsequent error probability analysis.

III. BIT ERROR RATE ANALYSIS

Let $B_{e|\gamma_{e2e}}(x)$ be the BER conditioned on γ_{e2e}, which depends on the employed modulation scheme. The average BER for the underlay DF cognitive network with the best relay selection scheme described in Section II can be obtained as

$$B_e = \int_0^\infty B_{e|\gamma_{e2e}}(x)\, f_{\gamma_{e2e}}(x)\, dx \tag{7}$$

where $f_{\gamma_{e2e}}(x)$ is the probability density function (pdf) of γ_{e2e}.

The following BER analysis framework is valid for[3] $M-$ary Quadrature Amplitude Modulation ($M-$QAM) with arbitrary values of modulation order $M = 2^h$. For square $M-$QAM with h even and rectangular $M-$QAM with h odd, $B_{e|\gamma_{e2e}}(x)$ is given by $2\Theta\left(\sqrt{M}, m, M; x\right)$ and $\Theta(G, u, M; x) + \Theta(J, u, M; x)$ in [37, eq. (16)] and [37, eq. (22)], respectively. There, $\Theta(s, v, M; x)$ is given by (8) (top of the next page) with $m = 3/(M-1)$, $u = 6/(G^2 + J^2 - 2)$, $G = 2^{(h-1)/2}$ and $J = 2^{(h+1)/2}$. Furthermore, the notations $\lfloor . \rfloor$ and $Q(.)$ are the floor function and the one dimensional Gaussian $Q-$function [38], respectively, which are both included as standard built-in functions in popular mathematical software packages such as MAPLE, MATLAB and MATHEMATICA.

Given $B_{e|\gamma_{e2e}}(x)$ and $f_{\gamma_{e2e}}(x)$, it immediately follows that for $M-$QAM constellations, B_e can be expressed as

$$B_e = \begin{cases} \Phi(G, u, M; \chi) + \Phi(J, u, M; \chi) & , h \text{ odd} \\ 2\Phi\left(\sqrt{M}, m, M; \chi\right) & , h \text{ even} \end{cases} \tag{9}$$

where $\chi = \{\lambda_{S,R_k}, \lambda_{R_k,D}, \Lambda_{S,R_k}, \Lambda_{R_k,D}, I, P\}$ includes the set of system operational parameters and the function $\Phi(s, v, M; \chi)$ is given by (10) at the top of the next page. It is noted that in (10), the function $\zeta(\beta; \chi)$ is expressed as

$$\zeta(\beta; \chi) = \int_0^\infty Q\left(\sqrt{\beta x}\right) f_{\gamma_{e2e}}(x)\, dx. \tag{11}$$

A. Exact Analysis

By integrating (11) once by parts and then performing the necessary change of variables and substituting (6) into the result, one obtains the following compact integral representation:

$$\begin{aligned} \zeta(\beta; \chi) &= \frac{\sqrt{\beta}}{2\sqrt{2\pi}}\int_0^\infty \frac{F_{\gamma_{e2e}}(x)}{\sqrt{x}e^{\frac{\beta x}{2}}}dx + \left(Q\left(\sqrt{\beta x}\right)F_{\gamma_{e2e}}(x)\right)\Big|_0^\infty \\ &= \frac{1}{\sqrt{2\pi}}\int_0^\infty F_{\gamma_{e2e}}\left(\frac{t^2}{\beta}\right)e^{-\frac{t^2}{2}}dt \end{aligned} \tag{12}$$

[3]The BER of other modulation schemes such as $M-$ary Phase Shift Keying ($M-$PSK) can be analyzed in a similar manner.

$$F_{\gamma_{e2e}}(x)=\prod_{k=1}^{K}\left\{1-\left(1-\frac{e^{-\lambda_{S,R_k}\Lambda_{S,R_k}I/P}}{1+\Lambda_{S,R_k}I/x}\right)\left(1-\frac{e^{-\lambda_{R_k,D}\Lambda_{R_k,D}I/P}}{\Lambda_{R_k,D}I/x+1}\right)e^{-\left(\lambda_{S,R_k}+\lambda_{R_k,D}\right)x/P}\right\} \tag{6}$$

$$\Theta(s,v,M;x)\triangleq\frac{2}{s\log_2 M}\sum_{g=1}^{\log_2 s}\sum_{i=0}^{\left(1-2^{-g}\right)s-1}(-1)^{\left\lfloor\frac{i2^{g-1}}{s}\right\rfloor}Q\left(\sqrt{(2i+1)^2 vx}\right)\left(2^{g-1}-\left\lfloor\frac{i2^{g-1}}{s}+\frac{1}{2}\right\rfloor\right) \tag{8}$$

$$\Phi(s,v,M;\chi)\triangleq\int_0^\infty\Theta(s,v,M;x)f_{\gamma_{e2e}}(x)\,dx=\frac{2}{s\log_2 M}\sum_{g=1}^{\log_2 s}\sum_{i=0}^{\left(1-2^{-g}\right)s-1}\frac{(-1)^{\left\lfloor\frac{i2^{g-1}}{s}\right\rfloor}\zeta\left([2i+1]^2 v;\chi\right)}{\left(2^{g-1}-\left\lfloor\frac{i2^{g-1}}{s}+\frac{1}{2}\right\rfloor\right)^{-1}} \tag{10}$$

$$\zeta(\beta;\chi)=\frac{1}{\sqrt{2\pi}}\int_0^\infty\prod_{k=1}^{K}\left\{1-\frac{\left(1-\frac{t^2e^{-\lambda_{S,R_k}\Lambda_{S,R_k}I/P}}{t^2+\beta\Lambda_{S,R_k}I}\right)\left(1-\frac{t^2e^{-\lambda_{R_k,D}\Lambda_{R_k,D}I/P}}{t^2+\beta\Lambda_{R_k,D}I}\right)}{e^{\frac{\left(\lambda_{S,R_k}+\lambda_{R_k,D}\right)t^2}{\beta P}}}\right\}e^{-\frac{t^2}{2}}dt \tag{13}$$

which can be equivalently expressed according to (13) at the top of this page. Unfortunately, it is extremely difficult, if not impossible, to obtain a closed-form solution for the above integral for arbitrary operational parameters K, λ_{S,R_k}, $\lambda_{R_k,D}$, Λ_{S,R_k}, $\Lambda_{R_k,D}$, I and P. However, even though (13) is not expressed in closed form, substituting (13) in (10) and then into (9) yields an exact single integral-form BER expression that to the best of the authors' knowledge has not been reported in the open literature. Furthermore, the resulting expression can be rather useful in analyzing the BER performance and its numerical evaluation is not problematic due to singularities and convergence issues. The latter holds due to the presence of the exponential term with negative arguments in the numerator and the shifted arguments in the denominator of (13).

B. Asymptotic Analysis

Deriving the diversity order and coding gain of the considered underlay DF cognitive networks with best relay selection requires investigation of the BER in the high SNR regime. To this end, we assume $I=\tau P$, where τ is a positive real constant, and define the average SNR as $\overline{\gamma}=P$ according to [42]. Hence, by performing the necessary change of variables, (13) can be rewritten as in (14) (top of the next page). It is recalled here that $e^{\alpha/x}\overset{x\to\infty}{\approx}1+\frac{\alpha}{x}$ where α is a constant. Therefore, by substituting accordingly in (14) and ignoring small-valued terms, one obtains (15) at the top of the next page. Using the fact that $\bar{\gamma}\to\infty$, the t^2 terms in the denominators of (15) can be omitted. As such, the above expression can be further approximated according to (16). Notably, the $\mathcal{T}$ integral in (16) can be solved in closed form with the aid of [39, eq. (3.461.2)], namely as

$$\mathcal{T}=\frac{(2K-1)!!\sqrt{2\pi}}{2}. \tag{17}$$

Substituting (17) into (16) yields

$$\zeta(\beta;\chi)\overset{\bar{\gamma}\to\infty}{\approx}\left(\frac{1}{\bar{\gamma}}\right)^K\frac{\mathcal{J}}{2\beta^K} \tag{18}$$

where

$$\mathcal{J}=\frac{\prod_{k=1}^{K}\left(\frac{e^{-\lambda_{S,R_k}\Lambda_{S,R_k}\tau}}{\Lambda_{S,R_k}\tau}+\frac{e^{-\lambda_{R_k,D}\Lambda_{R_k,D}\tau}}{\Lambda_{R_k,D}\tau}+\lambda_{S,R_k}+\lambda_{R_k,D}\right)}{[(2K-1)!!]^{-1}}. \tag{19}$$

By inserting (18) in (10), one obtains (20) at the top of the next page. To this effect and by performing the necessary change of variables, the following compact representation for the BER of $M-$QAM in the high-SNR regime is deduced

$$B_e\overset{\bar{\gamma}\to\infty}{\approx}\begin{cases}\mathcal{G}_o/\bar{\gamma}^K & ,h\text{ odd}\\ \mathcal{G}_e/\bar{\gamma}^K & ,h\text{ even}\end{cases} \tag{21}$$

where $\mathcal{G}_o$ and $\mathcal{G}_e$ are given at the top of the next page.

It is recalled here in the high SNR regime, B_e can be expressed in terms of the diversity order, G_d, and the coding gain, G_c, as $B_e\overset{\bar{\gamma}\to\infty}{\approx}(G_c\bar{\gamma})^{-G_d}$ according to [24]. As such, it is straightforward to infer from (21) that underlay DF cognitive networks with best relay selection achieve the full diversity order of $G_d=K$ offered by all available secondary relays; this result coincides with [3, Lemma 2]. As discovered in [20], the diversity order of cooperative networks with K relays and best relay selection is K. Hence, as $\bar{\gamma}\to\infty$, the considered cognitive network becomes *non-cognitive* and the diversity order is the same with [20]. Moreover, the coding gain is given by

$$G_c=\begin{cases}\mathcal{G}_o^{-1/K} & ,h\text{ odd}\\ \mathcal{G}_e^{-1/K} & ,h\text{ even}\end{cases} \tag{24}$$

C. Special Case: Closely Located Relays

We assume that all involved relays are located close to eachother such that: *i*) the fading powers between S and all relays are identical, i.e. $\lambda_{S,R_k}=\lambda_1$, $\forall k=1,2,...,K$; *ii*) the fading powers between D and all relays are equal, i.e. $\lambda_{R_k,D}=\lambda_2$, $\forall k=1,2,...,K$; and *iii*) the fading powers between PU and all relays are the same, i.e. $\lambda_{R_k,P_{Rx}}=\lambda_4$, $\forall k=1,2,...,K$. For notation simplicity, although not necessary for the derivation that follows, we also denote $\lambda_{S,P_{Rx}}=\lambda_3$ and we assume

$$\zeta\left(\beta;\chi\right)=\int\limits_{0}^{\infty}\prod_{k=1}^{K}\left\{1-\left(1-\frac{t^{2}e^{-\lambda_{S,R_k}\Lambda_{S,R_k}\tau}}{t^{2}+\beta\Lambda_{S,R_k}\tau\bar{\gamma}}\right)\left(1-\frac{t^{2}e^{-\lambda_{R_k,D}\Lambda_{R_k,D}\tau}}{t^{2}+\beta\Lambda_{R_k,D}\tau\bar{\gamma}}\right)e^{-\frac{\left(\lambda_{S,R_k}+\lambda_{R_k,D}\right)t^{2}}{\beta\bar{\gamma}}}\right\}\frac{e^{-\frac{t^{2}}{2}}}{\sqrt{2\pi}}dt \tag{14}$$

$$\zeta\left(\beta;\chi\right)\overset{\bar{\gamma}\to\infty}{\approx}\frac{1}{\sqrt{2\pi}}\int\limits_{0}^{\infty}\prod_{k=1}^{K}\left(\frac{t^{2}e^{-\lambda_{S,R_k}\Lambda_{S,R_k}\tau}}{t^{2}+\beta\Lambda_{S,R_k}\tau\bar{\gamma}}+\frac{t^{2}e^{-\lambda_{R_k,D}\Lambda_{R_k,D}\tau}}{t^{2}+\beta\Lambda_{R_k,D}\tau\bar{\gamma}}+\frac{\left(\lambda_{S,R_k}+\lambda_{R_k,D}\right)t^{2}}{\beta\bar{\gamma}}\right)e^{-\frac{t^{2}}{2}}dt \tag{15}$$

$$\zeta\left(\beta;\chi\right)\overset{\bar{\gamma}\to\infty}{\approx}\left(\frac{1}{\bar{\gamma}}\right)^{K}\frac{1}{\sqrt{2\pi}}\prod_{k=1}^{K}\left(\frac{e^{-\lambda_{S,R_k}\Lambda_{S,R_k}\tau}}{\beta\Lambda_{S,R_k}\tau}+\frac{e^{-\lambda_{R_k,D}\Lambda_{R_k,D}\tau}}{\beta\Lambda_{R_k,D}\tau}+\frac{\lambda_{S,R_k}+\lambda_{R_k,D}}{\beta}\right)\underbrace{\int\limits_{0}^{\infty}t^{2K}e^{-\frac{t^{2}}{2}}dt}_{\triangleq\mathcal{T}} \tag{16}$$

$$\Phi\left(s,v,M;\chi\right)\overset{\bar{\gamma}\to\infty}{\approx}\left(\frac{1}{\bar{\gamma}^{K}}\right)\frac{\mathcal{J}}{v^{K}s\log_2 M}\sum_{g=1}^{\log_2 s}\sum\nolimits_{i=0}^{\left(1-2^{-g}\right)s-1}\frac{(-1)^{\left\lfloor\frac{i2^{g-1}}{s}\right\rfloor}}{(2i+1)^{2K}}\left(2^{g-1}-\left\lfloor\frac{i2^{g-1}}{s}+\frac{1}{2}\right\rfloor\right) \tag{20}$$

$$\mathcal{G}_o=\sum_{g=1}^{\log_2 G}\sum\nolimits_{i=0}^{\left(1-2^{-g}\right)G-1}\frac{\mathcal{J}\left(-1\right)^{\left\lfloor\frac{i2^{g-1}}{G}\right\rfloor}\left(2^{g-1}-\left\lfloor\frac{i2^{g-1}}{G}+\frac{1}{2}\right\rfloor\right)}{(2i+1)^{2K}Gu^{K}\log_2 M}+\sum_{g=1}^{\log_2 J}\sum\nolimits_{i=0}^{\left(1-2^{-g}\right)J-1}\frac{\mathcal{J}\left(-1\right)^{\left\lfloor\frac{i2^{g-1}}{J}\right\rfloor}\left(2^{g-1}-\left\lfloor\frac{i2^{g-1}}{J}+\frac{1}{2}\right\rfloor\right)}{(2i+1)^{2K}Ju^{K}\log_2 M} \tag{22}$$

$$\mathcal{G}_e=\frac{2\mathcal{J}}{\sqrt{M}m^{K}\log_2 M}\sum_{g=1}^{\log_2\sqrt{M}}\sum\nolimits_{i=0}^{\left(1-2^{-g}\right)\sqrt{M}-1}\frac{(-1)^{\left\lfloor\frac{i2^{g-1}}{\sqrt{M}}\right\rfloor}}{(2i+1)^{2K}}\left(2^{g-1}-\left\lfloor\frac{i2^{g-1}}{\sqrt{M}}+\frac{1}{2}\right\rfloor\right) \tag{23}$$

the general case where $\lambda_1\neq\lambda_2\neq\lambda_3\neq\lambda_4$. The adopted assumption on the geographical closeness of the relays is quite reasonable, particularly in wireless sensor networks where neighbouring sensor nodes form a cluster [36], and widely accepted and recently exploited, e.g. see [9, 11, 24, 25, 31–35] and references therein. Based on this assumption, (6) can be re-expressed by the following simplified representation:

$$F_{\gamma_{e2e}}\left(x\right)=\left\{1-\frac{\left(1-\frac{e^{-\lambda_1\Lambda_1 I/P}}{\Lambda_1 I/x+1}\right)\left(1-\frac{e^{-\lambda_2\Lambda_2 I/P}}{\Lambda_2 I/x+1}\right)}{e^{\left(\lambda_1+\lambda_2\right)x/P}}\right\}^{K} \tag{25}$$

where $\Lambda_1=\Lambda_{S,R_k}=\lambda_{S,P_{Rx}}/\lambda_{S,R_k}=\lambda_3/\lambda_1$ and $\Lambda_2=\Lambda_{R_k,D}=\lambda_{R_k,P_{Rx}}/\lambda_{R_k,D}=\lambda_4/\lambda_2$. To this effect, by consecutively applying the binomial expansion [39, eq. (1. 111)] in (25), one deduces (26) (top of the next page) where the binomial coefficient is defined as $C_K^a\triangleq\frac{K!}{a!(K-a)!}$. Based on this, the pdf of γ_{e2e} can be obtained by taking the first derivative of $F_{\gamma_{e2e}}\left(x\right)$, which yields (27). Therefore, by substituting (27) into (11), one obtains the closed form expression as (28), at the top of the next page, where $\sigma=a\left(\lambda_1+\lambda_2\right)/P$ and

$$\Psi\left(\alpha,\beta,b,c;\varepsilon_1,\varepsilon_2\right)=\int\limits_{0}^{\infty}\frac{e^{-\alpha x}\,Q\left(\sqrt{\beta x}\right)}{\left(x+\varepsilon_1\right)^{b}\left(x+\varepsilon_2\right)^{c}}dx. \tag{29}$$

Evidently, deriving a closed-form expression for B_e is subject to the analytical evaluation of (29). To the best of our knowledge, an exact closed-form expression for (29) does not exist. Therefore, we present hereinafter a simple and accurate closed-form approximation for (29) which can be utilized in analyzing the BER performance of the underlay DF cognitive networks with best relay selection straightforwardly and without essentially requiring time-consuming computer simulations. To this end, we firstly insert $\mathrm{erfc}(z)\triangleq 2Q(\sqrt{2}z)$ into [40, eq. (14)] to yield the approximation $Q\left(\sqrt{\beta x}\right)\approx\frac{1}{4}\left(\frac{1}{3}e^{-\frac{\beta}{2}x}+e^{-\frac{2\beta}{3}x}\right)$. By substituting accordingly in (29), one obtains

$$\begin{aligned}\Psi\left(\alpha,\beta,b,c;\varepsilon_1,\varepsilon_2\right)=&\frac{1}{12}\mathrm{T}\left(\alpha+\frac{\beta}{2},b,c;\varepsilon_1,\varepsilon_2\right)\\&+\frac{1}{4}\mathrm{T}\left(\alpha+\frac{2\beta}{3},b,c;\varepsilon_1,\varepsilon_2\right)\end{aligned} \tag{30}$$

where the function $\mathrm{T}\left(\alpha,b,c;\varepsilon_1,\varepsilon_2\right)$ is defined as

$$\mathrm{T}\left(\alpha,b,c;\varepsilon_1,\varepsilon_2\right)=\int\limits_{0}^{\infty}\frac{e^{-\alpha x}}{\left(x+\varepsilon_1\right)^{b}\left(x+\varepsilon_2\right)^{c}}dx. \tag{31}$$

It is straightforward to infer that $\mathrm{T}\left(\alpha,b,c;\varepsilon_1,\varepsilon_2\right)=1/\alpha$ when $b=c=0$. Otherwise, its exact closed-form expression is given for different cases as follows.

- *Case* 1: $\varepsilon_1=\varepsilon_2$.

$$F_{\gamma_{e2e}}(x)=\sum_{a=0}^{K}\sum_{n,m=0}^{a}\sum_{b=0}^{n}\sum_{c=0}^{m}\frac{C_K^aC_a^nC_a^mC_n^bC_m^c(-1)^{a+n+m+b+c}}{(\Lambda_1 I)^{-b}(\Lambda_2 I)^{-c}e^{(n\lambda_1\Lambda_1+m\lambda_2\Lambda_2)I/P}}\frac{e^{-a(\lambda_1+\lambda_2)x/P}}{(x+\Lambda_1 I)^b(x+\Lambda_2 I)^c} \tag{26}$$

$$f_{\gamma_{e2e}}(x)=\sum_{a=0}^{K}\sum_{n,m=0}^{a}\sum_{b=0}^{n}\sum_{c=0}^{m}\frac{C_K^aC_a^nC_a^mC_n^bC_m^c(-1)^{a+n+m+b+c+1}}{(\Lambda_1 I)^{-b}(\Lambda_2 I)^{-c}e^{(n\lambda_1\Lambda_1+m\lambda_2\Lambda_2)I/P}}\frac{\frac{a(\lambda_1+\lambda_2)}{P}\frac{(x+\Lambda_1 I)^{-b}}{(x+\Lambda_2 I)^{c}}+b\frac{(x+\Lambda_2 I)^{-c}}{(x+\Lambda_1 I)^{b+1}}+c\frac{(x+\Lambda_1 I)^{-b}}{(x+\Lambda_2 I)^{c+1}}}{e^{\frac{a(\lambda_1+\lambda_2)x}{P}}} \tag{27}$$

$$\zeta(\beta;\chi)=\sum_{a=0}^{K}\sum_{n,m=0}^{a}\sum_{b=0}^{n}\sum_{c=0}^{m}\frac{\sigma\Psi(\sigma,\beta,b,c;\Lambda_1 I,\Lambda_2 I)+b\Psi(\sigma,\beta,b+1,c;\Lambda_1 I,\Lambda_2 I)+c\Psi(\sigma,\beta,b,c+1;\Lambda_1 I,\Lambda_2 I)}{(-1)^{a+n+m+b+c+1}(C_K^aC_a^nC_a^mC_n^bC_m^c)^{-1}(\Lambda_1 I)^{-b}(\Lambda_2 I)^{-c}e^{(n\lambda_1\Lambda_1+m\lambda_2\Lambda_2)I/P}} \tag{28}$$

For this special case, a closed-form expression for $\mathrm{T}(\alpha,b,c;\varepsilon_1,\varepsilon_2)$ and is given by

$$\mathrm{T}(\alpha,b,c;\varepsilon_1,\varepsilon_2)=\mu(\alpha,b+c;\varepsilon_1) \tag{32}$$

where

$$\begin{aligned}\mu(\alpha,d;\varepsilon)&=\int_0^\infty\frac{e^{-\alpha x}}{(x+\varepsilon)^d}dx=e^{\alpha\varepsilon}\int_\varepsilon^\infty\frac{e^{-\alpha y}}{y^d}dy\\&=\frac{(-1)^d\mathrm{Ei}(-\alpha\varepsilon)}{\alpha^{1-d}e^{-\alpha\varepsilon}\Gamma(d)}+\sum_{w=0}^{d-2}\frac{(-1)^w\alpha^w\varepsilon^{w-d+1}}{\prod_{n=1}^{w+1}(d-n)}.\end{aligned} \tag{33}$$

In deriving (33), the last integral was obtained in closed form with the aid of [39, eq. (358.4)] while $\mathrm{Ei}(x)=-\int_{-x}^{\infty}\frac{e^{-t}}{t}dt$ denotes the exponential integral function [39, eq. (8.211)], which is a built-in function in most mathematical software packages.

- *Case* 2: $\varepsilon_1\neq\varepsilon_2$.

Since b and c are positive integers, either b or c can be zero. Therefore, the following subcases hold:

– *Subcase* A: $b=0$ and $c>0$. It follows straightforwardly that

$$\mathrm{T}(\alpha,b,c;\varepsilon_1,\varepsilon_2)=\int_0^\infty\frac{e^{-\alpha x}}{(x+\varepsilon_2)^c}dx=\mu(\alpha,c;\varepsilon_2). \tag{34}$$

– *Subcase* B: $b>0$ and $c=0$. In this subcase, we have

$$\mathrm{T}(\alpha,b,c;\varepsilon_1,\varepsilon_2)=\int_0^\infty\frac{e^{-\alpha x}}{(x+\varepsilon_1)^b}dx=\mu(\alpha,b;\varepsilon_1). \tag{35}$$

– *Subcase* C: $b>0$ and $c>0$. We firstly apply the partial fractions identity for decomposing the following rational function as

$$\frac{1}{(x+\varepsilon_1)^b(x+\varepsilon_2)^c}=\sum_{d=1}^{b}\frac{\mathcal{A}_d}{(x+\varepsilon_1)^d}+\sum_{g=1}^{c}\frac{\mathcal{B}_g}{(x+\varepsilon_2)^g} \tag{36}$$

where

$$\mathcal{A}_{b-j+1}=\frac{(-1)^{j-1}\prod_{l=0}^{j-2}(c+l)}{(j-1)!(\varepsilon_2-\varepsilon_1)^{c+j-1}},\quad j\in[1,b] \tag{37}$$

$$\mathcal{B}_{c-j+1}=\frac{(-1)^{j-1}\prod_{l=0}^{j-2}(b+l)}{(j-1)!(\varepsilon_1-\varepsilon_2)^{b+j-1}},\quad j\in[1,c]. \tag{38}$$

To this effect, by substituting (36) into (31) one obtains,

$$\mathrm{T}(\alpha,b,c;\varepsilon_1,\varepsilon_2)=\sum_{d=1}^{b}\mathcal{A}_d\mu(\alpha,d;\varepsilon_1)+\sum_{g=1}^{c}\mathcal{B}_g\mu(\alpha,g;\varepsilon_2). \tag{39}$$

By substituting (32) for $\Lambda_1=\Lambda_2$ and (34), (35) or (39) for $\Lambda_1\neq\Lambda_2$ in (30) and then in (28), a closed-form approximate expression for $\zeta(\beta;\chi)$ is obtained. Using this expression in (10) and finally in (9), a closed-form approximate expression for the average BER of $M-$QAM is deduced that will be shown in the next section to be highly accurate for all tested cases. To the best of the author's knowledge, the presented closed-form approximation holding for closely spaced relays has not been reported before in the open technical literature.

IV. NUMERICAL RESULTS

This section is devoted to the validation of the presented analytical results for the BER performance of the considered underlay DF cognitive networks with best relay selection over Rayleigh fading channels. Without loss of generality, two typical modulation schemes are considered, namely, $2-$QAM, also known as Binary Phase Shift Keying (BPSK), for odd h, and $4-$QAM, also known as Quadrature Phase Shift Keying (QPSK), for even h.

A. General Scenario: Arbitrarily Located Relays

This subsection illustrates numerically evaluated results for the analytical expressions presented in Subsections III-A and III-B. Towards this end, we select an arbitrary network topology as shown in Fig. 2. The fading power for the $t\to r$ channel is $\lambda_{t,r}^{-1}=d_{t,r}^{-\alpha}$ according to [43], where α is the path-loss exponent and $d_{t,r}$ is the distance between transmitter t and receiver r. In the sequel, $\alpha=3$ is considered for limiting case-studies. Figure 3 demonstrates the BER performance of under-

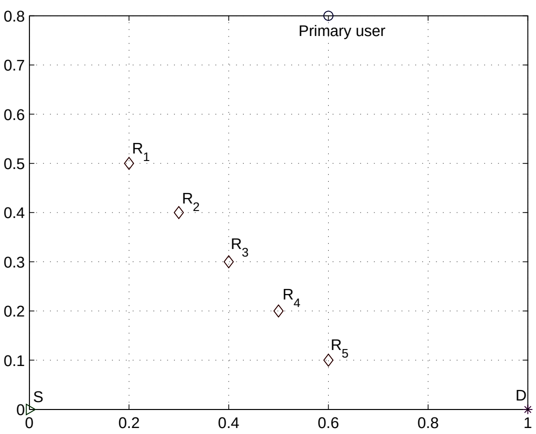

Fig. 2. Network topology for arbitrarily located relays.

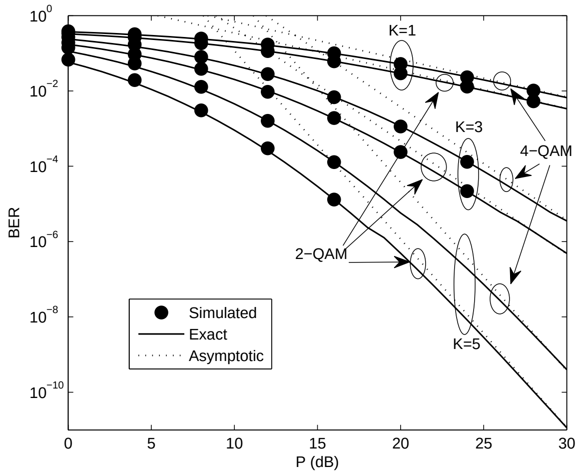

Fig. 3. BER performance versus the maximum transmit power-to-noise variance ratio P for arbitrarily located relays in Fig. 2.

lay DF cognitive networks with best relay selection with respect to the variation of the maximum transmit power-to-noise variance ratio $P = \bar{P}/N_0$ for $I = \tau P$ with $\tau = 0.5$. Different number of relays, $K = \{1, 3, 5\}$, corresponds to various relay sets, $\{R_1\}$, $\{R_1, R_2, R_3\}$, $\{R_1, R_2, R_3, R_4, R_5\}$, respectively. It is observed that the exact analysis in (13) matches perfectly with the Monte Carlo simulation while coinciding the asymptotic analysis in (18) at large values of P, validating the accuracy of the derived expressions. Moreover, the performance is significantly improved as K increases. This comes from the fact that the larger the K, the higher the diversity order achieved by the system and thus, the smaller corresponding BER. Furthermore, the results are rather reasonable in the sense that the system performance is better with lower modulation levels.

B. Special Case: Closely Located Relays

We indicatively consider the special case of closely located relays, as described in Subsection III-C. To this end, we consider the following simulation parameters: $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 6$, $\lambda_4 = 7$, and $I = \bar{I}/N_0 = 20$ dB.

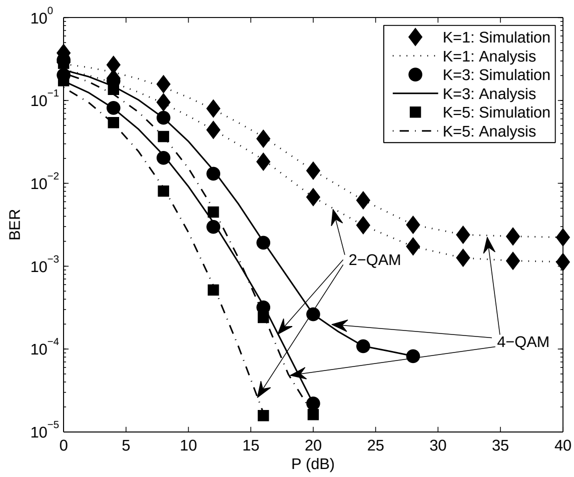

Fig. 4. BER performance versus the maximum transmit power-to-noise variance ratio P for closely located relays.

Figure 4 illustrates the BER behaviour of underlay DF cognitive networks with best relay selection with respect to P for different number of relays K. It is seen that the analytical results are in nearly excellent agreement with the corresponding simulated results. This confirms that even though the proposed expression given by (28) is an approximation, it is particularly tight and accurate. Furthermore, the performance of these networks is significantly improved as P increases. This is quite reasonable since P upper bounds the transmit power of SUs and hence, the larger the P, the larger the transmit power, which ultimately reduces the corresponding BER. Nevertheless, like underlay DF cognitive networks without relay selection (e.g. see [44] and references therein), the BER performance of underlay DF cognitive networks with best relay selection saturates at large values of P. As seen in Fig. 4, the performance saturation phenomenon[4] occurs for $K = \{1, 3\}$. This phenomenon emerges from the fact that the transmit power of the SU is subject to both maximum transmit power and interference power constraints. In other words, its transmit power is constrained by the minimum value of the maximum transmit power P and the maximum interference power I. As a result, for large values of P, the corresponding transmit power is completely determined by I, resulting in unchanged BER levels for any increase of P.

Furthermore, it is observed in Fig. 4 that, as in conventional relaying networks, the number of relays K appears to have a significant impact on the performance of underlay DF cognitive networks with best relay selection. As seen in Fig. 4, increasing K enhances considerably the BER performance, especially at large values of P. Indicatively, for a target BER of 2×10^{-2} and the $2-$QAM modulation, relay selection achieves the SNR gains of about 8 dB and 9.5 dB, compared to scenarios with no relay selection (single-relay case), for $K = 3$ and $K = 5$, respectively. This SNR gain increases at lower BER targets;

[4]The same observation is also expected for $K = 5$. However, for $K = 5$, the performance saturation occurs at very low BERs and hence, it is exhaustive and time consuming to run Monte Carlo simulations at those very low BERs to validate the analytical results. As a result, in Fig. 4 we have obtained BER results till 10^{-5} and as shown the saturation phenomenon can not be observed for K=5.

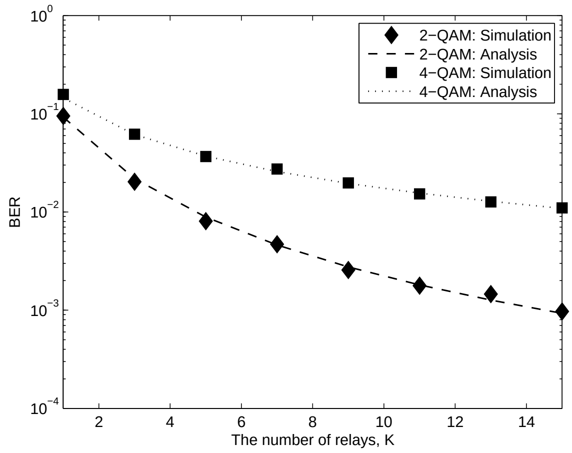

Fig. 5. BER performance versus the number of relays, K.

for example, the SNR gain of relay selection with $K = 5$ over $K = 3$ increases from 1.5 dB to 3.3 dB when the BER target varies from 2×10^{-2} to 3×10^{-4}, respectively. This owes to the fact that the higher the K, the higher the corresponding diversity order. Furthermore, the modulation level drastically impacts the BER performance.

Figure 5 illustrates the BER performance of underlay DF cognitive networks with best relay selection with respect to the number of relays and $P = 8$ dB. It is shown that the analytical and simulated results are in good agreement, which verifies the validity of the proposed expression in (28). Also, the results are reasonable since the BER reduces as modulation level decreases and as the number of relays increases. We define the performance improvement, PG_M, with respect to the increase in the number of relays from K_1 to K_2 for a certain modulation level M as the ratio of the BER corresponding to K_1, $B_e(K_1)$, to the BER corresponding to K_2, $B_e(K_2)$, i.e. $PG_M = B_e(K_1)/B_e(K_2)$. It is shown that performance improvements with respect to the increase in the number of relays is better achievable for lower modulation constellations. For example, $PG_2 = 23.5149$ for $2-$QAM in contrary to $PG_4 = 5.6469$ for $4-$QAM when K increases from 3 to 15.

V. CONCLUSION

This work was devoted to the analysis of the BER performance of underlay DF cognitive networks with best relay selection over Rayleigh fading channels for both the general case of arbitrarily located relays and the special case of closely located relays. For the former case, we present an exact single integral-form BER expression and derived the diversity order and coding gain for best relay selection scenarios while for the latter case, we presented a tight closed-form approximation for the corresponding BER. The algebraic representation of the presented results is relatively convenient to handle both analytically and numerically and it was shown that the BER performance of underlay DF cognitive networks with best relay selection is significantly improved as the number of relays increases.

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