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Comparison of Spatial and Temporal Averaging on Ultrafast Imaging in Presence of Quantization Errors

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Abstract—In compound plane wave imaging (CPWI), multiple plane waves are used to insonify the imaging region with different steering angles. The compounding operation is effectively a spatial averaging filter that reduces the speckles of the image and increases the image contrast and its lateral resolution. Although spatial averaging often improves CPWI image quality, quantization errors which depend on sampling frequency and element spacing (pitch), introduced during beam steering reduce this improvement. In this study, the effect of spatial and temporal averaging on speckle noise reduction, contrast resolution and spatial resolution in ultrafast ultrasound imaging is evaluated. The overall results from the simulations shows that the maximum effect of quantization errors on speckle noise is 0.18 dB, on the image contrast is 0.27 dB, on axial resolution is 2.38% and finally on lateral resolution is 1.44%. On the other hand, plane wave imaging (PWI) employing temporal averaging technique which is not bound with quantization errors relatively produces high contrast to noise ratio (CNR) and speckle signal to noise ratio (SSNR) at 40 MHz for both centre frequency compared to CPWI.

I. INTRODUCTION

The two main modalities in ultrafast imaging are PWI and CPWI. In plane wave imaging, all the transducer elements are excited simultaneously without any focusing or steering delay. The created planar wave fronts will insonify the whole imaging area at once to create an image frame. There is a significant drawback in plane wave imaging which is the drop in image quality. The reason behind this downside in PWI is because of the lack of transmit focusing. Thus, to improve the quality image without sacrificing the high frame rate too much, compound plane wave imaging was introduced.

Image compounding is the process of addition of several image frames into a single composite image in an attempt to reduce speckle noise, hence improving the visual quality for better diagnosis [1] [2]. As for CPWI, multiple plane waves are insonified with different steering angles in sequence, and the received signal is compound coherently or incoherently to produce a high quality image [3] [4]. The time delay, \( \tau_n \) associated to each element for steering the plane wave at an angle \( \beta \) is given by

\[
\tau_n = \frac{dn \sin \beta}{c}
\]

Where \( n \) is number of elements in the transducer, \( d \) is the distance between adjacent elements, also known as pitch and \( c \) is average speed of sound in tissue. Even though the compounding process can improve the image quality, when the plane wave is steered to any value of angle other than zero, quantization starts to appear on the wave front of the emitted field. The quantization effect is tightly related to minimum achievable time step, which is a function of the sampling frequency.

II. QUANTIZATION ERRORS

The ideal steered plane wave should be smooth curves, but in the real world it is unrealisable, mainly due to the limitation of the discrete elements. Delay profiles calculated using (1) for any steering angle will be rounded or quantized to a minimum discrete time interval results in rounding errors. The difference between theoretical delay profiles and quantized is known as phase quantization errors, or just as quantization errors [5]. The quantization errors for each elements, \( \tau_e \) is given by

\[
\tau_e = |\tau_n - \tau_s| \\
\tau_s = |\frac{\|\tau_n f_s\|}{f_s}|
\]

Where \( f_s \) is sampling frequency, \( \tau_n \) is simulation time delay and \( \tau_s \) is quantized or rounded time delay for a certain desired steering angles. A number of papers have discussed the quantization error problem which arises in PAI. Studies conducted by [6] and [7] mentioned that the quantization errors can cause side lobes to start to appear and can reduce the dynamic range of imaging. Meanwhile, [7] and [8] have analysed the effect of quantization errors on image dynamic range and mention the presence of grating lobes at critical angles. In another study conducted by [8], it is suggested that to decrease the quantization error effect, phase lock loops (PLL) can be used to increase the sampling frequency clock and thereby reduce the minimum time delay profile. Even though a significant number of papers have reported the presence and degradation caused by quantization errors, none of them have conducted studies on rounding errors or measured the performance of the final image quality produced. At the same time, all previous quantization error effect studies have been focused on PAI and not on CPWI. Thus this study has been carried out on CPWI image quality by measuring SSNR, CNR and its spatial resolution when the quantization errors are present in spatial or angular compounding.

At the same time, evaluation has been made to PWI where it employ temporal averaging. In temporal averaging technique, a number of plane waves insonified at 0° at the same ROI.

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At the same time, evaluation has been made to PWI where it employ temporal averaging. In temporal averaging technique, a number of plane waves insonified at 0° at the same ROI.
The idea was to obtained independent RF signal without introducing quantization errors on the wave front. The same simulation setups as in Table II have been used in determining the performance of temporal averaging technique.

The errors present in steered plane wave can be classed as correlated or periodic errors since the errors occur periodically across an aperture as shown in Fig. 1. This Correlated errors occurs as a result of the beams being steered with a linear delay profile as described by (1). An 128 element transducer with a steering angle of 0.2, a sampling frequency of 100 MHz and a centre frequency of 5 MHz, was realised in simulation. In Fig. 1, theoretical and quantized delay calculated for said probe is shown. The error present between both delays is also shown. The errors is sinusoidal.

III. SIMULATION AND PERFORMANCE EVALUATION

In order to study the effect of quantization errors on CPWI, two different simulations have been carried out. The first was to determine the steering angle which will cause maximum quantization error. The second was to use the steering angles calculated in first simulation to quantify the image quality on cyst and wires phantoms.

A. Steering Angle for maximum quantization errors

In this simulation, equation (2) and (3) has been used to calculate the time delay differences or quantization errors between theoretical and quantized setup for steering angles from −45 to +45. The simulation setup is shown in Table II. The quantization errors produced for a sampling frequency of 100 MHz and centre frequency of 5 MHz is shown in Fig. 2. It can be seen that the maximum quantization errors occurring just after and before zero quantization errors occurs and continues to appear periodically for every n/fs where n is integer. In order to study the effect of quantization errors efficiently, the first maximum quantization error to appear is ignored since the steering angles is too small and near to the normal. If a small steering angle near to zero is used for compounding, the chances of producing a low quality image are high, as in PWI. Thus in this studies, the second angles which produced maximum quantization errors has been chosen. Meanwhile others cases are shown in Table I.

A single image was formed by compounding three different plane waves steered at angles calculated in Table I. Three phantom cysts with a diameter of 5 mm and three wire phantoms located separately at 30, 40 and 50 mm depth from the transducer face are simulated. A total amount of 50000 scattering has been used in the simulation to achieve equal amount of scattering distribution throughout the ROI. To evaluate the effect of quantization errors, SSNR, CNR and spatial resolution of the images were computed. The SSNR is used to determine the speckle pattern formation on the image. Speckle is a granular texture which corresponds to the constructive and destructive interference of echoes received from scatters [10]. Speckle noise measured with SSNR values on the region of interest (ROI) as given by

\[
\text{SSNR(dB)} = 20\log_{10}\left(\frac{\mu_{\text{ROI}}}{\sigma_{\text{ROI}}}\right)
\]

Where \(\mu_{\text{ROI}}\) is the mean and \(\sigma_{\text{ROI}}\), is the standard deviation of the image amplitude on the ROI. Meanwhile CNR

![Fig. 1. Quantized and Ideal delay profile for steering angles of 0.2.](image1)

![Fig. 2. Total time difference between Quantized and theoretical delay profile for a sampling frequency of 100 MHz and a centre frequency of 5 MHz.](image2)
is used to expresses the detectability of the object contrast between ROI inside the cyst and its background.

\[
\text{CNR(dB)} = 20\log_{10}\left(\frac{\mu_{\text{ROI}} - \mu_{\text{Back}}}{\sqrt{(\sigma_{\text{ROI}}^2 + \sigma_{\text{Back}}^2)}}\right)
\]

To measure the SSNR and CNR of the cysts at different depth on the image produced, 2 different regions with same dimension of 4 mm x 4 mm have been selected for each measurement. The first ROI is inside the cyst while the other ROI is located outside the cyst at the same depth. This is to ensure that the attenuation caused by frequency doesn’t affect the measurements. The SSNR was measured at the highlighted regions R2, R4 and R6 as shown in Fig. 3. While the CNR was measured between regions R1 and R2, R3 and R4, and R5 and R6. Meanwhile, in order to measure the spatial resolution of the wire phantoms, a full width half maximum technique (FWHM), -6 dB has been employed [11].

![Geometry of the cyst (anechoic) with wire phantoms. The highlighted regions were used to measure the performance.](image)

IV. RESULT

Results calculated for CPWI and PWI by using equation (4) and (5) for CNR and SSNR are represented in graphical form in Fig. 4. The CNR values for centre frequency of 5 and 7 MHz with sampling frequency of 40, 100 and 160 MHz are shown in Fig. 4(d), (e) and (f). The significant differences in SSNR values becomes visible at sampling frequency of 40 MHz. Difference in SSNR values between both delays for centre frequency of 5 MHz at 30, 40 and 50 mm are 0.04, 0.08 and 0.06 dB. As for centre frequency of 7 MHz through out the same depth the difference in SSNR for both delays are 0.09, 0.18 and 0.01 dB. There are no significant changes in SSNR values between ideal and quantized delays for sampling frequency 100 and 160 MHz. Although quantization errors inherited in CPWI and not on PWI, but the results for high sampling frequency (100 and 160 MHz) are still in favour of CPWI.

Differences in axial and lateral resolution between ideal and quantized delays in the highlighted region R7, R8 and R9 in Fig.3 for sampling frequency of 40, 100 and 160 MHz and centre frequency of 5 and 7 MHz is presented in Table III. It clearly shows that the percentage difference between the two delays become smaller as the sampling frequency increases from 40 to 160 MHz.

![Differences in axial and lateral resolution between ideal and quantized delays](image)

V. DISCUSSION

The goal of this study was to evaluate the influence of quantization error on compounded image quality. To achieve this, several performance indicators have been employed to measured the results obtained from the simulation. The simulations have been carried out with those parameters as given in Table I and Table II and presented in Fig. 4. The differences for CNR and SSNR values between the two delays for centre frequency of 5 and 7 MHz shows reduction pattern when the sampling frequency increases. It gives an indication that the difference between the two images obtained from theoretical and quantized delay becomes less or the correlation between them becoming higher. According to [1], speckles in ultrasound images only can be reduced by combining partially correlated or non-correlated images of the same ROI, produced by transducers with different spatial or angular locations. Thus if the correlation calculated between the images produced by the two delays shows very high (≈ 1) correlation then it means that there were less or no changes in speckle formation between them. The high correlation values also give an indicate that the quantization errors do not influence the steering angles between the two delays. Fig. 5 shows the correlation calculated for all CPWI cases which have been simulated.
shows very high correlation values for sampling frequencies of 100 and 160 MHz compared to 40 MHz.

Generally, centre frequency of 7 MHz shows significantly less deviation in SSNR and CNR when compared to 5 MHz due to quantization errors. This is due to the reason that pitch has been set to λ. The λ value increases when the centre frequency decreases according to $\lambda = c/f_0$. Changes in axial and lateral resolution very dependence on formation of speckle patterns. Since both delays does not differ much, there are only small changes in spatial resolution and the different become smaller as the sampling frequency increases.

VI. Conclusion

The quantization errors directly proportional to minimum achievable time step which is a function of the sampling frequency. Thus, increasing the sampling frequency will eventually reduce the quantization errors. It is also proven that temporal averaging which is not bound with quantization errors relatively produces high CNR and SSNR at low sampling frequency(40 MHz) for both centre frequency compared to CPWI. The overall results shows that the quantization errors does not have a significant effect on the final image quality of CPWI for the angular range of $-10.02^\circ$ to $+10.02^\circ$.

REFERENCES