Nonlinear Current-Limiting Control for Grid-tied Inverters

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Abstract—A current-limiting controller with nonlinear dynamics is proposed in this paper for single-phase grid-tied inverters. The inverter is connected to the grid through an LCL filter and it is proven that the proposed controller can achieve accurate real and reactive power regulation. By suitably selecting the controller parameters, it is shown by using the nonlinear input-to-state stability theory that the inverter current remains below a given value at all times. This is achieved without external limiters, additional switches or monitoring devices and the controller remains a continuous-time system guaranteeing the boundedness of the system states. Guidelines for selecting the controller parameters are also given to provide a complete controller design procedure. Simulation results of a single-phase grid-tied inverter are presented to verify the desired power regulation of the proposed controller and its current-limiting capability.

I. INTRODUCTION

The large-scale integration of renewable energy sources to the power network during the last decades has started to affect the stable operation of the grid. This has created essential requirements for the grid-connected units to maintain the stability of the network. Since an inverter is usually required to integrate the renewable energy sources to the grid, several controllers have been proposed in the literature for grid-tied inverters to achieve accurate real and reactive power regulation [1], [2], [3]. Although in most of the times, unity power factor is obtained, modern power networks require a flexibility in controlling the reactive power of the inverter as well [1], [2], [4], [5].

Nevertheless, the stability properties of most of the existing controllers for grid-tied inverters have not been adequately exploited. The main reason is the increased complexity of the closed-loop system due to the nonlinear dynamics resulting from the calculation of the real and the reactive power. Most of the existing approaches use small-signal modeling of the system and linearization methods [6], [7], [8], while recently, nonlinear analysis has been conducted in order to strengthen the stability theory [9], [10], [11]. However, several assumptions are usually considered, such as a purely inductive network or constant load and line impedances, while the inner voltage and current control loops are often neglected [12].

Since in most of the cases, the grid is assumed to be stiff, i.e., with relatively constant voltage and frequency, the stability of the grid-tied inverter is directly related to the injected current, which should remain below a given value at all times. Although external limiters and saturation units can be added into the traditional control approaches to achieve the current-limiting property, these can lead to undesired oscillations and instability, due to the lack of a rigorous stability proof [13], [14]. Advanced nonlinear controllers, such as passivity-based or feedback linearization methods, can guarantee the asymptotic performance and the current-limitation, but their dependence on the system parameters and their complicated structure make them difficult to be implemented in practice [9], [15], [16], [17], [18], [19]. As a result, the proof of stability for grid-tied inverters operating with an inherent current-limiting property and independently from the system parameters, considering the nonlinear dynamic model of the system, represents a challenging task and is investigated in the current paper.

A nonlinear control strategy for single-phase grid-tied inverters is proposed in this paper to guarantee closed-loop system stability in the sense of boundedness and a given limit for the inverter current using the nonlinear dynamic model description. The inverter is assumed to be connected to the grid through an LCL filter, where the grid is assumed stiff or at least with bounded voltage and frequency close to their rated values. It is shown that the proposed controller can guarantee accurate real and reactive power regulation to the reference values. Particularly, a dynamic virtual resistance, which changes according to a nonlinear expression, and a phase shifting are designed and implemented based on the plant dynamics. Using nonlinear input-to-state stability theory, it is analytically proven that using a suitable controller parameter selection, a given limit for the inverter current can be guaranteed at all times, independently from the reference values of the real and the reactive power. This also leads to the proof of stability in the sense of boundedness for the inverter current, the grid current and the output capacitor voltage. In this way, the main tasks are achieved with an additional mathematical analysis that limits the inverter current below a given value, thus protecting the inverter and the filter at all times. The significant difference between the proposed controller and the existing virtual impedance methods is that the dynamic form of the controller is embedded into the virtual resistance and no additional voltage signals are added in the control design for the real power regulation that further complicate the closed-loop system analysis. Additionally, no external limiters or monitoring systems are required for limiting the inverter current, with the current-limiting being...
an inherent property of the proposed controller, as it is proven for the nonlinear closed-loop system. To complete the design procedure, a guidance for selecting the controller parameters is also presented. Extensive simulation results are provided to verify the current-limiting property of the proposed controller as well as its performance for several changes of the reference values.

The rest of the paper is organized as follows. In Section II, the dynamic model of the grid-tied inverter is presented and the main problem addressed in the present paper is formulated. In Section III, the nonlinear current-limiting controller is proposed and analyzed. The current-limiting property is proven and a framework for selecting the controller parameters is also presented. In Section IV, simulation results are provided for a grid-tied inverter under the proposed controller, while in Section V, some conclusions are drawn.

II. DYNAMIC MODELING AND PROBLEM FORMULATION

The system under consideration is a single-phase inverter connected to the grid via an LCL filter as shown in Fig. 1. The LCL filter consists of the inductances \( L \) and \( L_g \) with small parasitic resistances in series \( r \) and \( r_g \), respectively, and a capacitor \( C \) with a large parasitic resistance \( R_c \) in parallel. The inverter output voltage and current are denoted as \( v \) and \( i \), respectively, \( v_c \) is the grid voltage and \( v_g \) and \( i_g \) are the grid voltage and current, respectively. Here, the grid is considered stiff and as a result \( v_g = \sqrt{2}V_g \sin \omega_g t \), where \( V_g \) is the root-mean-square (RMS) grid voltage and \( \omega_g \) is the grid angular frequency, although these can vary slightly from their rated values.

The dynamic model of the system is given by the following equations:

\[
\begin{align*}
L \frac{di}{dt} & = -ri + v - v_c \\
C \frac{dv_c}{dt} & = i - v_c - i_g \\
L_g \frac{di_g}{dt} & = v_c - r_g i_g - v_g,
\end{align*}
\]

which is obviously linear with state vector \( x = [i \ v_c \ i_g]^T \) and control input the inverter voltage \( v \), while \( v_g \) represents an uncontrolled external input.

For grid-tied inverters, the main task it to design a controller that achieves accurate real and reactive power regulation to some reference values \( P_{set} \) and \( Q_{set} \), respectively. The measured real and the reactive powers \( P \) and \( Q \) are usually obtained at the capacitor node as the average values of the instantaneous power expressions over a period \( T \), which for a single-phase inverter become:

\[
\begin{align*}
P & = \frac{1}{T} \int_{t}^{t+T} v_c(\tau)i(\tau)d\tau, \\
Q & = \frac{1}{T} \int_{t}^{t+T} v_c(\tau)i(\tau)d\tau,
\end{align*}
\]

where \( v_{eq} \) is the capacitor voltage delayed by \( \frac{\pi}{2} \) rad. It is obvious that the power expressions are nonlinear due to the multiplication of the system states, resulting in a nonlinear closed-loop system that is difficult to analyze in terms of stability. This is the main reason why most of the existing methods investigate the linearized model (small-signal) [6], [7], [8]. Therefore, for a solid theory, stability analysis should be conducted on the nonlinear system. The most challenging issue in grid-tied inverter stability is the limitation of the injected current below a given value. This is crucial for the stable and reliable operation of the system, since it should be proven at all times, i.e., during transients or changes of the system parameters, to avoid damage of the inverter and further instabilities at the power network.

The purpose of the proposed paper is to develop a nonlinear control scheme that acts independently from the system parameters, achieves the desired power regulation and guarantees a given limit for the inverter current based on the nonlinear dynamic model of the closed-loop system.

III. THE PROPOSED CURRENT-LIMITING CONTROLLER

A. Characteristics of the controller

In order to achieve the required performance with an inherent current-limiting function, the following controller is proposed

\[
v = v_c + (1 - w_q)(v_g \cos \delta + v_{eq} \sin \delta - w_i), \quad (4)
\]

where \( v_{eq} = \sqrt{2}V_g \cos \omega_g t \) and the variables \( w, w_q \) and \( \delta \) represent the controller states with dynamics:

\[
\begin{align*}
w & = -c_w (P_{set} - P) w_q^2, \\
w_q & = c_w (w - w_m) w_q (P_{set} - P) - k_w \left( \frac{(w-w_m)^2}{\Delta w_m^2} + w_q^2 - 1 \right) w_q, \\
\delta & = -c_\delta (Q_{set} - Q), \quad (7)
\end{align*}
\]

with \( c_w, c_\delta, w_m, \Delta w_m \) and \( k_w \) being positive constants. The initial conditions of \( w, w_q \) and \( \delta \) are defined as \( w_0 = w_m, w_q(0) = 1 \) and \( \delta_0 = 0 \), respectively. Note that \( v_{eq} \) can be obtained using a traditional PLL. The PLL dynamics are assumed much faster than the plant and the controller dynamics, which is a common assumption in the analysis of power converters [7], [12].

For system (5)-(6), by considering the Lyapunov function candidate

\[
W = \frac{(w-w_m)^2}{\Delta w_m^2} + w_q^2, \quad (8)
\]
its time derivative yields

$$\dot{W} = -2k_w \left( \frac{(w - w_m)^2}{\Delta w^2_m} + w^2_q - 1 \right) w^2_q.$$  \hspace{1cm} (9)

According to the initial conditions $w_0$ and $w_q0$, it results in

$$W = 0, \forall t \geq 0,$$

which means that

$$W(t) = W(0) = 1, \forall t \geq 0.$$  

This implies that both $w$ and $w_q$ start and stay on the ellipse

$$W_0 = \left\{ w, w_q \in R : \frac{(w - w_m)^2}{\Delta w^2_m} + w^2_q - 1 = 0 \right\},$$

at all times as shown in Fig. 2. By choosing $w_m > \Delta w_m > 0$, the ellipse is defined in the right-half plane resulting that $w \in [w_{min}, w_{max}] = [w_m - \Delta w_m, w_m + \Delta w_m] > 0$ for all $t \geq 0$. Now, using the transformation

$$w = w_m + \Delta w_m \sin \phi,$$

$$w_q = \cos \phi,$$

it gives

$$\dot{\phi} = -c_w (P_{set} - P) w_q,$$  \hspace{1cm} (10)

which means that the states $w$ and $w_q$ will travel on the ellipse $W_0$ with angular velocity $\dot{\phi}$. This indicates that when $P = P_{set}$, then $\dot{\phi} = 0$ and both $w$ and $w_q$ can converge to some constant values $w_c$ and $w_{qc}$, respectively, corresponding to the desired equilibrium point.

It should be underlined that by starting from point $(w_m, 1)$ on the $w - w_q$ plane, the controller states $w$ and $w_q$ will be restricted only on the upper semi-ellipse of $W_0$. This is due to the fact that the angular velocity $\dot{\phi}$ depends on $w_q$ from (10) and if the states try to reach the horizontal axis, then $w_q \rightarrow 0$ and $\dot{\phi} \rightarrow 0$ independently from the difference $P_{set} - P$. This will make the controller states slow down and remain on the upper semi-ellipse of $W_0$, avoiding a limit cycle behavior resulting from the controller dynamics, which would lead to a continuous oscillation around $W_0$. Therefore, it is reasonable to state that $w_q \in [0, 1]$ for all $t \geq 0$.

Additionally, the proposed controller introduces an integral structure in (7) to achieve the desired reactive power regulation. Particularly, $\delta$ corresponds to the desired phase shifting at the inverter voltage as it is better explained in the analysis that follows.

### B. Stability of the closed-loop system

Since $v_g = \sqrt{2} V_g \sin \omega_g t$, then taking into account the trigonometric identities the proposed controller (4) becomes

$$v = v_c + (1 - w_q)(\sqrt{2} V_g \sin(\omega_g t + \delta) - wi),$$  \hspace{1cm} (11)

which shows that the controller state $\delta$ introduces a necessary phase shifting to the inverter voltage. By applying the proposed controller (11) to the original plant dynamics (1), the inverter current equation results in

$$L \frac{di}{dt} = -(r + (1 - w_q)w) i + (1 - w_q)\sqrt{2} V_g \sin(\omega_g t + \delta).$$  \hspace{1cm} (12)

From the previous controller analysis, it holds true that $w \in [w_{min}, w_{max}] > 0$ and $w_q \in [0, 1]$ for all $t \geq 0$. Dynamic equation (12) dictates that the proposed controller introduces a dynamic virtual resistance at the output of the inverter given by the term $(1 - w_q)w$, which changes according to the nonlinear expressions (5)-(6).

For system (12), consider the Lyapunov function candidate

$$V = \frac{1}{2} Li^2.$$  \hspace{1cm} (13)

Its time derivative results in

$$\dot{V} = -(r + (1 - w_q)w) i^2 + (1 - w_q)\sqrt{2} V_g \sin(\omega_g t + \delta)$$

$$\leq -(r + (1 - w_q)w_{min}) i^2 + (1 - w_q)\sqrt{2} V_g |i||\sin(\omega_g t + \delta)|.$$  

This shows that $\dot{V} < 0$ when $|i| > \frac{(1 - w_q)\sqrt{2} V_g |\sin(\omega_u t + \delta)|}{r + (1 - w_q)w_{min}}$, proving that (12) is input-to-state stable (ISS) [20]. Since $(1 - w_q)|\sqrt{2} V_g |\sin(\omega_g t + \delta)|$ is bounded, then the inverter current $i$ is bounded for all $t \geq 0$. According to the ISS property, it holds true that

$$|i| \leq \frac{(1 - w_q)\sqrt{2} V_g}{r + (1 - w_q)w_{min}}, \forall t \geq 0,$$

if initially $i(0)$ satisfies the previous inequality. By choosing

$$w_{min} = \frac{V_g}{I_{max}}$$  \hspace{1cm} (14)

then

$$|i| \leq \frac{(1 - w_q)\sqrt{2} V_g}{r \frac{I_{max}}{V_g} + (1 - w_q)\sqrt{2} I_{max}}$$

since $(1 - w_q) > 0$ and $r \frac{I_{max}}{V_g} > 0$. The previous inequality holds for any $t \geq 0$ and for any constant positive $I_{max}$, and as a result

$$I < I_{max}, \forall t \geq 0,$$

where $I$ is the RMS value of the inverter current, proving that the proposed controller introduces an inherent current-limiting property independently from required power regulation, the nonlinear expressions of $P, Q$ and the dynamics of $\delta$. This is a crucial property since the inverter is protected at all times by limiting the output current, even during transients or if a large reference value $P_{set}$ is applied.

In order to investigate the stability in the sense of boundedness of the rest of the plant states, the dynamics of the
which can be seen as a linear time-invariant system of the form
\[ \dot{x} = Ax + u \]
with state \( x = [v_c \ i_g]^T \) and input \( u = [i_c - \frac{v_g}{L_g}]^T \). By choosing
\[ P = \begin{bmatrix} C & 0 \\ 0 & L_g \end{bmatrix} > 0 \]
it is proven that
\[ PA + AT \cdot P = \begin{bmatrix} -\frac{2}{R_c} & 0 \\ 0 & -2r_g \end{bmatrix} < 0 \]
which proves that \( A \) is Hurwitz and (15) is a bounded-input bounded-state system. Since \( v_g = \sqrt{2V_g \sin \omega_g t} \) is bounded and \( i \) is bounded from the ISS and the current-limiting properties, then both the capacitor voltage \( v_c \) and the grid current \( i_g \) are proven to remain bounded at all times.

C. Parameters design
As explained in the previous subsection, the term \((1 - \frac{1}{w_q})w\) represents a dynamic virtual resistance at the output of the inverter. Since the ISS analysis and the current-limiting property dictate that \( w_{\text{min}} \) is selected from (14) corresponding to maximum current \( I_{\text{max}} \), then \( w_{\text{max}} \) will correspond to a minimum current \( I_{\text{min}} \) as
\[ w_{\text{max}} = \frac{V_g}{I_{\text{min}}} \].

Note that even when the inverter is not connected to the grid, a small current flows through the LC filter and then \( I_{\text{min}} \) can be chosen as relatively small corresponding to this small current. This will lead to the calculation of \( w_m \) and \( \Delta w_m \) as
\[ w_m = \frac{V_g}{2} \left( \frac{1}{I_{\text{min}}} + \frac{1}{I_{\text{max}}} \right) \]
\[ \Delta w_m = \frac{V_g}{2} \left( \frac{1}{I_{\text{min}}} - \frac{1}{I_{\text{max}}} \right) \],
from the definition of the ellipse \( W_0 \).

Parameter \( k_w \) is an arbitrary positive constant since it is multiplied with the term \( \frac{(w-w_m)^2}{\Delta w_m^2} + w_q^2 - 1 \) in (6), which is zero on the ellipse \( W_0 \). In fact, the role of the \( k_w \) is to increase the robustness of the \( w_q \) dynamics in an actual implementation due to calculation errors or external disturbances.

Parameters \( c_w \) and \( c_\delta \) affect the dynamic performance of the controller. Particularly, \( c_w \) is found in the angular velocity (10) of the controller states \( w \) and \( w_q \). Since \( w \) and \( w_q \) start from point \((w_m,1)\), travel on the ellipse \( W_0 \) and they can reach the point \((w_{\text{min}},0)\) at the limit of the current after a settling time \( t_s \), then consider a worst case scenario where the controller states travel on the arc of \( W_0 \) with central angle \( \frac{\pi}{7} \) rad and with a maximum angular velocity \( \frac{\pi}{2} \) rad/s. This is a worst case scenario because the angular velocity will decrease as soon as \( P \) approaches \( P_{\text{act}} \) according to (10). In this framework, starting from zero real power \( P \) and setting the maximum real power \( P_{\text{act}} = S_n \), where \( S_n \) is the rated power of the inverter, then by taking into account that \( w_q \leq 1 \) \((w_q = 1 \) at the worst case\), it yields from (10) that
\[ \frac{\pi}{2t_s} = \frac{c_w S_n}{\Delta w_m} \]
which gives
\[ c_w = \frac{\pi \Delta w_m}{2t_s S_n} \].

Similarly, parameter \( c_\delta \) affects the dynamics of \( \delta \) from (7). As proven from (11), the angle \( \delta \) is required for the shifting of the phase of the inverter voltage to achieve the required reactive power regulation. By neglecting the small phase shifting caused by the inductor \( L \) and assuming a worst case scenario where \( Q \) starts from zero and reaches \( S_n \) after a settling time \( t_s \), then this will correspond to a change of \( \Delta \delta = \frac{\pi}{2} \). This operation can approximately give \( \delta \approx \frac{\Delta \delta}{t_s} \), and then from (7) it is
\[ \frac{\pi}{2t_s} = c_\delta S_n \]
which gives
\[ c_\delta = \frac{\pi S_n}{2t_s} \].

It should be noted that both (19) and (20) represent a guidance for calculating \( c_w \) and \( c_\delta \), respectively, and have been obtained for a worst case scenario. In practice larger values can be chosen or equivalently smaller \( t_s \) can be used. This means that the values can be increased until a satisfactory response is achieved.

IV. Simulation results
For the verification of the proposed controller, a grid-tied single-phase inverter with an \( LCL \) filter is simulated using SimPower systems toolbox of Matlab/Simulink. The system and controller parameters are shown in Table I. A switching frequency of 15 kHz is used for the pulse-width-modulation of the inverter and the sinusoidal tracking algorithm PLL is applied to obtain the required \( v_{gq} \) for the controller design [1]. The controller parameters \( c_w \) and \( c_\delta \) are directly calculated from (19) and (20), respectively.
The system response is shown in Fig. 3. Initially the inverter is not connected to the grid and the inverter voltage \( v \) is set to \( v_g \) with a small negative reactive power being present due to the capacitance of the LCL filter (Fig. 3(a)). At the time instant \( t = 0.1 \) s, the inverter is connected to the grid and the controller is enabled with \( P_{set} = 50 \) W and \( Q_{set} = 0 \) Var resulting in a fast regulation of the real and the reactive power, as observed in Fig. 3(a). The time responses of the RMS values of the inverter current and the capacitor voltage are shown in Fig. 3(b) and (c), respectively. The controller state \( \delta \) is shown in Fig. 3(d) which is regulated at a small positive constant in order to apply the necessary shifting for the inverter voltage angle to cancel the effect of the \( L \) inductor and achieve the required unity power factor. At \( t = 0.4 \) s, the reference real power becomes \( P_{set} = 200 \) W and the inverter is regulated at the desired value after a short transient. At \( t = 0.6 \) s, the reference power increases to \( P_{set} = 600 \) W that violates the technical limits of the inverter in order to verify the current-limiting property of the inverter. As it is illustrated in Fig. 3(a), the real power is regulated to a lower value because as it can be seen from 3(b), the inverter current tries to violate its maximum value. Therefore, the current-limiting property of the inverter is clearly depicted and the unity power factor is maintained, as it is also clear from the steady-state response of the capacitor voltage and the inverter current in Fig. 3(e). At \( t = 0.8 \) s, \( P_{set} \) changes back to \( 200 \) W and the system returns to its previous values. To verify the ability of the inverter to regulate the reactive power, at \( t = 1.2 \) s the reactive power reference is set to \( Q_{set} = 100 \) Var and the reactive power is quickly regulated to its desired value. This is also shown from the steady-state response in Fig. 3(f). All of these suitably prove the capability of the proposed controller achieve the main tasks with an inherent current limitation. Finally, the analysis presented in Section III is verified in Fig. 4, where the controller states \( w \) and \( w_g \) converge to the required steady-state values in Fig. 4(a) and 4(b), respectively, and it is observed that they operate exclusively on the desired upper semi-ellipse of \( W_0 \) (Fig. 4(c)).

V. CONCLUSIONS

A nonlinear controller with a current-limiting property was proposed for single-phase grid-tied inverters with an LCL filter. The proposed controller can achieve the desired real and reactive power regulation with a guaranteed closed-loop stability in the sense of boundedness. Based on the nonlinear dynamics of the system and using input-to-state stability theory, a given limit for the inverter current is always proven independently from the power reference values. A guidance for selecting all the controller parameters was also presented to obtain the complete controller implementation procedure. The desired performance of the proposed current-limiting controller and the theoretical analysis were verified through extensive simulations.
REFERENCES


